New Developments in Difference Equations and Applications

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New Developments in Difference Equations and Applications

Professor Ming-Po Chen (1941–1997)

To

Professor Ming-Po Chen

Born in Taiwan, Professor Chen received his BS degree from Taiwan Normal University and PhD degree from Pennsylvania State University under the guidance of Professor George Andrews. He worked at Academia Sinica as a research fellow from 1977 until his death in 1997 and published more than 100 research papers in various fields of mathematics including combinatorics, analytic functions, differential equations, fractional calculus, variational inequalities and difference equations. He had more than 40 masters students and is remembered as one of the most respected mathematicians in Taiwan.

PREFACE

Although difference equations appeared well before their continuous counterparts, international conferences that bring together experts in this and related areas have been organized only in recent years. Such an occasion took place from September 1–5, 1997, when more than 70 mathematicians from 13 countries participated in the Third International Conference on Difference Equations and Applications in Taipei, Taiwan.

This volume contains a selection of papers on difference equations, most of which were presented at the conference and accepted after peer review. It covers the latest developments in a wide range of topics including stability theory, oscillation theory, combinatorics, numerical analysis, asymptotics, and partial difference equations, as well as applications to sciences. The conference that led to this book was initiated by the late Professor Ming-Po Chen and was organized by him and Professors Sui Sun Cheng, Jyh-Hao Lee and Kin-Ming Hui. It was supported by the National Science Council, the Mathematics Research Promotion Center of the Republic of China and Academia Sinica, which hosted the meeting. A large number of individuals including the staff of the Institute of Mathematics of Academia Sinica gave invaluable help during all phases of the conference. On behalf of all the participants, we thank them for their warm hospitality and for a job well done. Our gratitude also goes to those who contributed papers and to those who donated their time and expertise in refereeing the submitted papers, thus ensuring the high quality of these proceedings.

Our special gratitude goes to the late Professor M.-P. Chen, who was instrumental in making this conference a great success. Although his work in difference equations occupied only a small portion of his vast contributions to mathematics, which included combinatorics, analytic functions, fractional calculus, differential equations, special functions and variational inequalities, he was keen on initiating the organization of the conference. He remained enthusiastically involved throughout the entire convention, even after receiving a diagnosis of liver ailment. It was most unfortunate that he died in December 1997, three months after the convening of the conference. It is our honor to dedicate this book to his memory.

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CONTENTS

BOUNDEDNESS OF SOLUTIONS OF A PLANT-HERBIVORE SYSTEM

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Abstract We investigate the boundedness character of solutions of a Plant-Herbivore System.

1 INTRODUCTION

In this note we investigate the boundedness character of solutions of the following Plant-Herbivore System

$$\left.\begin{aligned} x_{n+1} &= \frac{\alpha x_n}{\beta x_n + e^{y_n}} \\ y_{n+1} &= \gamma(x_n + 1)y_n \end{aligned}\right\} \quad , n = 0, 1, \ldots \tag{1.1}$$

where $\alpha \in (1, \infty)$, $\beta \in (0, \infty)$, and $\gamma \in (0, 1)$ and the initial conditions x_0 and y_0 are arbitrary positive real numbers. This system was introduced and studied by Allen, Hannigan, and Strauss [1]. The model describes the interaction of the grape vine and the adult apple twig borer. A thorough description of the model can be found in [2].

System (1.1) has the equilibrium points $E_1 = (0, 0)$, $E_2 = (\frac{\alpha-1}{\beta}, 0)$ and when

$$\alpha + \beta - \frac{\beta}{\gamma} > 1 \tag{1.2}$$

it also has the equilibrium point $E_3 = (\frac{1}{\gamma} - 1, ln(\alpha + \beta - \frac{\beta}{\gamma}))$. See [3].

If $\alpha + \beta - \frac{\beta}{\gamma} \leq 1$, it is easy to see that every positive solution of System (1.1) is bounded. In fact, in this case it is shown in [1] that the equilibrium E_2 is globally asymptotically stable. Our goal here is to investigate the boundedness of solutions when (1.2) holds.

2 MAIN RESULT

In this section we show that when (1.2) holds every positive solution of System (1.1) is bounded. The first result shows that $\{x_n\}$ is bounded.

Theorem 2.1 *Assume that (1.2) holds. Let $\{x_n\}$, $\{y_n\}$ be a positive solution of System(1.1). Then there exists an $N \geq 0$ such that*

$$x_n \leq \frac{\alpha - 1}{\beta} \quad for \quad n \geq N. \tag{2.1}$$

Proof Set

$$\mathbf{S} = \left\{(x, y) : 0 < x \leq \frac{\alpha - 1}{\beta} \quad and \quad y > 0\right\}.$$

Now observe that the strip **S** is invariant under the map

$$T(x, y) = \left(\frac{\alpha x}{\beta x + e^y}, \gamma(x + 1)y\right). \tag{2.2}$$

Indeed if $(x,y) \in \mathbf{S}$, then

$$\frac{\alpha x}{\beta x + e^y} < \frac{\alpha x}{\beta x + 1} \leq \frac{\alpha(\frac{\alpha-1}{\beta})}{\beta(\frac{\alpha-1}{\beta}) + 1} = \frac{\alpha - 1}{\beta}$$

and so $T(x,y) \in \mathbf{S}$.

Hence every solution of System (1.1) which starts in $\mathbf{S}$ stays in $\mathbf{S}$ and so (2.1) holds. It remains to show that every positive solution of System (1.1) enters $\mathbf{S}$. To this end, assume for the sake of contradiction that

$$(x_n, y_n) \notin \mathbf{S} \quad for \quad n \geq 0.$$

Then clearly

$$\frac{1}{\gamma} - 1 < \frac{\alpha - 1}{\beta} < x_n \leq \frac{\alpha}{\beta} \qquad for \quad n \geq 0 \tag{2.3}$$

from which it follows that $\{x_n\}$ is bounded and

$$y_{n+1} = \gamma(x_n + 1)y_n > \gamma(\tfrac{1}{\gamma} - 1 + 1)y_n = y_n.$$

That is, $\{y_n\}$ is strictly increasing. Since in this region $\{x_n\}$ is decreasing and as System (1.1) has no equilibrium point outside $\mathbf{S}$, it follows that

$$\lim_{n\to\infty} y_n = \infty.$$

Hence from the first equation of System (1.1) and the fact that $\{x_n\}$ is bounded we find

$$\lim_{n\to\infty} x_n = 0$$

which contradicts (2.3). □

In the next result we assume that a positive solution $\{x_n\}$, $\{y_n\}$ of System (1.1) lies in $\mathbf{S}$ and show that $\{y_n\}$ is bounded.

Theorem 2.2 *Assume that (1.2) holds and that $\{x_n\}$, $\{y_n\}$ is a positive solution of System (1.1) such that*