STOCHASTIC MODELLING AND APPLIED PROBABILITY

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Average-Cost Control of Stochastic Manufacturing Systems

Springer

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With 10 Illustrations

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To our wives, Andrea, Ruoping, and Qian,

and our children, Chantal, Anjuli, Rui, Sheena, and Sean

Preface

This book is concerned with production planning and control in stochastic manufacturing systems. These systems consist of machines that are subject to breakdown and repair. The objective is to control the rate of production over time in order to meet the demand at a minimum long-run average cost. The exact optimal solution of such a problem is very difficult and in many cases impossible to obtain. To reduce the complexity, we consider the case in which the rates at which machine failure and repair events occur are much larger than the rate of fluctuation in the product demand. The idea behind our near-optimal decision-making approach is to derive a limiting control problem which is simpler to solve than the given original problem. The limiting problem is obtained by replacing the stochastic machine capacity process by the average total capacity of machines and by appropriately modifying the objective function. We use the optimal control of the limiting problem to construct the piecewise deterministic controls for the original problem, and show that these constructed controls are asymptotically optimal under certain assumptions on the cost functions involved.

Increasingly complex and realistic models of manufacturing systems with failure-prone machines facing uncertain demands are formulated as stochastic optimal control problems. Partial characterization of their solutions are provided when possible along with their decomposition based on event frequencies. In the latter case, two-level decisions are constructed in the manner described above and these decisions are shown to be asymptotically optimal as the average time between successive short-term events becomes much smaller than that between successive long-term events. The striking novelty of this approach is that this is done without solving for the optimal solution, which as stated earlier is an insurmountable task.

This book is a sequel to Sethi and Zhang [125]. It focuses on the long-run average-cost criteria in contrast to Sethi and Zhang who deal with discounted cost objectives. A discounted cost criterion emphasizes near-term system behavior, whereas a long-run average cost measures system performance in terms of the corresponding stationary distributions. Such criteria are often more desirable in practice for long-term production planning. In addition, from a mathematical point of view, analysis of control policies of long-run average cost problems are typically more involved than those with discounted cost. This book explores the relationship between control problems with a discounted cost and those with a long-run average cost in connection with near-optimal control.

The material covered in the book cuts across the disciplines of Applied Mathematics, Operations Management, Operations Research, and System and Control Theory. It is anticipated that the book would encourage development of new models and techniques in these disciplines. The book is written for operations researchers, system and control theorists, applied mathematicians, operations management specialists, and industrial engineers. Although some of the proofs require advanced mathematics, as a rule the final results are accessible to most of them.

We wish to thank Wendell Fleming, John Lehoczky, Ruihua Liu, Ernst Presman, Mete Soner, Wulin Suo, Michael Taksar, Houmin Yan, George Yin, and Xun-Yu Zhou, who have worked with us in the area of optimal and near-optimal controls of manufacturing systems. We are indebted to W. Fleming for invaluable discussions and advice over the years. We want to thank many of our students and associates including Yongjiang Guo, Jiankui Yang, and Yuyun Yang for their careful reading of the manuscript and assistance at various stages. We appreciate the assistance provided by Barbara Gordon in the preparation of the manuscript. Finally, we are grateful to the Natural Sciences and Engineering Research Council of Canada, the National Natural Sciences Foundation of China, the Hundred Talents Program of the Chinese Academy of Sciences, the Office of Naval Research, and the University of Texas at Dallas for their support of the research on which a large part of this book is based.

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Notation

This book is divided into eleven chapters and a set of six appendices. Each of the eleven chapters is divided into sections. In any given chapter, say Chapter 4, sections are numbered consecutively as 4.1, 4.2, 4.3, and so on. Similarly, mathematical expressions in Chapter 4, such as equations, inequalities, and conditions, will be numbered consecutively as (4.1), (4.2), (4.3), ..., throughout each chapter. Also, figures and tables in that chapter are numbered consecutively as Figure 4.1, Figure 4.2, ... On the other hand, theorems are numbered consecutively in each of the sections. Thus, in any given chapter, say Chapter 3, the third theorem in Section 4 would be stated as Theorem 4.3. In Chapter 3, this theorem would be cited also as Theorem 4.3, while in all other chapters, it would be referred to as Theorem 3.4.3. The same numbering scheme is used for lemmas, corollaries, definitions, remarks, algorithms, and examples.

Each appendix, say Appendix B, has no numbered sections. Mathematical expression in Appendix B will be numbered consecutively as (B.1), (B.2), (B.3), ... Theorems are numbered consecutively as Theorem B.1, Theorem B.2, ... The same numbering scheme is used for lemmas, corollaries, definitions, and remarks. Items in Appendix B will be cited throughout the book, just as labeled in that appendix.

All deterministic and stochastic processes considered in this book are assumed to be measurable processes.

We provide clarification of some frequently used terms in this book. By ε sufficiently small (or ε small enough), we mean an $\varepsilon \in (0, \varepsilon_0]$ for some $\varepsilon_0 > 0$. The term "Hamilton-Jacobi-Bellman equation" is abbreviated as the "HJB equation." The term Hamilton-Jacobi-Bellman equation in terms of directional derivatives is abbreviated as "HJBDD." These terms, without any qualification, shall mean an average cost version of the corresponding equations. Their discounted cost version, when needed, will be so qualified, or simply referred to as dynamic programming equations. The term "open-loop controls" refers to "nonfeedback controls." The terms "surplus," "inventory/shortage," and "inventory/backlog" are used interchangeably. The terms "control," "policy," and "decision" are used interchangeably.

We make use of the following notation in this book:

$\square$	indicates the end of a proof, example, definition, or remark
$\#$	denotes "the number of"
$\mapsto$	a mapping from one set to another
$\Rightarrow$	denotes "implies"
$\Re^n$	n-dimensional Euclidean space
$\langle \boldsymbol{x}, \boldsymbol{y} \rangle$	the scalar product of any two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ in $\Re^n$
$\lvert A \rvert$	$= \sum_{i,j} \lvert a_{ij} \rvert$ for a matrix $A = (a_{ij})$
$\lvert \boldsymbol{x} \rvert$	$= \lvert x_1 \rvert + \cdots + \lvert x_n \rvert$ for a vector $\boldsymbol{x} = (x_1, \ldots, x_n)$
A'	the transpose of a vector or matrix A
$\mathbf{0}$	$= (0, 0, \ldots, 0)$ or $(0, 0, \ldots, 0)'$
$\mathbf{1}$	$= (1, 1, \ldots, 1)$ or $(1, 1, \ldots, 1)'$
1_F	the indicator function of a set F
$\mathcal{A}, \mathcal{A}^0, \mathcal{A}^\varepsilon, \ldots$	sets of admissible controls
$\{\mathcal{F}_t\}$	filtration $\{\mathcal{F}_t, t \geq 0\}$
$\mathcal{F}_t, \mathcal{F}_t^\varepsilon, \ldots$	σ-algebras
$\mathcal{N}(x)$	the neighborhood of x
$\mathcal{S}$	set of stable controls
$C^1(\mathcal{O})$	set of all continuously differentiable functions on $\mathcal{O}$
$C, C_g, C_h, \ldots$	positive multiplicative constants
$C_1, C_2, \ldots$	positive multiplicative constants
$f_{\boldsymbol{x}}$	the gradient of a scalar function f at $\boldsymbol{x}$ if it exists
$D^+ f(\boldsymbol{x})$	the superdifferential of f at $\boldsymbol{x}$
$D^- f(\boldsymbol{x})$	the subdifferential of f at $\boldsymbol{x}$
$E\xi$	the expectation of a random variable ξ
F^c	the complement of a set F
$F_1 \cap F_2$	the intersection of sets F_1 and F_2
$F_1 \cup F_2$	the union of sets F_1 and F_2
$L^2([s, T])$	the space of all square-integrable functions on $[s, T]$
$P(\xi \in \cdot)$	the probability distribution of a random variable ξ
V	the value function
W	the potential function

$(a_1, \ldots, a_l) > \mathbf{0}$	means $a_1 > 0, \ldots, a_l > 0$
$(a_1, \ldots, a_l) \geq \mathbf{0}$	means $a_1 \geq 0, \ldots, a_l \geq 0$
$\boldsymbol{a} \geq \boldsymbol{b}$	means $\boldsymbol{a} - \boldsymbol{b} \geq \mathbf{0}$ for any vectors $\boldsymbol{a}$ and $\boldsymbol{b}$
a^+	$= \max\{a, 0\}$ for a real number a
a^-	$= \max\{-a, 0\}$ for a real number a
$a_1 \wedge \cdots \wedge a_\ell$	$= \min\{a_1, \ldots, a_\ell\}$ for any real numbers a_i, $i = 1, \ldots, \ell$
$a_1 \vee \cdots \vee a_\ell$	$= \max\{a_1, \ldots, a_\ell\}$ for any real numbers a_i, $i = 1, \ldots, \ell$
a.e.	almost everywhere
a.s.	almost surely
$\mathrm{diag}(\boldsymbol{a})$	the diagonal matrix of $\boldsymbol{a} = (a_1, \ldots, a_n)$
$\mathrm{co}(F)$	the convex hull of a set F
$\overline{\mathrm{co}}(F)$	the convex closure of a set F
$\dfrac{df(x_0)}{dx}$	$= \left.\dfrac{df(x)}{dx}\right\rvert_{x=x_0}$
$\exp(Q)$	$= e^Q$ for any argument Q
$f^{-1}(\cdot)$	the inverse of a scalar function $f(\cdot)$
$\mathrm{ri}(F)$	the relative interior of a set F
$\log x$	the natural logarithm of x
$\ln x$	the logarithm of x with base e
$\boldsymbol{k}, \boldsymbol{u}, \boldsymbol{x}, \boldsymbol{z}, \ldots$	all Latin boldface italic letters stand for vectors
$\boldsymbol{u}(\cdot)$	the process $\{\boldsymbol{u}(t) : t \geq 0\}$ or simply $\boldsymbol{u}(t)$, $t \geq 0$
$\boldsymbol{x} \mapsto A\boldsymbol{x}$	linear transformation from $\Re^n$ to $\Re^m$; here $\boldsymbol{x} \in \Re^n$ and A is an $(m \times n)$ matrix
$\beta_g, \beta_h, \ldots$	positive exponential constants
λ	the minimum average cost
$\boldsymbol{\nu}$	equilibrium distribution vector of a Markov process
$\dfrac{\partial g(\boldsymbol{x}, t)}{\partial \boldsymbol{x}}$	the partial derivative of $g(\boldsymbol{x}, t)$ with respect to $\boldsymbol{x}$
$\dfrac{\partial g(\boldsymbol{x}_0, t)}{\partial \boldsymbol{x}}$	$= \left.\dfrac{\partial g(\boldsymbol{x}, t)}{\partial \boldsymbol{x}}\right\rvert_{\boldsymbol{x}=\boldsymbol{x}_0}$
$\partial_{\boldsymbol{p}} g(\boldsymbol{x}, t)$	the directional derivative of $g(\boldsymbol{x}, t)$ at $(\boldsymbol{x}, t)$ in direction $\boldsymbol{p}$
$\nabla f(\boldsymbol{x})$	gradient of function f at $\boldsymbol{x}$
$(\Omega, \mathcal{F}, P)$	the probability space
ε	a capacity process parameter (assumed to be small)
$\rho > 0$	the discount rate
$\sigma\{k(s) : s \leq t\}$	the σ-algebra generated by the process $k(\cdot)$ up to time t
$O(y)$	a function of y such that $\sup_y \lvert O(y)\rvert / \lvert y\rvert < \infty$
$o(y)$	a function of y such that $\lim_{y\to 0} o(y)/y = 0$

Contents

Part I:

Introduction and Models of Manufacturing Systems

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Concept of Near-Optimal Control

1.1 Introduction

This book is concerned with manufacturing systems involving machines that are subject to breakdown and repair. The systems under consideration range from single or parallel-machine systems to flowshops and jobshops. These systems exhibit an increasing complex structure of processing of products being manufactured. The objective is to control the rate of production over time in order to meet the demand at the minimum long-run average cost that includes the cost of production and the cost of inventory/shortage.

The exact optimal solution of such a problem is quite complex and difficult, perhaps impossible, to obtain. To reduce the complexity, we consider the case in which the rates, at which machine failure and repair events occur are much larger than the rate of fluctuation in the product demand. The idea behind our near-optimal decision-making approach is to derive a limiting control problem which is simpler to solve than the given original problem. The limiting problem is obtained by replacing the stochastic machine capacity process by the average total capacity of machines and by appropriately modifying the objective function. We use the optimal control of the limiting problem to construct the piecewise deterministic controls for the original problem, and we show these constructed controls are asymptotically optimal under certain assumptions on the cost functions involved.

The specific points to be addressed in this book are results on the asymptotic optimality of the constructed solution and the extent of the deviation