LINEAR CIRCUIT ANALYSIS

TIME DOMAIN, PHASOR, AND LAPLACE TRANSFORM APPROACHES

RAYMOND A. DeCARLO / PEN-MIN LIN

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PREFACE

PRESENT AND PAST

For several decades in the United States, most electrical engineering undergraduates have taken two semesters of linear circuit analysis in their sophomore year. Those in other engineering disciplines often have taken only one semester. Standard topics include laws and techniques of resistive circuit analysis, time domain transient analysis of first and second order linear circuits, a phasor approach to sinusoidal steady state analysis, resonant and magnetically coupled circuits, transformers, elementary 3-phase circuits and some Fourier series analysis.

The late 70's and the decade of the 80's marked a time of revolutionary growth in undergraduate electrical engineering education. Discrete time system and computer engineering concepts grew into integral parts of the engineer's required graduation toolbox. Software programs such as PSpice®, MATLAB® and its toolboxes, Theorist®, Mathematica®, and a host of others appeared to streamline the computational drudgery of engineering analysis and design. Paralleling this were advances in research and technology. IC implementations of active filters and especially switched capacitor filters have become shelf items in supply houses. Matrix-based numerical algorithms for large scale circuit simulation, parameter optimization, layout, and design are widely available. Our research colleagues have dramatically deepened our ability to analyze, design, and control nonlinear circuits and systems.

APPROACHING THE YEAR 2000

In order to meet the needs of the next decade, our approach was to more efficiently package the traditional circuits courses in an up-to-date framework without sacrificing rigor. For example, we have introduced the matrix formulation of node and mesh equations for solution with software programs such as MATLAB® or its equivalent, and have added a special section on the Modified Nodal Method of circuit analysis. The ubiquitous presence of active circuits built around the op amp, prompted us to integrate op amps throughout the text. In support of this we have a more careful treatment of circuits containing controlled sources and op amps; in particular a careful restatement of the traditional Thévenin and Norton Theorems usually stated (rigorously) in many texts only for passive circuits. We have unified the treatment of RL and RC first order circuits and emphasized the use of the characteristic equation for the solution of second order circuits in contrast to the usual formula-development for each of the parallel and series cases. Classical phasor analysis is introduced in step-by-step fashion beginning with a review of its complex variable foundation. These ideas underlie our treatment of ac steady state power considerations and applications to power systems which we treat in the text.

Many texts introduce a notion of generalized phasor analysis. Our imperative was to replace this often ill-used approach with an early introduction to Laplace transforms without first covering the Fourier series and Fourier transform. This allows us to recast the usual second semester circuits topics (resonant and magnetic circuits, two ports, filters, etc.) with a systems flavoring underpinned on the foundation of the Laplace transform. For example, with resonant circuits we lead the student quickly through the ordinary material spiced with an application or two and then unify the ideas under the umbrella of a bandpass transfer function. In fact, when teaching this material we ordinarily begin with the bandpass transfer function so that the student sees the unity from the onset.

This menu allows us to expand the boundaries of traditional coverage without additional investment in time while simultaneously developing advanced skills: students are able to tackle both transient and steady state circuit analysis for any excitation with a rational Laplace transform besides the usual steps, ramps, exponentials, sinusoids and sinusoidally modulated exponentials for circuits of small to moderate order. This framework offers students an entire semester of problem-solving practice in the Laplace transform context, thereby significantly honing their analytical skills. This is not achievable in the more traditional setting for teaching the course. The approach also enables us to unify the frequency and time-domain approaches to circuit analysis in the systems context by covering continuous-time convolution. Here we introduce the basic definitions, the integration and graphical approaches, and show where it might be useful for circuit analysis.

With the Laplace transform, the concepts of impedance and transfer function are cleanly and rigorously introduced. Further, there is the potential for the incorporation of synthesis techniques into homework exercises otherwise precluded in the traditional treatment. In sum, this approach better prepares students for the rigors of the signals and systems course of the junior year and frees up time in the junior year for the exploration of discrete time systems concepts.

To better implement our approach we developed a software program entitled tfc which automates the entire approach to Laplace transform analysis. This, of course, can be complemented by MATLAB® commands which also accommodate Laplace transform analysis and convolution.

To allow flexibility in curriculum development and to introduce (nonlinear) electronic circuits at a very elementary level, we provide two optional chapters on piecewise linear analysis of diode and transistor amplifier circuits. The piecewise linear approach enables the analysis of many simple and useful electronic circuits via the techniques of linear resistive circuits. It permits us to address the growing need to expose students to nonlinear behavior in a simple understandable way.

Personal Reasons

All of the above reasons not withstanding, starting in 1986 and continuing to this day, we have had a genuine desire to significantly improve the content and complexion of the basic circuits courses. We wanted to tie circuits concepts to real world devices such as microwave ovens, stereo amplifiers, adapters for portable computers, etc. With the suggestions and help of our editor, we did this with chapter openers. Further, we wanted to present challenging homework exercises and to present the material in a way consistent with our perception of research and development over the last decade.

I. KEY CONCEPTS

1. Balanced Emphasis on Concepts and Calculation

Quality software programs numerically automate important aspects of circuit analysis and design, relieving engineers of tedious and often impossible hand calculations. However, numerical algorithms implemented as canned programs are no substitute for an understanding of the basic circuit principles and properties which govern circuit behavior nor for a firm understanding of the steps necessary for the solution of a problem, because from our perspective, there are five educational tasks of a circuits course:

1. Create an environment where the student has the opportunity to learn the basic vocabulary, principles, analysis methods and design techniques of circuit theory as gleaned from the accumulated experience of past engineers and physicists.
2. Create an environment for applying the principles of mathematics and physics to engineering problems.
3. Develop a qualitative understanding of circuit behavior.
4. Foster the development of rational thinking patterns in the context of circuit analysis and design in order to prepare students for solving a broad variety of engineering problems.
5. Help develop the student's ability to critically evaluate their chosen problem-solving technique and the accuracy of their answer.

In order to meet these needs we have included some advanced sections marked with an asterisk (*) and a variety of problems and exercises from the simple to the difficult. (The more difficult exercises are also flagged with an asterisk.) Some problems apply the ideas of the text while a few enhance or extend the ideas described in the text. Although numerical approaches and problems are included throughout the text, we have included two appendices, one for each of the traditional semesters of circuits, on numerical problems for exploration by the student. These use software packages such as MATLAB®, tfc (a supplement to this text and described earlier), and SPICE. Of course, there are many other programs that can be substituted for these.

We believe that using such programs not only allows the student to more easily calculate numbers but reinforces the delineated properties of circuit behavior covered in the text. On the other hand, the principles and properties developed throughout the text allow the students to assess the reasonableness and accuracy of answers computed using a software program. Software programs are not infallible and it is always possible to construct an example which will cause a program to fail to produce meaningful numbers.

2. Text Exercises and Homework Problems

The text contains a wide variety of analysis, design, and computer-oriented problems. We have tried to provide a balance between problems emphasizing concepts and those emphasizing the mechanics of computing solutions. Design problems and software assisted problems are marked in the text with ◆ and ◆ respectively.

3. Chapter Openers/Real World Applications

The book includes a variety of real world applications that introduce and motivate almost every chapter in the book. Students immediately preview how circuits concepts underlie a wide variety of applications. Some of these are:

(a) Car Heater Fan Speed Control (Chapter 2)
(b) Digital-to-Analog Converter (Chapter 3)
(c) Protection Circuits Against Overvoltage and Polarity Reversal (Chapter 6)
(d) Capacitive Voltage Regulator (Chapter 7)
(e) Sawtooth Waveform Generation (Chapter 8)
(f) Microwave Oven (Chapter 9)
(g) Capacitive-bridge Pressure Sensor (Chapter 10)
(h) Fluorescent Light (Chapter 14)
(i) DC Motor (Chapter 15)
(j) Averaging by a Finite Time Integrator Circuit (Chapter 16)
(k) How a Touch Tone Phone Signals the Numbers Dialed (Chapter 17)
(l) Rectifier Circuit (Chapter 22)

Each of these openers is discussed in a simplified form either within a section of the associated chapter or as a chapter problem. In addition, we have included career boxes as an added feature so students will be better informed about career opportunities in the field.

4. In-depth Coverage

The traditional topics are covered in a more modern format, utilizing available software for problem solving where necessary. As mentioned in the Preface, we introduce a matrix-based approach to nodal and mesh analysis and include a special section on the Modified Nodal approach to circuit analysis. Teamed up with MATLAB® or its equivalent, the student has a powerful analysis tool not directly available to engineering students of past decades. (A MATLAB® supplement which expands upon the numerical implementation of the course material is available through Prentice Hall.) Op amp coverage is integrated throughout the text. Rigorous treatments of Thévenin and Norton theorems for active circuits are given. The Laplace transform is introduced after phasor analysis and utilized through the remainder of the text. This allows a unifying systems approach to impedance concepts, circuit transfer functions, convolutional approaches to circuit analysis, resonance, magnetic circuits, two ports and filtering. The software program, tfc, for use on a PC is available from Prentice Hall to enhance the Laplace transform analysis required in the text. The program computes roots, Laplace transforms, partial fraction expansions, and inverse Laplace transforms; it also has a provision for the manipulation of two-port parameters. Fourier series with applications to power supplies and distortion in an amplifier ends our treatment.

5. Review and Development of Text

Extensive review of both volumes and extensive class testing of Volume II.

6. Chapter Pedagogy

(a) Each chapter begins with an overview of chapter contents and ends with a summary and a glossary of terms and concepts used throughout the chapter.
(b) Throughout each chapter "key concept boxes" highlight important definitions, laws, and properties of circuit analysis and design.
(c) Many examples illustrate concepts and techniques. Each example is set off with its conclusion clearly marked.
(d) More advanced sections and problems are marked with an asterisk for students desiring more challenging material.

7. Presentation of Piecewise Linear Analysis

Presentation of piecewise linear analysis of diode and transistor amplifier circuits as a first step in approaching nonlinear circuits: This material is optional and may be omitted without any loss of continuity.

8. Software for Laplace Transform Analysis

Free software for Laplace transform and two-port analysis for IBM PC or equivalent is available from the publisher.

9. Instructor's Manual

An Instructor's Manual is available from the publisher.

10. MATLAB® Supplement

Jim Gottling of the Department of Electrical Engineering of The Ohio State University has put together a MATLAB® supplement that can be used in conjunction with the text.

II. HOW TO USE THIS BOOK

This is the only circuits book available in one comprehensive book and two separate volumes. The purpose is to give instructors and students some cost effective options.

1. Volume Selection

Volume I best fits a one semester course primarily for those not taking a second course with a price tag much lower in cost than competing two semester texts. Volume II is for those taking only the second course or for those using a different text for the first course. Alternately, one can choose a combined volume for those taking a two semester sequence or equivalent.

As mentioned earlier, most electrical engineering sophomores take two semesters of linear circuit analysis. Those in other engineering disciplines often take only one semester. The two-volume book contains an abundance of material that can be used in a variety of ways. An instructor can choose the proper ingredients for the intended course. Some possible plans are given in the following table:

	Course length	Intended Course Structure	Chapters used
1.	Two semesters (3 credit hours each)	Traditional linear circuit analysis	Semester 1 vol. 1, with ch. 6 and 12 omitted Semester 2 vol. 2, ch. 13 to 20, plus ch. 21 or ch. 22
2.	Three quarters (2 credit hours each)	Traditional linear circuit analysis	quarter 1 vol. 1, ch. 1, 2, 3, 4, 5, 7, 8 quarter 2 vol. 1, ch. 9, 10, 11, and vol. 2, ch. 13, 14, 16, 17 quarter 3 vol. 2, ch. 18 to 21
3.	One-semester (4 credit hours)	Introductory linear circuit analysis and Elementary electronic circuits	vol. 1, ch. 1 through 12
4.	One-semester (3 credit hours)	Introductory linear circuit analysis	vol. 1, with ch. 6 and 12 omitted
5.	One-semester (3 credit hours)	Laplace transform analysis of linear circuits	vol. 2, ch. 13 through 21

For the first semester of option 1 or for option 4, the lecture by lecture topics can be chosen as:

1	General circuit element, charge, current
2	Voltage, sources, power
3	Resistance, Ohm's Law, power reprise
4	Kirchhoff's Laws, single loop, node circuits
5	R combinations, v & i division
6	Op amp basics
7	Nodal analysis
8	Nodal analysis (cont.)
9	Mesh analysis
10	Superposition and linearity
11	Linearity (cont.) & source transformations
12	Review
13	**TEST #1**
14	Thévenin's and Norton's Theorems
15	Thévenin's and Norton's Theorems (cont.)
16	Inductance
17	Capacitance
18	L and C combinations, duality
19	First order undriven circuits: RL case and RC case; use of R_{th}; initial conditions from past dc excitations; sequential switching
20	First order undriven circuits (continued); recovery of stored energy; differential equation, and mathematical solution for first order circuits with constant inputs
21	RC emphasis; examples/applications
22	Unit step function; DC response of first order circuits; the 3-parameter formula; exponential decay and growth curves; inductive and capacitive circuit examples; piecewise constant inputs
23	Transient analysis of first order circuits (cont.); unit step, natural and forced response; solution to real exponential input; integral solution for arbitrary inputs
24	Switching & elapsed time calculation; negative time constant and stability
25	RC op-amp circuits
26	Differential equation models of parallel and series RLC circuits; derivation of solution for the undriven case
27	Review
28	**TEST #2**
29	Solution of 2nd order differential; equation; models for undriven circuits continued: overdamped, critically damped, and underdamped cases
30	Solution of 2nd order differential; equation; models for circuits driven by constant input excitations
31	Sinusoidal forcing function
32	Complex forcing function
33	Phasors: KVL & KCL with phasors
34	Impedance & admittance
35	Sinusoidal steady state (SSS) analysis using phasors; general case requiring (matrix) mesh and node equations
36	SSS phasor diagrams; examples of applications
37	Frequency response: magnitude and phase plots; lowpass and highpass examples
38	Instantaneous power absorbed by a general 2-terminal element and special cases of R, L, and C; average power over a specified time interval; average power when $p(t)$ is periodic; sinusoidal steady state average power
39	Maximum power transfer in SSS; adjustable impedance/resistive load; application in communication circuits

40	Review
41	**TEST #3**
42	Effective (rms) value; definition and examples of sine and triangular waves; counter-example to superposition of power; RMS value of response when all inputs are at different frequencies
43	Complex power, VA; reactive power, var conservation of Power
44	Balanced 3-phase circuits; economic consideration in power transmission
45	**FINAL EXAM**

For the second semester of option 1 or for option 5, the lecture-by-lecture topics can be chosen as:

1	Motivation for studying Laplace transform
2	Laplace transform, transforms of basic signals
3	Inverse transform, partial fraction expansion
4	Basic properties
5	Solution of linear DE's with initial conditions
6	Z(s),Y(s), series-parallel & other manipulations
7	Transfer function
8	Equivalent circuits for L and C with I.C.
9	Nodal and mesh analyses in s-domain
10	Switching in linear circuits
11	Switched capacitor circuits
12	H(s),poles, zeros, s-plane plot and stability
13	**Test #1**
14	Decomposition of the complete response
15	Sinusoidal steady state analysis
16	Frequency response from pole-zero plot
17	Impulse response, h(t); relation to H(s); additional Laplace properties and review
18	Convolution, definition, and integral evaluation; relationship to L.T.; time domain derivation
19	Circuit response calculation using convolution; graphical convolution; additional properties
20	Additional properties; convolution algebra;
21	SPICE, and tfc
22	Parallel resonance, calculation of Ωr
23	Parallel resonance, exact analysis
24	Other resonance forms; approximate analysis
25	Pole-zero approach to resonant circuit analysis
26	Frequency and amplitude scaling
27	Mutual inductance, the dot convention
28	Mutual inductance: s-domain and phasor equations
29	**Test #2**
30	Coefficient of coupling stored energy
31	Ideal transformer
32	Coupled inductors modeled with ideal transformer
33	One-port networks
34	y parameters
35	Equivalent circuit & analysis of terminated 2-port
36	z parameters
37	h parameters
38	t parameters; reciprocity; conversion table
39	2-port interconnections

40	**Test #3**
41	Indefinite admittance parameters
42	Lowpass Butterworth response; loss in dB; from $\lvert H(j\omega)\rvert$ to $H(s)$
43	Calculation of filter order and cut-off frequency; basic passive realization 2nd order filters
44	Basic active realization of 2nd order filters

For non-electrical engineering majors, spending three semester credit hours in linear circuit analysis, use Volume I with Chapters 6 and 12 omitted. For non-electrical engineering majors, spending four semester credit hours in linear circuit analysis and basic electronic circuits, use Volume I in its entirety.

Of course, many other arrangements are possible. The ingredients are there. An instructor can choose a proper combination for the intended course. For example, if Fourier Series analysis needs to be covered, then other topics (such as two-port interconnections) may be omitted.

2. Special Notes on Terminology

Although the Institute of Electrical and Electronic Engineers (IEEE) has a standard dictionary for electrical terms, the engineering literature does not always adhere to the standards. In one sense, students need to know the standard terminology and also the variations which is what they will encounter when they work in the real world. This variation also shows up in the book in some places. For example, we have used the following terms synonymously:

node	terminal
circuit	network
dependent source	controlled source

Although the Standard International unit for conductance is Siemens, the term "mho" (ohm spelled backward) is still widely used in engineering literature and manufacturers' data sheets. This book used the term mho and the symbol inverted Ω as the unit for conductance.

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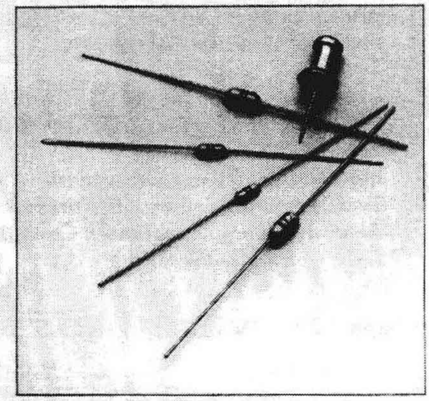
Dedication

To our wives
Chris and Louise,
and to
Frank and Adele DeCarlo

CHAPTER 1



CHAPTER OUTLINE

CONTENTS

Note: **This book is also available in individual volumes entitled, *Linear Circuit Analysis: Time Domain and Phasor Approach Volume One (Chapter 1–12 and Appendices A1–A3)* and *Linear Circuit Analysis: A Laplace Transform Approach Volume Two (Chapters 13–22 and Appendices B1–B3)***

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