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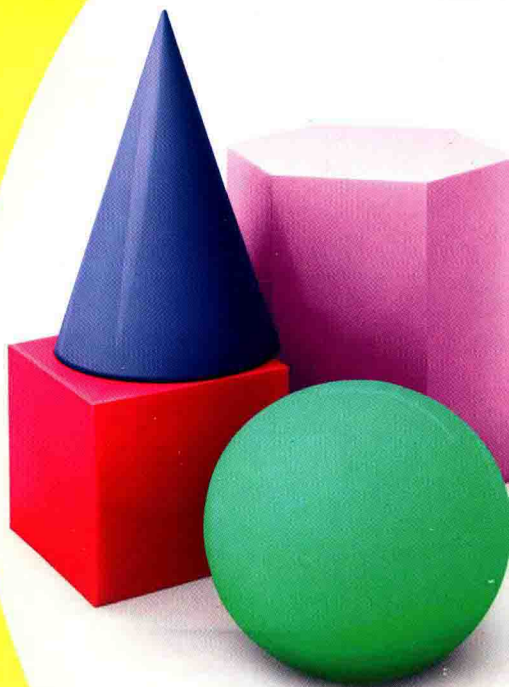
- Core concepts about the geometry of shapes and geometry proofs

cal theorems, postulates, definitions

principles and formulas need to know

**Mark Ryan**

*Author of Calculus For Dummies*



# *Geometry Essentials* FOR DUMMIES®

by Mark Ryan



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## Geometry Essentials For Dummies®

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## About the Author

A graduate of Brown University and the University of Wisconsin Law School, **Mark Ryan** has been teaching math since 1989. He runs The Math Center ([www.themathcenter.com](http://www.themathcenter.com)) in Winnetka, Illinois, where he teaches high school math courses, including an introduction to geometry and a workshop for parents based on a program he developed, *The 10 Habits of Highly Successful Math Students*. In high school, he twice scored a perfect 800 on the math portion of the SAT, and he not only knows mathematics but also has a gift for explaining it in plain English. He practiced law for four years before deciding he should do something he enjoys and use his natural talent for mathematics. Ryan is a member of the Authors Guild and the National Council of Teachers of Mathematics.

*Geometry Essentials For Dummies* is Ryan's sixth book. *Everyday Math for Everyday Life* (Grand Central Publishing) was published in 2002; *Calculus For Dummies* (Wiley), in 2003; *Calculus Workbook For Dummies* (Wiley), in 2005; *Geometry Workbook For Dummies* (Wiley), in 2006; and *Geometry For Dummies*, 2nd Edition (Wiley) in 2008. His math books have sold over a quarter of a million copies.

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We're proud of this book; please send us your comments at <http://dummies.custhelp.com>. For other comments, please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993, or fax 317-572-4002.

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# Introduction

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**G**eometry is a subject full of mathematical richness and beauty. The ancient Greeks were into it big time, and it's been a mainstay in secondary education for centuries. Today, no education is complete without at least some familiarity with the fundamental principles of geometry.

But geometry is also a subject that bewilders many students because it's so unlike the math that they've done before. Geometry requires you to use deductive logic in formal proofs. This process involves a special type of verbal and mathematical reasoning that's new to many students. The subject also involves working with two- and three-dimensional shapes. The spatial reasoning required for this is another thing that makes geometry different and challenging.

*Geometry Essentials For Dummies* can be a big help to you if you've hit the geometry wall. Or if you're a first-time student of geometry, it can prevent you from hitting the wall in the first place. When the world of geometry opens up to you and things start to click, you may come to really appreciate this topic, which has fascinated people for millennia.

## About This Book

*Geometry Essentials For Dummies* covers all the principles and formulas you need to analyze two- and three-dimensional shapes, and it gives you the skills and strategies you need to write geometry proofs.

My approach throughout is to explain geometry in plain English with a minimum of technical jargon. Plain English suffices for geometry because its principles, for the most part, are accessible with your common sense. I see no reason to obscure geometry concepts behind a lot of fancy-pants mathematical mumbo-jumbo. I prefer a street-smart approach.

This book, like all *For Dummies* books, is a reference, not a tutorial. The basic idea is that the chapters stand on their own as much as possible. So you don't have to read this book cover to cover — although, of course, you might want to.

## Conventions Used in This Book

*Geometry Essentials For Dummies* follows certain conventions that keep the text consistent:

- ✓ Variables and names of points are in *italics*.
- ✓ Important math terms are often in *italics* and are defined when necessary. Italics are also sometimes used for emphasis.
- ✓ Important terms may be **bolded** when they appear as keywords within a bulleted list. I also use bold for the instructions in many-step processes.
- ✓ As in most geometry books, figures are not necessarily drawn to scale — though most of them are.

## Foolish Assumptions

As I wrote this book, here's what I assumed about you:

- ✓ You're a high school student (or perhaps a junior high student) currently taking a standard high school-level geometry course, or . . .
- ✓ You're a parent of a geometry student, and you'd like to understand the fundamentals of geometry so you can help your child do his or her homework and prepare for quizzes and tests, or . . .
- ✓ You're anyone who wants to refresh your recollection of the geometry you studied years ago or wants to explore geometry for the first time.
- ✓ You remember some basic algebra. The good news is that you need very little algebra for doing geometry — but you do need some. In the problems that do involve algebra, I try to lay out all the solutions step by step.

## Icons Used in This Book



Next to this icon are definitions of geometry terms, explanations of geometry principles, and a few other things you should remember as you work through the book.



This icon highlights shortcuts, memory devices, strategies, and so on.



Ignore these icons, and you may end up doing lots of extra work or getting the wrong answer or both. Read carefully when you see the bomb with the burning fuse!



This icon identifies the theorems and postulates — little mathematical truths — that you use to form the logical arguments in geometry proofs.

## Where to Go from Here

If you're a geometry beginner, you should probably start with Chapter 1 and work your way through the book in order, but if you already know a fair amount of the subject, feel free to skip around. For instance, if you need to know about quadrilaterals, check out Chapter 6. Or if you already have a good handle on geometry proof basics, you may want to dive into the more advanced proofs in Chapter 5.

And from there, naturally, you can go

- ✓ To the head of the class
- ✓ To Go to collect \$200
- ✓ To chill out
- ✓ To explore strange new worlds, to seek out new life and new civilizations, to boldly go where no man (or woman) has gone before.

If you're still reading this, what are you waiting for? Go take your first steps into the wonderful world of geometry!

# The 5<sup>th</sup> Wave

By Rich Tennant



"I hear you think you got all the angles figured. Well, maybe you do and maybe you don't. Maybe the ratios of the lengths of corresponding sides of an equiangular right-angled triangle are equal, then again, maybe they're not - let's see your equations."



# Chapter 1

# An Overview of Geometry

## *In This Chapter*

- ▶ Surveying the geometric landscape: Shapes and proofs
- ▶ Understanding points, lines, rays, segments, angles, and planes
- ▶ Cutting segments and angles in two or three congruent pieces

**S**tudying geometry is sort of a Dr. Jekyll-and-Mr. Hyde thing. You have the ordinary geometry of shapes (the Dr. Jekyll part) and the strange world of geometry proofs (the Mr. Hyde part).

Every day, you see various shapes all around you (triangles, rectangles, boxes, circles, balls, and so on), and you're probably already familiar with some of their properties: area, perimeter, and volume, for example. In this book, you discover much more about these basic properties and then explore more advanced geometric ideas about shapes.

Geometry proofs are an entirely different sort of animal. They involve shapes, but instead of doing something straightforward like calculating the area of a shape, you have to come up with a mathematical argument that proves something about a shape. This process requires not only mathematical skills but verbal skills and logical deduction skills as well, and for this reason, proofs trip up many, many students. If you're one of these people and have already started singing the geometry-proof blues, you might even describe proofs — like Mr. Hyde — as monstrous. But I'm confident that, with the help of this book, you'll have no trouble taming them.