INTRODUCTORY COLLEGE MATHEMATICS

HACKWORTH
and
HOWLAND

SAUNDERS SERIES IN MODULAR MATHEMATICS

Geometric Measures

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PREFACE

Geometric Measures

This book is one of the sixteen content modules in the Saunders Series in Modular Mathematics. The modules can be divided into three levels, the first of which requires only a working knowledge of arithmetic. The second level needs some elementary skills of algebra and the third level, knowledge comparable to the first two levels. *Geometric Measures* is in level 2. The groupings according to difficulty are shown below.

Level 1	Level 2	Level 3
Tables and Graphs	*Numeration*	*Real Number System*
Consumer Mathematics	*Metric Measure*	*History of Real Numbers*
Algebra 1	*Probability*	*Indirect Measurement*
Sets and Logic	*Statistics*	*Algebra 2*
Geometry	*Geometric Measures*	*Computers*
		Linear Programming

The modules have been class tested in a variety of situations: large and small discussion groups, lecture classes, and in individualized study programs. The emphasis of all modules is upon ideas and concepts.

Geometric Measures emphasizes skills in the measurement and computation of lengths, areas, and volume. The module is appropriate for education and liberal arts students. It is also well suited for math-science and technical students. In any case, *Geometric Measures* is appropriate for the freshman or sophomore student.

The module begins by discussing units of measure and the inexactness of every measure. Then *Geometric Measure* develops the concept of maximum and minimum error of measurement. After discussing the concept of perimeter paths and the principle of conservation, the module proceeds to the topics of perimeter, area, and volume of geometric figures.

In preparing each module, we have been greatly aided by the valuable suggestions of the following excellent reviewers: William Andrews, Triton College, Ken Goldstein, Miami-Dade Community College, Don Hostetler, Mesa Community College, Karl Klee, Queensboro Community College, Pamela Matthews, Chabot College, Robert Nowlan, Southern Connecticut State College, Ken Seydel, Skyline College, Ara Sullenberger, Tarrant County Junior College, and Ruth Wing, Palm Beach Junior College. We thank them, and the staff at W. B. Saunders Company for their support.

Robert D. Hackworth
Joseph W. Howland

NOTE TO THE STUDENT

OBJECTIVES:

Upon completing this unit the reader is expected to be able to demonstrate the following skills and concepts:

1. an understanding of the approximation involved in every geometric measurement

2. an understanding of the effects of approximate measures on computations of perimeters, areas, and volumes.

3. an ability to find perimeters of polygons and circles

4. an ability to find areas of polygons and circles

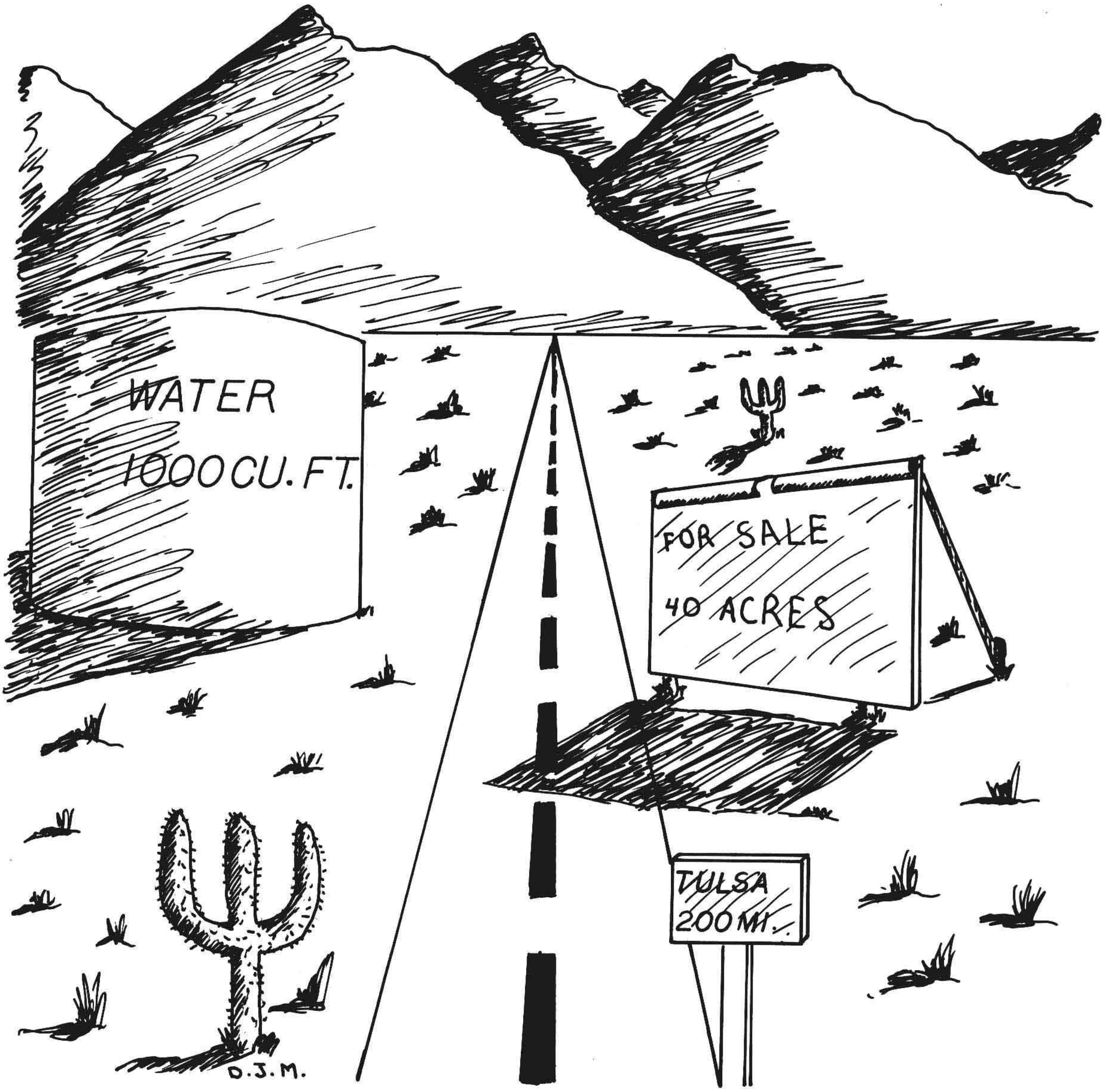
5. an ability to find volumes of rectangular solids and cylinders.

Three types of problem sets, with answers, are included in this module. Progress Tests appear at the end of each section. These Progress Tests are always short with three to six problems. The questions asked in Progress Tests always come directly from the material of the section immediately preceding the test.

Exercise Sets appear less frequently in the module. More problems appear in an Exercise Set than in a Progress Test. These problems arise from all sections of the module preceding the Exercise Set. Section I problems in each Exercise Set were chosen to match the objectives of the module. Section II problems of each Exercise Set are challenge problems.

A Self-Test is found at the end of the module. The Self-Test contains problems representative of the entire module.

In learning the material, the student is encouraged to try each problem set as it is encountered, check all answers, and restudy those sections where difficulties are discovered. This procedure is guaranteed to be both efficient and effective.


WATER
1000 CU. FT.
FOR SALE
40 ACRES
TULSA
200 MI.
D.J.M.

CONTENTS

GEOMETRIC MEASURES

LINEAR MEASURES

The most common and simplest type of geometric measure is called a linear measure. Direct linear measurements are often made with devices such as rulers, tape measures, and meter sticks. Each of the line segments shown in Figure 1 could be measured by a ruler or meter stick.

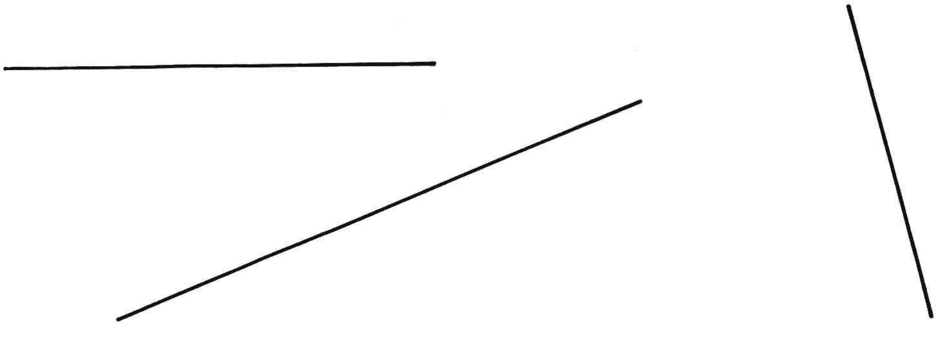


Figure 1

Each of the preceding figures is a straight line segment, but linear measures are not limited by the "straightness" of the geometric figures. The length of each of the lines in Figure 2 can also be described by a linear measure.

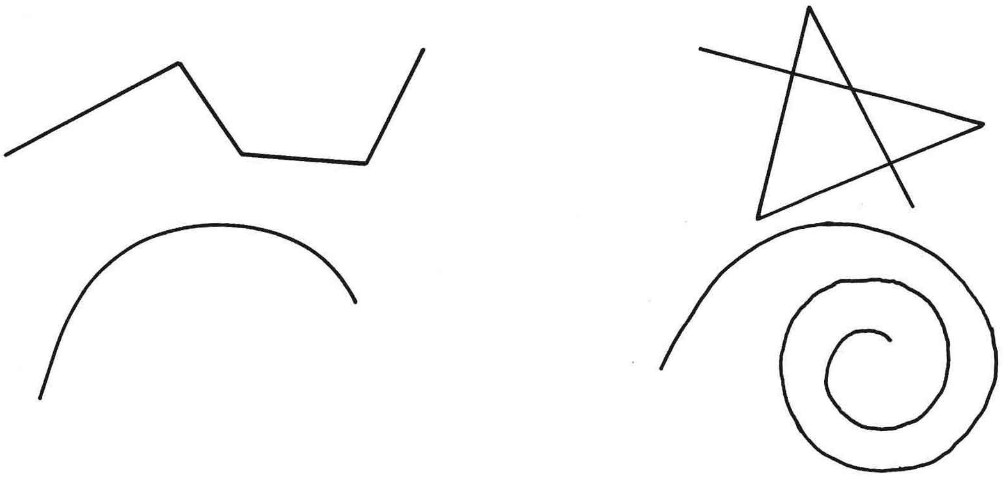


Figure 2

Measuring the "paths" or "curves" of Figure 2 may pose difficulties for those who believe linear measurements must be applied only to straight lines, but the concept of linear measures is more concerned with length than it is with a consistency of direction.

The central idea of linear measure is length or distance. It is best learned by actually measuring the lengths of some line segments. Below is a line segment with a ruler shown directly beneath it.

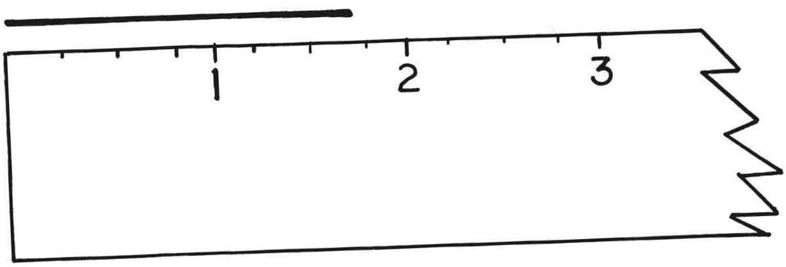


Figure 3

Figure 3 shows a ruler with its 1 and 2 inch marks clearly labeled. There are, however, other marks on the ruler in addition to the inch marks. These other marks on the ruler are very important because they, not the inch marks, indicate the unit of measurement for this particular ruler. To determine the unit of measure, count the number of equal spaces between the 1-inch and 2-inch marks. Since there are four spaces the unit of measure is $\frac{1}{4}$ inches or quarter/inches. The line segment of Figure 3 ends nearest to the third mark between 1-inch and 2-inch. The line segment's "correct" measurement in Figure 3 is $1\frac{3}{4}$ inches.

In Figure 4 is a line segment with the same length as that in Figure 3. This time the ruler is marked differently. That is, the ruler has a different unit of measure.

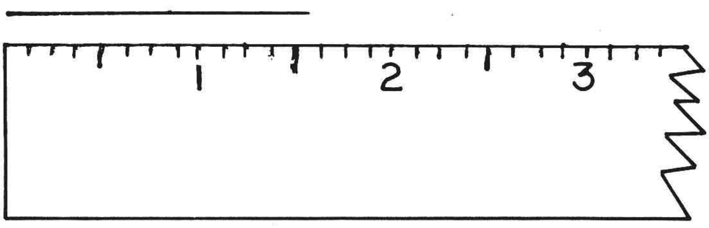


Figure 4

What is the unit of measure for the ruler in Figure 4? What is the correct measurement for the line segment in Figure 4? The correct answer for the first question is $\frac{1}{8}$ inch because there are eight spaces between the 1-inch and 2-inch marks. The "correct" measurement is $1\frac{5}{8}$ inches because the nearest (closest) mark to the end of the line segment is the fifth mark between the 1-inch and 2-inch labels.

The reader may be puzzled by the fact that the line segment of Figure 3 was "correctly" measured as $1\frac{3}{4}$ inches, but the same segment measured in Figure 4 had a "correct" measurement of $1\frac{5}{8}$ inches. This discrepancy illustrates a fact about measurements that is rarely understood. The "correctness" of a linear measure is relative to the use of a common unit of measure. Two measures of the same length using the same unit of measure should be equal, but two measures of the same length using two different units of measure will be different. Unequal measures for the same length?? That is true. All measurements are approximations, their accuracy determined by their units of measure. Therefore, measurements using different units of measure will provide different, unequal, approximations.

The measurement of a line segment with a ruler marked in half-inches may be $7\frac{1}{2}$ inches, but that is an approximation. The same line segment, if measured with a ruler marked off in tenths of an inch, may be 7.4 inches long. Both measurements may be correct with respect to the rulers used. Both measures are approximations because the accuracy is limited by the sizes of the units of measure.

In Figure 5 is shown a portion of a meter stick and a ruler using $\frac{1}{16}$ inch as its unit of measure. Use the ruler and try to find the exact measure of 6 centimeters.

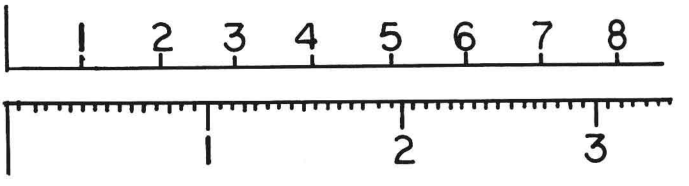


Figure 5

$2\frac{5}{16}$ or $2\frac{6}{16}$ inches are answers that might be read from Figure 5 for a length of 6 centimeters. These answers are approximations. By conversion tables, correct 4 decimal places, for changing centimeters to inches, 6 centimeters is approximately 2.3622 inches. The reader using the ruler of Figure 5 will arrive at an approximation for 6 cm that is both different from 2.3622 inches and less accurate.

To sum up, or review, the measuring concepts previously discussed, the learner should now know: (1) all measurements are approximations, (2) the degree of accuracy of a measurement is dependent upon the size of the unit of measure.

Exact numbers are never the result of measurements. Counting objects is a method of generating exact answers, but measures are always approximations.

Progress Test 1

1. The number 100 is an approximation for which of the following:
   a. There are 100 members in the United States Senate.
   b. The boys raced 100 yards.

True or false?

2. Two people using differently marked rulers may correctly measure the same line segment and arrive at different answers.

3. Two people using the same ruler may correctly measure the same line segment and arrive at different answers.

4. There is an error involved in every measurement.

MAXIMUM ERROR OF MEASUREMENT

The next measuring concept to be presented is the maximum error in a measurement. Using a ruler marked off in $\frac{1}{8}$ inch units, the ruler measures to the nearest $\frac{1}{8}$ inch. Using a ruler marked off in units of one centimeter, the ruler measures to the nearest one centimeter. In both cases, as well as all measuring situations, the measurement will be an approximation. However, the maximum degree of error in the measurement can always be established from the unit of measurement. Since measurements are made to the nearest unit, the maximum error in a correctly made measurement is plus or minus one-half the unit of measure. For example, when using $\frac{1}{2}$ inch as a unit, the maximum error is $\frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{4}$ inch. The segments in Figure 6 should all be correctly labeled 2 inches long, even though the longest may actually be $2\frac{1}{4}$ inches long and the shortest $1\frac{3}{4}$ inches long, the three segments in Figure 6 are all 2 inches long using the ruler shown. Notice that the segments can vary as much as $\frac{1}{4}$ inch on either side of the 2-inch mark and still be called 2 inches long.

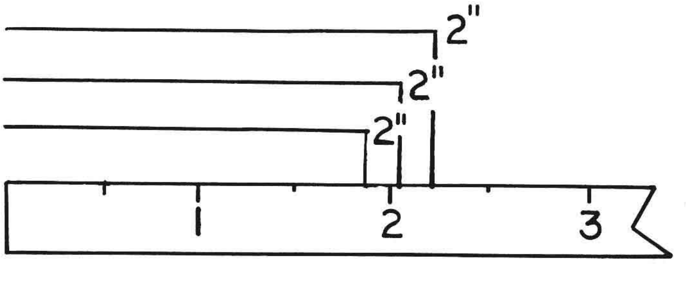


Figure 6

When $\frac{1}{2}$ inch is the unit the segments will not be correctly labeled 2 inches long if they vary more than $\frac{1}{2}$ of $\frac{1}{2}$ or $\frac{1}{4}$ inch from the 2 inch mark.

Using $\frac{1}{8}$ inch as the unit of measure, the maximum error in a measurement is one-half of $\frac{1}{8}$ inch $(\frac{1}{2} \cdot \frac{1}{8})$ or $\frac{1}{16}$ inch. Using one centimeter as the unit of measure, the maximum error in a measurement is one-half of one centimeter $(\frac{1}{2} \cdot \frac{1}{1})$ or $\frac{1}{2}$ cm (5 mm).

The maximum error in a measurement, sometimes called the degree of tolerance, is extremely important to any act of measurement. An amateur gardener planning to plant a row of corn may be willing to accept a measuring error of five feet in establishing the length of his row. With 5 feet as the maximum error of measurement, the gardener could use 10 feet as his unit of measure. One-half of 10 feet $(\frac{1}{2} \times 10)$ is 5 feet, the maximum error. Another way of viewing the problem is that two times 5 feet, the acceptable error, is 10 feet, the unit of measure.

In general, the maximum error of measurement is used to determine the unit of measure. A cabinet maker who can tolerate a maximum error of $\frac{1}{16}$ inch can use $\frac{1}{8}$ inch $(2 \times \frac{1}{16})$ as his unit of measure. A machinist who desires a maximum error of 0.0005 inch may use 0.001 inch (2 x 0.0005) as his unit of measure.

In measurements, the numbers $\frac{3}{4}$ inches, $\frac{6}{8}$ inches, and $\frac{12}{16}$ inches have three different meanings. Despite the fact that $\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$, the measurements $\frac{3}{4}$ inch and $\frac{6}{8}$ inch are different because the measurement $\frac{3}{4}$ inch indicates $\frac{1}{4}$ inch as the unit of measure, and the measurement $\frac{6}{8}$ inch indicates $\frac{1}{8}$ inch as the unit of measure. The measurement $\frac{12}{16}$ inch indicates $\frac{1}{16}$ inch as the unit of measure. The measurement $\frac{3}{4}$ inch has $\frac{1}{8}$ inch as the maximum error. $\frac{6}{8}$ inch has $\frac{1}{16}$ inch as the maximum error. $\frac{12}{16}$ inch has $\frac{1}{32}$ inch as the maximum error. $\frac{3}{4}$ inch, $\frac{1}{8}$ inch, and $\frac{12}{16}$ inch are different measurements with different degrees of accuracy.

Unfortunately, many people who communicate linear measurements do not understand and/or practice the ideas discussed here. Consequently, measurements like 200 yards are often mentioned despite the fact that such a measure is very ambiguous with respect to the unit of measured used. 200 yards may mean a unit of 100 yards with an associated maximum error of 50 yards. Then again, it could mean a unit of 10 yards with an associated maximum error of 5 yards. Or again, the 200 yard measurement could have one yard as the unit of measure with an associated maximum error of $\frac{1}{2}$ yard. This ambiguity can be removed from any measure by the by the simple expedient of naming the unit of measure whenever it is not clearly implied by the measurement. If a measurement of 5 feet was acquired with a unit of $\frac{1}{10}$ foot then the accuracy of the measurement could be made clear by the numeral 5.0 feet. If a measurement of 19 feet was acquired with a unit of $\frac{1}{8}$ inch then the accuracy of the measurement could be made clear by the numeral 19 feet, $\frac{0}{8}$ inches.

If a machinist is asked to mill a part with radius 2.0 centimeters, the accuracy of the measure will be to the nearest 0.1 centimeter. If the radius was requested as 2.000 centimeters, the accuracy would be to the nearest 0.001 centimeter. The difference between the accuracy demanded by 2.0 centimeters compared to 2.000 centimeters will be reflected in the machinist's costs.

Progress Test 2

1. If the unit of measure is $\frac{1}{4}$ inch, what is the maximum error of measurement?

2. A carpenter working on a job figured he could allow for a maximum error of measurement of $\frac{1}{2}$ inch. What unit of measure has $\frac{1}{2}$ inch as its maximum error of measurement?

3. Why are $\frac{6}{12}$ inch and $\frac{1}{2}$ inch different measurements?

4. Write the ambiguous measure "30 yards" to show that the unit of measure was one inch.

PRINCIPLE OF CONSERVATION

The last concept of linear measurement to be discussed here is called the Principle of Conservation. The idea behind this concept is that the measure of a linear figure is not altered if it is bent, divided, curved, and/or straightened. For example, an 18.0 inch piece of string may be curled into different curved shapes, wrapped around different shapes, coiled like a spring, or cut into smaller pieces of string, but there still remains 18.0 inches of string.

It is the Principle of Conservation that claims the same linear measurement can be representative of all the shapes shown below in Figure 7.

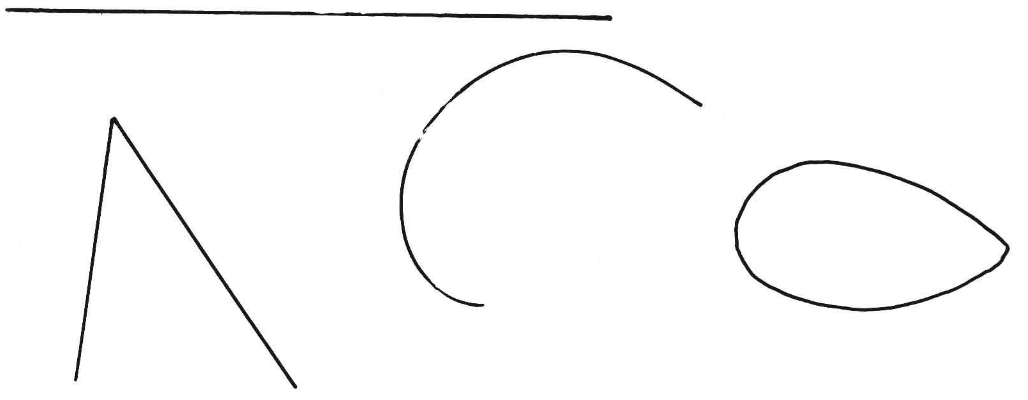


Figure 7

The Principle of Conservation simply states that the linear measure of an inelastic object does not change if the shape of the object is altered.

Progress Test 3

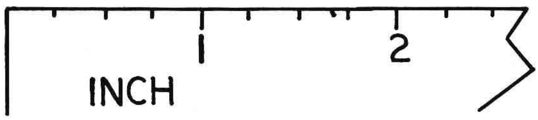
1. A string is tied into a circle 18 inches around. If the string is cut in one place and stretched out in a line, then how long will it be?

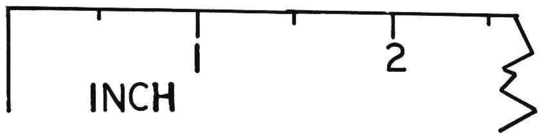
2. A flat piece of cardboard is 36 centimeters long. If it is folded to form the sides of a square, how far will it be around the outside of the square?

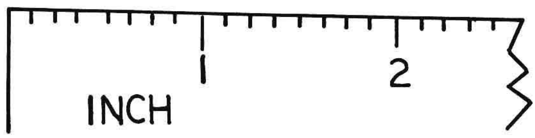
3. Does the Principle of Conservation apply to a rubber band's length regardless of the shape it forms? Why?

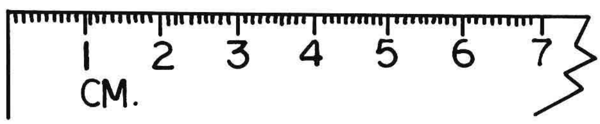
Exercise Set 1

I 1. Determine the unit of measure for each of the following:

a. 

b. 

c. 

d. 

2. Trace the segments below on the edge of a sheet of paper and use the rulers above to measure them.

a.

b. 

3. Give the maximum error that would exist using the following units of measure:

   a. 1 inch
   b. $\frac{1}{4}$ inch
   c. 1 centimeter
   d. $\frac{1}{32}$ inch
   e. $\frac{1}{8}$ inch
   f. .1 inch

4. For each maximum error given below find the corresponding unit of measure.

   a. $\frac{1}{4}$ inch
   b. .05 inch
   c. $\frac{1}{32}$ inch
   d. $\frac{1}{128}$ inch
   e. $\frac{1}{2}$ meter
   f. .005 centimeter

5. After a pole-vaulter made his vault, the height to the bar was measured with a tape marked off in inches. The height was called 18 feet, 6 inches. Find the lowest and highest the bar could be and still be measured as 18 feet, 6 inches from the ground.

6. The ____________ __ ______________ states that the length of a piece of string does not change whether the string lies flat on a table or is wound into a ball.

II. Challenge Problems

1. Which of the following are approximations?

   a. John Doe is 5 feet $10\frac{3}{4}$ inches tall.
   b. John Doe scored 100 on a biology test.
   c. John Doe had blood pressure of 140 over 80.
   d. John Doe has 27 classmates in his mathematics course.

2. a. Use Figure 5 to measure the length of 3 centimeters on the ruler.

   b. 3 cm is approximately 1.1813 inches. If 1.1813 inches is not your answer to part a, explain the difference.