Calculus and Linear Algebra

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Calculus and Linear Algebra

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to Morton

Preface

The gap between mathematics as practiced by mathematicians and mathematics as taught to freshmen has widened considerably in recent years. This has occurred because of the constancy of the latter in the face of sweeping changes in the former. Nor is mathematics alone in this respect. Physics, engineering, economics, and other quantitative disciplines all exhibit ever wider estrangement between the subject as viewed by a professional research worker and as seen by a beginning undergraduate.

The choice of calculus as the proper subject for first-year mathematics has been dictated in part by the needs of the other science courses which are normally taken and in part by its use in advanced mathematics courses. These considerations are still valid, but they have undergone subtle changes.

First, not too long ago mathematics was a rather unpopular major subject, and so a calculus instructor could safely assume that he was addressing a group of students whose primary interests lay elsewhere. Now, an astonishing number of incoming students are primarily interested in mathematics. This, it seems to me, imposes an obligation on the first-year course to display a branch of mathematics other than analysis, so that students who have not decided upon a major will get a better idea of the flavor of the subject and so that prospective mathematics majors will be better aware of and better equipped for the road that lies ahead of them.

Second, after intensive urging by the mathematical community, the high schools are moving rapidly to upgrade their mathematics offerings. It is now by no means uncommon to encounter students with a full year of calculus already behind them in high school. This poses a challenge and an opportunity to the universities: the challenge of revising their own curricula to match the changes in secondary school education which they themselves have instigated, and the opportunity to reach deep, substantive areas of mathematics much earlier in the undergraduate years.

Third, the needs of the sciences have also changed. A physicist of today is likely to use as much, or more, of algebra than analysis. The ideas of abstract algebraic structures, and of linear algebra in particular, pervade the quantum theory and the theory of elementary particles, as well as many of the other rapidly advancing branches of physics. In engineering, algebra rules the roost in information theory, theory of electrical circuits and networks, analysis of structures, logical design, and many other areas. To a lesser, but still important, extent, the quantitative areas of economics lean heavily on linear algebra. Altogether, then, while the calculus retains its primacy overall, it cannot be denied that algebraic concepts have come increasingly toward the forefront.

These considerations all point in the same direction, toward the earliest possible introduction of abstract algebra, and particularly linear algebra, coupled with a pruning out of the calculus course of those topics which can be readily handled at the secondary school level. This book was written in response to these needs.

The material covered in the first two-thirds of the book constitutes a course in the calculus of functions of one variable. Stress is laid on the conceptual aspects of the subject. Differentiation and integration are developed more or less simultaneously so that their interrelations can be more clearly seen. The first few sections contain motivational material which seems to be quite useful for orientation.

I have compromised with the thorny question of epsilons and deltas. I feel that they are too important to be neglected and at the same time too difficult to be carried through consistently. Therefore, I introduce the formalism quite early, just to get across the point of view. It is not, however, used meaningfully in proofs until later, during the work on infinite series, where it is indispensable. In this way, the idea has a chance to percolate for a while before stringent demands are made on it.

The work on linear algebra is concentrated on the objective of reaching a full understanding of the theory of simultaneous linear equations. This requires a good deal of sophisticated material, which is not omitted. Surprisingly enough, the students have found this part of the course much easier than the calculus. Considerable effort is expended in developing the idea of an abstract structure, and this is then applied to the theory of equations. The final chapter provides a brief introduction to the calculus of functions of several variables. It appears in this volume largely because the intertwining of algebraic and analytical concepts which occurs there offers the reader an opportunity to see vectors, determinants, and matrices at work on the problems of calculus.

It is my belief that in the years to come such a first-year course as this will become more commonplace. If so, then the means will be at hand for helping some of the other sciences to narrow the gaps in their curricula. For the "queen of the sciences" to lead her subjects would hardly be surprising.

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Foundations of the Calculus

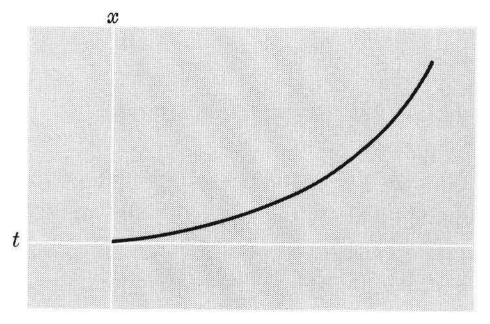
1.1. Differential Calculus

Calculus has been called "the study of change." Therefore, in this first section we will examine some particular examples of changing phenomena in order to see how we can use the calculus to gain a better understanding of these phenomena. Strictly speaking, these first few sections are unnecessary. They are intended only for the purpose of motivating certain concepts which might otherwise seem somewhat artificial. We first plan to give a typical problem in the differential calculus, then a typical problem in integral calculus. In each case we will find that the basic processes of the calculus, namely, differentiation and integration, are forced upon us if we try to answer the questions which are posed in a natural way.

In the differential calculus we are generally given a relationship which describes how one variable depends upon another. For instance, we may be told how the population of a certain country depends upon time, or how the speed of a projectile depends upon its distance above the ground. We are then asked to find out how the rate of change of the first variable depends upon the second. Thus, being given the position of a car at every instant of time, we may be asked to find the rate of change of its position, i.e., its speed, at every instant of time. Or, being given its speed, we may be asked for the rate of change of its speed, i.e., its acceleration, as it depends on time. After looking at such examples, we will then consider the opposite kind of problem, in which the rate of change is given and we wish to recover the original dependence. Thereby, we will be led to the integral calculus.

Proceeding now to a particular problem, let us imagine an automobile which starts from rest at a certain time which we will call $t = 0$. It then travels in a straight line in such a way that its distance from its starting point after 1 sec is 1 ft, after 2 sec is 4 ft, after 3 sec is 9 ft, etc. In general, if t is any real, nonnegative number, we suppose that after t sec the car has moved t^2 ft from its initial position. A graph of the distance the car has traveled after t sec, plotted against t, is shown in Fig. 1.1.

Figure 1.1



Since we now know exactly where the car is at every instant it seems that we should be able to answer any conceivable question about its motion. Certainly we can answer such questions about its position. How long does the car take to move 2 ft from its rest position? Evidently 1.414 . . . sec.

Exercises

1. How long does the car take to move 3 ft from its rest position? L ft from its rest position? Two ft from its position at time $t = 2$? L ft from its position at time $t = 2$? Five ft from its position at time $t = t_0$? L ft from its position at time $t = t_0$?

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