SECOND EDITION

T·A·N

Calculus

FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES

SECOND ◆ EDITION

CALCULUS FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES

S. T. Tan
Stonehill College

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PREFACE

Calculus for the Managerial, Life, and Social Sciences, Second Edition is a brief edition of the author's *Applied Calculus, Second Edition.* This book is suitable for use in a one-semester or two-quarter introductory calculus course for students in the managerial, life, and social sciences.

As with the first edition, the objective of *Calculus for the Managerial, Life, and Social Sciences,* is two-fold: (1) to write a textbook that is readable by students and (2) to make the book a useful teaching tool for instructors. We hope that with the present edition we have come one step closer to realizing our goal. The second edition of this text incorporates many suggestions by both users of the first edition and reviewers of the current edition.

The following list includes some of the many important features of the book.

◆ **Level of Presentation** Our approach is intuitive and we state the results informally. However, we have taken special care to ensure that this does not compromise the mathematical content and accuracy. Proofs of certain results are given, but they may be omitted if desired. The problem-solving approach is stressed throughout the book. Numerous examples and solved problems are used to amplify each new concept or result in order to facilitate students' comprehension of the new material. Figures are used extensively to help students visualize the concepts and ideas being presented.

◆ **Applications** The text is application-oriented. Many interesting, relevant, and up-to-date applications are drawn from the fields of business, economics, social and behavioral sciences, life sciences, physical sciences, and other fields of general interest. Some of these applications have their source in newspapers, weekly periodicals, and other magazines. Applications are found in the illustrative examples in the main body of the text as well as in the exercise sets. In fact, one goal of the text is to include at least one real-life application in each section (whenever feasible).

◆ **Exercises** Each section of the text is accompanied by an extensive set of exercises that contains an ample set of problems of a routine, computational nature that will help students master new techniques. The routine problems are followed by an extensive set of application-oriented problems that test students' mastery of the topics. Each chapter of the text also contains a set of review exercises. Answers to all odd-numbered exercises appear in the back of the book.

◆ **Coverage of Topics** The book contains more than enough material for the usual applied calculus course. Thus, the instructor may be flexible in choosing the topics most suitable for his or her course.

CHANGES IN THE SECOND EDITION

This edition contains an improved treatment of many of the topics in the previous edition, as well as a few additions and changes:

◆ A precalculus review has been added in the Preliminary Chapter. It is suggested that students give it a quick perusal and review only those sections they need.

◆ Self-Check Exercises have been added at the end of each section. These exercises, with completely worked-out solutions appearing at the end of each exercise set, give students a chance to test themselves on their understanding of the material.

◆ A new format has been adopted for the second edition. This includes boxed definitions, methods, and procedures.

◆ Many more examples and exercises have been included. The examples provide further illustrations of the concepts. The exercises are of varied degrees of difficulty ranging from rote to more challenging problems.

◆ Examples and exercises requiring the use of a calculator have been labeled.

◆ The section on limits has been rewritten and one-sided limits added.

◆ Topics on integration have been reorganized with earlier introduction of the definite integral and the Fundamental Theorem of Calculus.

SUPPLEMENTS

◆ A *Partial Solutions Manual* is available to both students and instructors. It includes solutions to selected exercises in the book.

◆ An *Answer Book and Test Bank* is free to adopters of the book. It includes all the answers to exercises in the book, as well as various alternate forms of chapter tests for instant reproduction.

◆ A *computerized test bank* is likewise available upon adoption. Instructors are encouraged to consult the publisher for further information.

◆ A combined graphics and computation *software package* also accompanies the book. Further details are available through the publisher.

ACKNOWLEDGMENTS

I wish to express my personal appreciation to each of the following reviewers whose many suggestions have helped make a much improved book.

Reviewers of the second edition:

Kedrick Hartfield, *Mercer University;* Karla Neal, *Louisiana State University;* Priscilla Putnam-Haindl, *Jersey City State College;* and P. L. Sperry, *University of South Carolina.*

Reviewers of the previous edition:

Daniel Anderson, *University of Iowa;* Charles Clever, *South Dakota State University;* William Coppage, *Wright State University;* Lyle Dixon, *Kansas State University;* Bruce Edwards, *University of Florida at Gainesville;* Charles S. Frady, *Georgia State University;* Howard Frisinger, *Colorado State University;* Larry Gerstein, *University of California at Santa Barbara;* Lowell Leake, *University of Cincinnati;* Maurice Monahan, *South Dakota State University;* Lloyd Olson, *North Dakota State University;* Richard Porter, *Northeastern University;* and Thomas N. Roe, *South Dakota State University.*

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S. T. Tan

SECOND ◆ EDITION

Calculus for the Managerial, Life, and Social Sciences

CONTENTS

* Sections marked with an asterisk are not prerequisites for later material.

PRELIMINARY CHAPTER

PRECALCULUS REVIEW

P.1 REAL NUMBERS

The Real Number Line ◆ Properties of Inequalities
◆ Absolute Value

This chapter reviews some of the basic concepts and techniques of algebra that are essential in the study of calculus. The material in this review will help you work through the examples and exercises in this book. You can read through this material now and do the exercises in those areas where you feel a little "rusty," or you can review the material on an as-needed basis as you study the text. We begin our review with a discussion of real numbers.

THE REAL NUMBER LINE

The real number system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division.

Real numbers may be represented geometrically by points on a line. Such a line is called the **real number,** or **coordinate, line** and can be constructed as follows. Arbitrarily select a point on a straight line to represent the number zero. This point is called the **origin.** If the line is horizontal, then a point at a convenient distance to the right of the origin is chosen to represent the number one. This determines the scale for the number line. Each positive real number lies at an appropriate distance to the right of the origin, and each negative real number lies at an appropriate distance to the left of the origin, as shown in Figure P.1.

In this manner, a **one-to-one correspondence** is set up between the set of all real numbers and the set of points on the number line. That is,

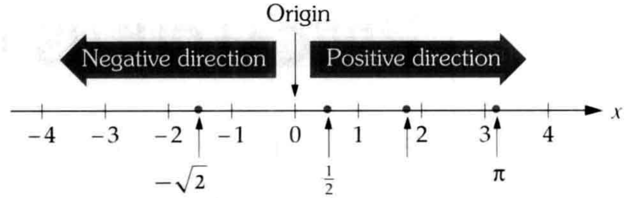


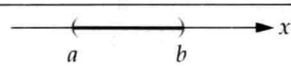
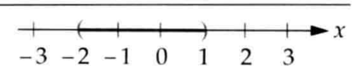
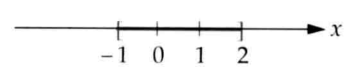
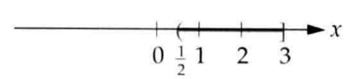
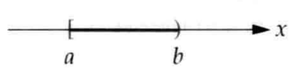
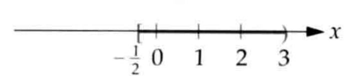
Figure P.1 *The real number line*

exactly one point on the line is associated with each real number. Conversely, exactly one real number is associated with each point on the line. The real number that is associated with a point on the real number line is called the **coordinate** of that point.

Throughout this book, we will often restrict our attention to certain subsets of the set of real numbers. For example, if x denotes the number of cars rolling off an assembly line each day in an automobile assembly plant, then x must be nonnegative, that is, $x \geq 0$. Taking this example one step further, suppose management decides that the daily production in the plant must not exceed 200 cars. Then x must satisfy the inequality $0 \leq x \leq 200$. More generally, we will be interested in the following subsets of real numbers.

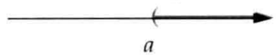
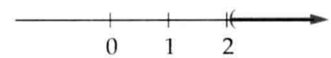
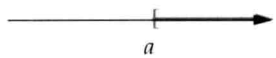
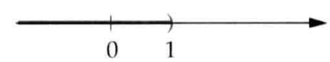
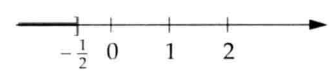
The set of all real numbers that lie *strictly* between two fixed numbers a and b is called an open interval (a, b). It consists of all real numbers x that satisfy the inequalities $a < x < b$, and it is called "open" because neither of its endpoints is included in the interval. A closed interval contains *both* of its endpoints. Thus the set of all real numbers x that satisfy the inequalities $a \leq x \leq b$ is the closed interval $[a, b]$. Notice that square brackets are used to indicate that the endpoints are included on this interval. Half-open intervals contain only *one* of their endpoints. Thus, the interval $[a, b)$ is the set of all real numbers x that satisfy $a \leq x < b$, whereas the interval $(a, b]$ is described by the inequalities $a < x \leq b$. Examples of these intervals are illustrated in Table P.1.

Table P.1 Finite Intervals

Interval	*Graph*	*Example*	
Open (a, b)		$(-2, 1)$	
Closed $[a, b]$		$[-1, 2]$	
Half-open $(a, b]$		$(\frac{1}{2}, 3]$	
Half-open $[a, b)$		$[-\frac{1}{2}, 3)$	

In addition to finite intervals, we will encounter **infinite intervals.** Examples of infinite intervals are the half-lines (a, ∞), $[a, \infty)$, $(-\infty, a)$, and $(-\infty, a]$ defined by the set of all real numbers that satisfy $x > a$, $x \geq a$, $x < a$, and $x \leq a$, respectively. The symbol ∞, called *infinity*, is not a real number. It is used here only for notational purposes in conjunction with the definition of infinite intervals. The notation $(-\infty, \infty)$ is used for the set of real numbers x, since, by definition, the inequalities $-\infty < x < \infty$ hold for any real number x. Infinite intervals are illustrated in Table P.2.

Table P.2 Infinite Intervals

Interval	*Graph*	*Example*	
(a, ∞)		$(2, \infty)$	
$[a, \infty)$		$[-1, \infty)$	
$(-\infty, a)$		$(-\infty, 1)$	
$(-\infty, a]$		$(-\infty, -\frac{1}{2}]$	

PROPERTIES OF INEQUALITIES

In practical applications, intervals are often found by solving one or more inequalities involving a variable. In such situations, the following properties may be used to advantage.

PROPERTIES OF INEQUALITIES

If a, b, and c are any real numbers, then

		Example
Property 1	If $a < b$ and $b < c$, then $a < c$.	$2 < 3$ and $3 < 8$, so $2 < 8$
Property 2	If $a < b$, then $a + c < b + c$.	$-5 < -3$, so $-5 + 2 < -3 + 2$; that is, $-3 < -1$
Property 3	If $a < b$ and $c > 0$, then $ac < bc$.	$-5 < -3$, and since $2 > 0$, we have $(2)(-5) < (2)(-3)$; that is, $-10 < -6$
Property 4	If $a < b$ and $c < 0$, then $ac > bc$.	$-2 < 4$, and since $-3 < 0$, we have $(-2)(-3) > (4)(-3)$; that is, $6 > -12$