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The Synchronized Dynamics of Complex Systems

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Preface

Cui dono lepidum novum libellum
arida modo pumice expolitum?
Corneli, tibi: namque tu solebas
meas esse aliquid putare nugas,
iam tum cum ausus es unus Italorum
omne aevum tribus explicare chartis
doctis, Iupiter, et laboriosis.
Quare habe tibi quicquid hoc libelli,
qualecumque quod, o patrona virgo,
plus uno maneat perenne saeclo.
Catullo, carme 1[1]

There is an unwritten rule that states that a book should start with an epigraph.
I frankly never understood if this is because a book has really to start this way.
It is however a matter of fact that sometimes this observance flows into the choice of some apodeictic sentence, unravelling the meaning of which requires in most cases a great effort of the reader.
Having to obey to such an usance, I had to think a lot for finding the proper way to start.
I don't like those *ipse dixit* sentences. Either, indeed, they have a clear meaning (and therefore one can express that concept without having to quote a sentence written by another), or they have a sort of fuzzy meaning, open to many different interpretations (and in this case they are, therefore, useless).
To give an example, this is what I learned from reading a recent book by an Italian writer (Alessandro Baricco) where it is reported what a music critic (not at all a berk) wrote on the authoritative journal *The Quarterly Musical Magazine and Review* one year after the first representation of the Beethoven Ninth Symphony:

[1] Whom I will dedicate to this new amusing booklet,
just cleaned by the dry pumice?
To you, Cornelio, and indeed you were used
to think that my "nugae" were worth something
already when you dared, the only one among Italians,
explaining all centuries within three papers,
duct, Iupiter, and that costed you a lot of work.
Therefore, have something of this booklet,
whatever you want, so as, virgin protector,
it could last perpetually for more than one century.

"Elegance, pureness and moderation, that were the basic principles of our art, have progressively knuckled under a new style, flimsy and hasteful, that these times of airy abilities have adopted. Brains that, as for education and use, are unable to think to something else than clothes, fashion, gossip, novel reading and moral squandering, don't get at tasting the more elaborated and less feverishness pleasures of science and art. Beethoven writes for those brains, and it seems that for this he is getting a rather big success, if I have to believe to the plaudits that, from all sites, flourish for his last work."

Now, I share fully the comment of Alessandro Baricco when he says that what makes one smile is precisely the fact that the Ninth Symphony today is considered a stronghold of traditional music against new tastes. That music began a flag, a hymn, a supreme underpinning of our civilization, and permeates our lives in unimaginable ways: on 1982, when Philips had to fix the standard for the size of a compact disk (about 12 cm radius), they decided that the new support had to be able to store in its entirety the best piece of music ever written, Beethoven's Ninth Symphony. *Sic transit gloria mundi!*

So, having discarded such a kind of incipit, I decided to use another option: to collect the various sentences that meant to me something during my life (not necessarily related to synchronization) and to start each chapter of the present book with one of them.

And the most obvious choice for starting this Preface was the first *carme* of Catullo, a dedication for his collection of *nugae*, that accompanied my adolescence and my classic studies, and that genuinely reflects the genesis of the present book.

During the last fifteen years, indeed, I had the chance of being part of a tight scientific community working on synchronization processes in nonlinear dynamical systems.

I am greatly indebted to all members of such community for the friendly and warm atmosphere that characterized all our discussions about science, and that contributed to establish with any one of them not only a professional relationship, but also a profound fellowship, that will last regardless on the differences that will characterize our future scientific interests.

As always happens in science, our ideas are questionable, whereas my deep debt of gratitude for their friendship is not.

Ideally, this book is then dedicated to all my colleagues.

Primarily, it comes to my mind Hector Mancini. It is a rare fortune for a man to have the chance to meet a such beautiful person, who always cared about me much beyond what the professional relationship would have required, always heedful to understand my needs, and glad to help. The time I spent with him at the Department of Physics and Applied Mathematics of the University of Navarra taught me what means having a friend, with whom sharing the good and the bad moments of the life.

Together with him, I would like to mention the younger colleagues of the same Department, who shared with me the enthusiasm of my first studies on this subject: Diego Maza Ozcoidi, Javier Burguete, Wenceslao Gonzalez-Viñas, Jean Bragard. Our endless discussions in front of the white-board of the Department's coffee room will last in my memory as one of the most inspiring moments of my career. I strongly hope to have left there, together with a good Italian Expresso coffee machine, also the attitude of realizing such a brain storming processes that were so helpful and important to delineate joint activities and to inspire new research lines.

Immediately after, I would like to thank gratefully those colleagues with whom, some time after, I shared the effort of writing a long monographic review manuscript on synchronization, published on Physics Reports. I feel specially indebted to Jüergen Kurths, Diego Valladares, Grigory Osipov and Chansong Zhou. The present book is an extension of that review paper, and contains a lot of the material we collected and discussed together. To all of them goes my deepest gratitude for the so many scientific discussions we shared, and for the so much of the present subject I learned from them.

And how to forget the many other colleagues who have accompanied my studies during the last years, and with whom I shared discussions, comments, remarks and other good and funny moments: Kenneth Showalter who always supported my research; Lou Pecora, a real genius and a real gentleman; Arkady Pikovski who, all the times we meet, addresses me in a perfect Italian: "Tutto bene?" ("Is everything OK?"); Michael Rosenblum, Michael Zaks, Vadim Anishchenko and the many vodka's that accompanied our meetings; Rajarshi Roy who always likes to show pictures of mine in his excellent talks; my great Israeli friends Itamar Procaccia and Eshel ben Jacob who helped me in so many circumstances, Eckehard Schöll, Celso Grebogi and the great moments we had together; Ulrich Parlitz, Eric Kostelich, Vito Latora, Gabriel Mindlin, Maxi San Miguel, Vicente Perez Muñuzuri, Regino Criado, Mike Shlesinger.

And the greatest young scientists I had the chance of meeting in Spain, such as Yamir Moreno Vega, Jordi Garcia Ojalvo, Irene Sendiña Nadal, Inmaculada Leyva, Ines Perez Mariño, Javier Buldú who contributed with so many beautiful ideas to my reasonings and thinking.

Together with my advisor Fortunato Tito Arecchi, it is my pleasure to mention all the other colleagues with whom I collaborated on issues related to the subject of synchronization: PierLuigi Ramazza, Stefania Residori, Gianni Giacomelli, GianPiero Puccioni, Riccardo Meucci, Marco Ciofini, Antonino Giaquinta, Alessandro Farini, Fréderic Plaza, Manuel Velarde, Antonino Labate, Roberto Genesio, Livio Narici, Silvia Soria, Ying Chen Lai, Enrico Allaria, Maria Luisa Ramón, Santiago Madruga, Manuel Matías, Andrea Vallone, Antonio Pelaez, Sergio Casado, Umberto Bortolozzo, Tom Carroll, Fred Feudel, Ricardo López Ruiz, Murilo Baptista, Luc Pastur, Kresimir Josic, Alexander Hramov, Alexey Koronovskii, Andreas Amann, Francesco Sorrentino, Mario Di Bernardo, George Hentschel, Alexander Pisarchik, Mark Spano, Bruce Gluckmann, Luigi Fortuna, Mattia Frasca.

I definitely have to recognize that my personal contribution to the field stands much below what I could receive from being part of such a wonderful scientific team, in terms of the many human friendly relationships I had the chance of establishing with these people and with the many others that unavoidably I've forgotten to mention.

I remember with particular gratefulness all my students, from the first one Santiago de San Román (and our endless card games), to the last one Dong-Uk Hwang, to all the others: Carolina Mendoza, Mario Chavez, Italo Bove, Guillermo Huérta Cuellar. In most cases, they are now established researchers, and I feel particularly indebted to them for the innumerable inspirations and ideas we had the chance of sharing, as well as for all those unrepeatable moments that characterized our collaboration.

My fond thought goes, also, to two other colleagues who are not anymore with us, and whose absence we all feel: Lorenz Kramer and Carlos Pérez García. With the first I had the chance of interacting when I was yet in my early stage of the career, reaching to appreciate his incredible knowledge of nonlinear science, together with his friendly attitude in sharing his mastery on the subject with the younger colleagues. With the second I passed a much longer time when we both were Professors in the same Department of Physics in Spain, learning so much about my professional activity.

Furthermore, this book is dedicated to all persons who made (and make) it possible for me to follow my passion for science.

I will never forget two of my Professors whom I had the chance of having during my classic studies at the end of the eighties: Profs. Chiara Asselle and Paolo Chiarelli.

Prof. Chiara Asselle was my professor of Latin and ancient Greek at that time. Still now, more than 20 years after I finished that part of my education, my dreams are populated by her fantastic classes of Latin and Greek literature. Definitely, my way of getting involved in research, and of getting curious about discovering what is beyond reality, is so influenced by what I could learn from her passionate explanations of the philosophy and lyric developed in those ancient years.

Prof. Paolo Chiarelli was my professor of mathematics and physics in those early years, and he is actually the main responsible for my final preference for scientific studies in the University. It was a real fortune to have him as a professor during those years, as he really initiated me to the beauty of physics, and transmitted me the passion for research, as well as the attitude of scientifically investigating the world around me.

Moreover, I would like now to express my gratitude to the Italian Ambassador in Israel His Excellence Sandro de Bernardin, to the Director of the Italian Cultural Institute Simonetta della Seta, to the Councellor of the Embassy Davide La Cecilia, to the commercial Councellor Tiziana D'Angelo, and to all the personnel of the Italian Embassy in Tel Aviv, with whom I will share the pleasure of working during the next future.

And finally, I want to thank most gratefully all the members of my family: my mother Laura and father Mario, my wife Cristina and my daughter Chiara. They always supported my activities in the strongest as possible terms, helping me in all required circumstances. I am totally aware that my entire scientific career couldn't have developed without their continuous bearing.

But now, it is time to start.

Contents

Chapter 1

Introduction

The larger is chaos
the larger the option for a man
to control the natural phenomena.

It was a great sunny day, that 10th of June 2000 in London.

Thousands and thousands of citizens, the City's Authorities, the most of the Press and local and international broadcasting channels were attracted to participate to a special event: the Opening Ceremony of the London Millennium Bridge.

But, nobody even imagined that they would have instead assisted to one of the most spectacular manifestations of what a nonlinear synchronization phenomenon is.

As soon as the many people, indeed, put over the bridge and started to walk, something suddenly happened: the bridge started to wobble from side to side!

The following is the report of the event that is still appearing in the BBC News Site:[1]

"Some 80,000 people crossed the bridge on its opening day and those on the southern and central spans detected vibrations. The bridge began to sway and twist in regular oscillations. The worst of the movement occurred on the central span where the deck was moving by up to 70 mm. The frequency of the oscillations increased, leaving people unnerved and unsteady. The engineers insisted the bridge wouldn't fall down but closed it completely after an attempt to limit numbers proved unworkable. Engineers discovered that the sideways forces of the pedestrians' footsteps created a slight horizontal wobble in the bridge. As the structure began moving, pedestrians adjusted their gait to the same lateral rhythm as the bridge. The adjusted footsteps magnified the motion—just like when four people all stand up in a small boat at the same time. As more pedestrians locked into the same rhythm, the increasing oscillations led to the dramatic swaying captured on film. The sideways motion has been seen before, most notably in 1975 on the Auckland Harbour Bridge in New Zealand."

[1] http://news.bbc.co.uk/hi/english/static/in_depth/uk/2000/millennium_bridge

A movie is also found in the same Web site, that witnesses the dramatic locking effect of the pedestrian footsteps, whose synchronization is the cause of the observed Millennium Bridge's motion.

Though in that occasion the wonder and surprise of both experts and citizens was big, the understanding of how collective (synchronized) phenomena set up in the evolution of coupled dynamical systems goes back historically up to the earlier days of physics.

Already in 1665, indeed, Christiaan Huygens introduced the novel concept of "sympathy" observing the collective (synchronized) rhythm of two pendulum clocks suspended by the side of each other, that swung with exactly the same frequency and in a 180 degrees out of phase configuration [1].

Huygens further noticed that such an anti-phase synchronous state was robust against perturbations (if disturbing the pendulums, the "sympathy" state was eventually restored and persisted indefinitely), and deduced that the fundamental cause of this effect was the very tiny coupling coming from the imperceptible movements of the common frame supporting the two pendulums.

Furthermore, if one carefully observes natural phenomena, one immediately realizes that synchronous behavior of slightly interacting dynamical units is an ubiquitous phenomenon and permeates many aspects of our daily life: from the spectacular synchronization of the blinking behavior of fireflies of certain species that one can witness in some trees on the side of rivers in South-East Asia, to the synchronization in the calling behavior of Japanese rain frogs (whose origin is the attempt of the males of those species to better attract females for mating).

And, again, the perfect synchrony of the rotation and revolution periods of the Moon which is the cause of the fact that we always observe the same face of the Moon from the Earth, or the studies on women who spend a lot of time together and show evidence of developing menstrual synchrony, in the sense that their interaction can lead them to menstruate around the same day of the month.

Further examples are the interaction of vibrations of some adjacent organ pipes, which is known to cause, in certain occasions, a perfect unison or can even have the dramatic effects of reducing the sound of the pipes to almost silence, and the evidence that some normal and abnormal behavior of the human brain (including some brain diseases) are the result of a sudden and abrupt synchronization in the activity of a large number of neuronal populations.

There was even who imagined that a time synchronized jumping of millions of persons could have caused a permanent deviation in the revolution trajectory of the Earth around the Sun, leading to drive the planet into a new orbit: the web-site http://www.worldjumpday.org reports the results of a very funny social experiment, consisting in synchronizing the jump of millions of people on July the 20th, 2006 (the *World Jump Day*) at 11.39.13 GMT.

The same colloquial meaning for the word *synchronization* has its very old root in the ancient Greek. Synchronization comes, indeed, from the Greek $\sigma\grave{\upsilon}\gamma$

$\chi\rho\acute{o}\nu o\varsigma$ which means "to share the common time", and this original meaning has been maintained up to now as *agreement or correlation in time of different processes* [2].

The scientific definition of *synchronization* refers, instead, to a general process wherein two (or many) dynamical systems (either equivalent or nonequivalent) are coupled or forced (periodically or noisy), in order to realize a collective or *synchronous* behavior.

In particular, when one deals with nonlinear chaotic or complex dynamical units, the arousal of such collective (synchronized) dynamics is, in general, far from being trivial.

Indeed, a nonlinear dynamical unit is said to produce a *chaotic* motion when its evolution depends crucially on the initial conditions. This implies that even two identical (but separated or uncoupled) systems that would evolve from almost identical (i.e., only slightly differing) initial states, would give rise to two trajectories that would naturally and exponentially separate in time. As a consequence, one could say that chaotic systems are systems that intrinsically defy synchronization, as even when the systems are identical but start from slightly different initial conditions, the corresponding trajectories would evolve in time in a unsynchronized manner.

This relevant practical problem (in experiments or in nature one generically has initial states that are never known with infinite precision) started a great interest on the study of how a collective (synchronized) behavior can be set in coupled chaotic systems.

When one regards synchronization phenomena in coupled chaotic systems, one can assume at least three distinct points of view for classifying the different observed behaviors.

The first way is to classify the observed collective states depending on the nature of the synchronization phenomena.

And, indeed, in the context of coupled chaotic elements, many different synchronization states have been studied in the past years. They are: complete or identical synchronization (CS) [3–5], phase (PS) [6,7] and lag (LS) synchronization [8], generalized synchronization (GS) [9,10], intermittent lag synchronization (ILS) [8,11], imperfect phase synchronization (IPS) [12], and almost synchronization (AS) [13].

CS was the first discovered phenomenon and it can be considered as the simplest form of synchronization. Its emergence consists in a perfect hooking of the chaotic trajectories of two coupled identical systems, in such a way that they remain in step with each other in the course of the time [5]. This phenomenon strictly corresponds to the case for which the synchronization error (the difference between the states of the two coupled systems) vanishes asymptotically.

GS is instead a more complicated phenomenon which involves the coupling of nonidentical systems, and whose arousal corresponds to the emergence of a function (not necessarily the identity) that associates the output of one system to that of the other system [9,10]. As this function might well be not invertible, one immediately realizes that the concept of GS is an asymmetric concept, in that one can have situations where the state of one system can be predicted by measurements of the other system, but not vice versa.

An intermediate regime occurring in coupled nonidentical system is phase synchronization (PS), wherein a locking of properly defined phases is produced, without implying in general a corresponding high correlation in the amplitudes of the systems [6]. PS corresponds, therefore, to a weaker stage of synchronization, which generally occurs already at very small coupling strengths, if compared with the values needed to generate CS or GS.

LS implies the boundedness of the difference between the actual output of one system and the delayed output of the other. The shift in time τ_{lag} making the equivalence of the two systems is called *lag time* [8]. In its turn, it is evident that this phenomenon implies that the two outputs lock both their phases and amplitudes, but with the presence of a time lag [8].

ILS is a state in which the two systems verify LS for most of the time, but intermittent bursts of nonsynchronous behavior occur persistently [8,11], generally in correspondence with the passage of the system's trajectory in particular attractor regions where the LS state is locally unstable [8,11].

In analogy with ILS, IPS is a regime where phase slips (2π jumps in the phase difference of the two systems) occur persistently within a PS regime [12]. This synchronization regime is characteristic of coupled oscillators that are not *phase coherent*, i.e., such that they oscillate at a wide distribution of frequencies.

Finally, AS results in the asymptotic boundedness of the difference between a subset of the variables of one system and the corresponding subset of variables of the other system [13], without implying a specific kind of relationships between the variables not belonging to the two subsets.

Given a pair of coupled systems, one can observe a scenario of transitions among different types of synchronization. The first observation of such transitions was described for symmetrically coupled nonidentical systems and consisted in successive transitions between PS, LS and a regime similar to CS when increasing the strength of the coupling [8].

The second point of view is to distinguish different synchronization phenomena as a function of the nature of the coupling.

For instance, there is a great difference in the process leading to synchronized states, depending upon whether the coupling is symmetrical or asymmetrical. In particular, one should distinguish two main cases: the *unidirectional* and the *bidirectional* coupling configuration.

In the former case, there is a drive (or master) system whose output influences the behavior of a response (or slave) system. This implies that one system (the master) evolves uncoupled and drives the evolution of the other (the slave).