SECOND EDITION

Business Mathematics

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BUSINESS MATHEMATICS

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PREFACE TO THE FIRST EDITION

MATHEMATICS now enters into fields of study which were almost wholly non-mathematical only a few years ago. In business, elementary arithmetic was considered quite adequate for most purposes, although particularly alert and progressive businessmen might pride themselves on the statistical techniques which they or their assistants could deploy. In recent years all this has changed; operational research, the introduction of electronic computers and a generally more numerate and scientific approach to business problems have made mathematics an important and sometimes essential part of the businessman's equipment.

These changes are being reflected in the attitudes of the professional bodies and of the universities, technical colleges and other teaching institutions. Naturally enough, candidates for the qualifications offered by these bodies are increasingly often required to show competence in mathematics. A difficulty is, however, that many people who wish to follow a business career have had a far less thorough grounding in mathematics than, for instance, young people entering a scientific training. Nor, as yet, have many older people established in business a sufficient grasp of the newer techniques to help their junior colleagues. It is hoped that this HANDBOOK may have something to offer to both groups in that it attempts to supply an entry to the world of mathematics, particularly as applied to business, that is free of the obscurity and mystery which so often and so unnecessarily prevents otherwise capable people from attaining mathematical understanding.

PREFACE TO THE SECOND EDITION

THE ABILITY to make effective decisions is one of the most important factors in successful business behaviour. Business mathematics is concerned with improving the quality of decision-making in industry and commerce and it is not surprising that a high degree of mathematical ability has come to be accepted as a necessary part of a manager's equipment at every level of responsibility. This is not to deny that entrepreneurial flair and the ability to take risks, and to distinguish acceptable risks from unacceptable ones, are also important in running a successful business, but even here mathematics has a part to play. It is with these considerations in mind that the new edition of *Business Mathematics* has been prepared. While many familiar things remain, the emphasis has been placed on the mathematics of business decision, including decisions made under conditions of risk and uncertainty.

Important as they are, business decisions can be no better than the information on which they are based and much everyday mathematical work in business is concerned with the prior analysis and presentation of this information. Often, at this level, the methods employed are statistical in nature and over the last ten or fifteen years the sophistication of the techniques available and in common use has grown enormously. In part this has been due to the increase in computing power now able to be brought to bear at comparatively low cost, but it is also due to the fact that new entrants to industry and commerce are now much better trained in quantitative methods. These people, entering as junior management, are moving up the tree so that the general mathematical competence applied at middle management level is likely to increase. Consequently, the standard *required*, without question, from tomorrow's entrants will be higher.

The higher standards of mathematics applied in business have been reflected in the requirements of the professional bodies and in the curricula of the universities, polytechnics and other institutions concerned with education and training for business. Regrettably, though, the basic mathematical

foundations for the quite demanding courses offered by all these bodies have not always been securely laid. All too often fundamental concepts in mathematics have never been fully understood or have been forgotten by the time that a career choice is made. Difficulties of this sort are compounded when people established in the earlier stages of a business career find, at some point, that further progress and promotion are blocked because it is not possible to obtain a necessary qualification without mathematical competence. It may be the case, too, that some senior managers, trained in a different environment, find that in dealing with statisticians or operational research specialists their mathematical background is not adequate. It is hoped that this HANDBOOK may have much to offer to each of these groups. It attempts to open the world of mathematics, particularly as applied to business, without any of the mystery and obscurity which so often and so unnecessarily prevent otherwise capable people from understanding and using mathematical techniques.

The scheme of the book. Part I of the book deals with the basic concepts on which the mathematical ideas developed later are based. Because these concepts are so basic, they are accessible to everyone and yet they are so near to the heart of the subject that progress on these foundations can be very rapid. Bearing in mind the higher standards now expected, it is felt that these fundamental ideas ought to be extended to include the differential calculus as an elementary notion rather than as a more advanced method. The presentation, however, is still in fundamental terms.

Part II extends the basic ideas of Part I and in doing so illustrates the slightly more advanced concepts, as they are developed, by examples from business life. Integration, vectors, matrices and the elementary mathematics of probability and statistics are introduced. The statistical ideas, in line with the previously expressed belief that quite sophisticated methods are coming into general use, now include some of the special methods previously regarded as being of a more advanced type. Every opportunity has been taken, in introducing advanced work, to develop the student's competence in mathematics and to increase familiarity with the use of symbols. In Part III the mathematics acquired is used in the specialised field of Finance, with special attention being given

to discounting techniques and to the appraisal of investment projects.

Part IV specifically relates mathematical analysis to business decision-making. The essence of business decision is that it is concerned with unknown future conditions. Initially, and perhaps this is appropriate in the less secure business climate with which we are now faced, decisions are considered under uncertainty. Subsequently, ideas introduced in Part II are used in the assessment of the worth of business information and are then applied to probabilistic decision-making. The standard techniques of operational research are then outlined and are developed using both models involving probabilistic approaches and those concerning decisions made under the assumption of certainty. There is a steady development in the standard of mathematical skill required and in the use of symbols, so that the reader should have little difficulty in passing to more advanced work.

In Part V, both the statistical and the mathematical backgrounds are applied to a central business problem: that of forecasting future conditions. All businesses must attempt forecasts, whether by intuitive or by formal methods. This is currently a lively area of work and it is hoped that the final chapters will give some indication of this.

There are Progress Tests at the end of each chapter and worked answers to most of the questions are given in Appendix IV. Where there are no worked answers, the questions are marked with an asterisk and the relevant sections of the text are indicated.

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The Institute of Cost and Management Accountants
The Cambridgeshire College of Arts and Technology (H.N.D.)
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July 1979 L.W.T.S.

LIST OF SYMBOLS

Symbol (and name where appropriate)	*Meaning or use*
α (alpha)	often used to indicate a constant
β (beta)	used as a constant or coefficient; indicates a particular type of distribution in statistics
Δ (capital delta)	incremental sign
λ (lambda)	a scalar or constant; the arrival rate in queueing theory
μ (mu)	used as the population mean in statistics and the service rate in queueing theory
ρ (rho)	the traffic intensity
σ (small sigma)	population standard deviation
Σ (large sigma)	the summation sign
θ (theta)	usually indicates an angle
χ (chi)	χ^2 is used to indicate a statistical distribution
$\int$	the integral sign
!	the factorial sign as in $n!$ ("n factorial")
$\{\}$	used to indicate a set
$\|$	the set-builder sign—"such that"
$<$	less than
$\leqslant$	less than or equal to
$>$	more than
$\geqslant$	more than or equal to
$\cup$	union (of sets)
$\cap$	intersection (of sets)
$\wedge$	"and"
$\vee$	"or"
$Pr\ (a\|b)$	conditional probability; "the probability of a given b"
$\longrightarrow$	a mapping
$\longleftrightarrow$	a one-to-one mapping
$f(x)$	function
dy/dx $f'(x)$ $d/dx\ f(x)$	derivative signs
∂	partial derivative sign, as in $\partial z/\partial x$
A'	transpose of a matrix
π (pi)	Archimedes' constant ($= 3{\cdot}14159\ldots$)
e	the exponential constant ($= 2{\cdot}71828\ldots$)

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PART ONE

BASIC CONCEPTS AND METHODS

CHAPTER I

INTRODUCTION: MATHEMATICS IN BUSINESS

THE GROWING IMPORTANCE OF MATHEMATICS

1. Why use mathematics? In all fields of activity the scientific approach has been gaining ground in this century. Many activities that were previously handled by verbal analysis and description have proved to be more easily dealt with by mathematical techniques. Biology, botany and meteorology are examples of sciences that have yielded to mathematical analysis. The use of mathematics in physics, chemistry and the other natural sciences is of such long standing as to be commonplace.

Business problems, too, can be handled more efficiently:

(*a*) by using the scientific method; and
(*b*) by applying appropriate mathematical techniques where they have been developed.

To a large extent, these two things go together. Mathematical formulation of problems can give certainty in handling complex problems and can enforce a precision in stating the facts of a situation where these would otherwise be lost in emotion and argument. We should use mathematical methods where they can give clear solutions to business problems, because by doing so we can use the limited resources of a business more efficiently. This saves money.

2. The scientific method. If we try to describe the scientific method which has proved so effective in revealing the secrets of the physical world, we should have to list the several stages like this.

(*a*) *Observations* are made and these stimulate ideas about the process being observed.

(*b*) These ideas are expressed as clearly and formally as possible as a *hypothesis*.

(*c*) An experiment is devised to *test* the hypothesis.

(*d*) On the basis of the experiment, which provides new observations, *the hypothesis is either accepted or rejected*.

(*e*) *If it is rejected*, a new hypothesis may be formed, to be tested in turn.

In business, this process is not wholly possible. Experiments merely to test hypotheses are not often possible; moreover, we are usually more concerned to solve a specific problem rather than to find out the exact "truth" of a situation. There are many situations which *can* be dealt with scientifically, even if not in quite the same way.

3. The scientific method in business. If the managers of a business are completely satisfied, then, provided that they are competent, we may take it that there is no problem and the business can be run in the same old way. There are not many business situations that permit that degree of complacency. If the managers are not satisfied, we must track down what is wrong, that is to say we must *define the problem*. This is the most difficult step in the scientific handling of business problems.

The next thing to do is to make a detailed description of the situation or process concerned, together with all the interrelationships between its parts. We may refer to this as *making a model* of the situation. To do this we need to collect information.

When we have collected as much information as we need, and have checked that it is as accurate as we can make it (or can afford to make it: *see* Chapter XXIV), we can *manipulate our model* to see what action will lead to the best result. We *optimise* our model.

Having found the *theoretically* best thing to do, we next need to *formulate a plan* for action in the real world of factory, office or store. The plan must then be applied with sufficient built-in checks to let us know if it is working satisfactorily. If necessary, we must *modify the original plan* in the light of the information gained.

4. Mathematics and the scientific method. It is necessary to go into the place of the scientific method in business because