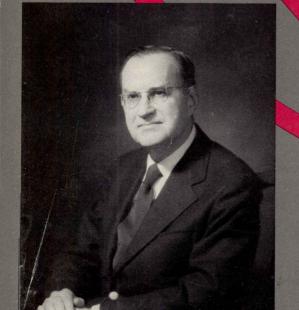# Welfare, Planning, and Employment

Selected Essays in Economic Theory

Abram Bergson

WELFARE, PLANNING, AND EMPLOYMENT
Selected Essays in Economic Theory

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PREFACE

Written over a period of more than four decades, the essays in this volume cannot really be said to have a single, common theme, but all fall in the realm of economic theory, and the various topics touched on are often closely related to each other. Thus, an initial group of writings, dealing with the concept of social welfare and the conditions for an economic optimum, falls squarely in the field of welfare economics. Essentially that is also true of the group that follows, comprised of essays relating to problems of measuring welfare and efficiency. A third category of writings focuses on rather different themes, socialist economics and public enterprise, but all these essays could be viewed as representing applications of welfare economics. The final group of essays on income and employment theory, however, is clearly positive rather than normative in character and so represents something of a break with the balance of the writings included.

The essays are said to have been selected. In fact, I include all but a few minor items among the writings in theory that I have published since the appearance, in 1966, of my *Essays in Normative Economics*. On the other hand, given the technique of reproduction that is used in the present volume, the marginal cost of incorporating additional materials, I understand, is relatively modest. In the circumstances, there seemed to be a case to include here three widely cited articles that were reprinted in the 1966 collection but are closely related to those published more recently. I refer to essays 1, 4, and 9. I have also taken the opportunity to reproduce now two early pieces—essays 14 and 15—that were not included in the 1966 collection. Although these two articles have received only modest attention since they were published, I allow myself to imagine that that may have been due to some extent to their appearance during World War II and the resultant professional absorption in more applied pursuits. The articles may retain interest as an early attempt to grapple with what has lately come to be a major concern in economic theory: integration of macro- and microeconomics.

Abram Bergson

Cambridge, Massachusetts
July 1981

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SUMMARY

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The object of the present paper is to state in a precise form the value judgments required for the derivation of the conditions of maximum economic welfare which have been advanced in the studies of the Cambridge economists,[2] Pareto and Barone, and Mr. Lerner.[3] Such a formulation, I hope, will clarify certain aspects of the contribution of these writers, and at the same time provide a basis for a more proper understanding of the principles of welfare.

I shall develop my analysis under a set of assumptions which in certain respects differ from those introduced in the welfare studies. It will be assumed throughout the discussion that the amounts of all the factors of production, other than labor, are fixed and, for convenience, non-depreciating. While a variable capital supply is included in some of the

1. I am very grateful to Mr. Paul Samuelson for suggestions on many points.
2. I use this caption to designate those economists whose names are directly attached to the Cambridge School — Marshall, Professor Pigou, Mr. Kahn — as well as others, such as Edgeworth, whose welfare analysis is in all essentials the same as that of the Cambridge group. But in the course of my discussion I shall refer mainly to the studies of the first group of economists. This will ease my task considerably, and, I believe, will involve no loss of generality.
3. The studies referred to are Marshall, Principles (all references to the Third — 1895 — Edition); Pigou, Economics of Welfare (all references to the Fourth — 1932 — Edition); Kahn, Economic Journal, March, 1935; Pareto, Cours d'Economie Politique (all references to the Lausanne — 1897 — Edition); Barone, The Ministry of Production in a Socialist State (Translated from the Italian article of the same title in Giornale degli Economisti, 1908; the translation appearing in Hayek, Collectivist Economic Planning); and Lerner, Review of Economic Studies, June and October, 1934.

welfare studies, this is not a well developed part of the analysis, and for our present purposes it will be desirable to confine to the simpler case the discussion of the evaluations required.[4] I shall assume, also, that the variables involved in the analysis — the amounts of the various commodities consumed and services performed — are infinitesimally divisible. This assumption will be interpreted more strictly than is usually done. Otherwise it is the postulate of the welfare writers, and its introduction here will involve no significant departure from their analysis. Finally, I shall assume that there are only two kinds of consumers' goods, two kinds of labor, and two factors of production other than labor in the community, and that each commodity is produced, with labor and the other factors, in a single production unit. This assumption is introduced only to simplify the notation employed. The discussion will apply, with no modification, to the many commodity, many factor, and many production unit case.[5]

I

Among the elements affecting the wefare of the community during any given period of time are the amounts of each of the factors of production, other than labor, employed in the different production units, the amounts of the various commodities consumed, the amounts of the different kinds of work done, and the production unit for which this work is performed by each individual in the community during that period of time. If we use A and B to denote the two kinds of labor; C and D to denote the two factors

4. On a simple model, similar to that of Barone, the analysis may be extended to the case of a variable capital supply.

5. The assumption that each commodity is produced in one production unit, it is true, excludes an element of "external economies" from the analysis. But in the present essay I am interested only in the maximum conditions for the community's welfare, and not in the departures from the maximum under a given institutional set-up. To the extent that, in the many production unit case, there are external economies, these will require no modification in the maximum conditions I shall present, for these conditions relate only to marginal *social* value productivities.

of production other than labor; and X and Y to denote the two consumers' goods; we may express this relationship in the form

(1.1) $W = W(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y, \cdots,$

$\qquad x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y, C^x, D^x, C^y, D^y, r, s, t, \cdots).$

Here C^x and D^x are the amounts of the non-labor factors of production C and D employed in the production unit producing the consumers' good X; C^y and D^y are the amounts of these factors employed in the production unit producing the consumers' good Y; x_i and y_i are the amounts of X and Y consumed by the ith individual; and a_i^x, b_i^x, a_i^y, and b_i^y are the amounts of each kind of work performed by him for each production unit during the given period of time.[6] The symbols $r, s, t, \cdots$, denote elements other than the amounts of commodities, the amounts of work of each type, and the amounts of the non-labor factors in each of the production units, affecting the welfare of the community.

Some of the elements $r, s, t, \cdots$, may affect welfare, not only directly, but indirectly through their effect on (say) the amounts of X and Y produced with any given amount of resources, e.g., the effects of a change in the weather. On the other hand, it is conceivable that variations in the amounts of commodities, the amounts of work of each type, and the amounts of non-labor factors in each of the production units also will have a direct and indirect effect on welfare; e.g., a sufficient diminution of x_i and y_i may be accompanied by an overturn of the government. But for relatively small changes in these variables, other elements in welfare, I believe, will not be significantly affected. To the extent that this is so a partial analysis is feasible.

I shall designate the function,

(1.2) $E = E(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y, \cdots,$

$\qquad x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y, C^x, D^x, C^y, D^y),$

which is obtained by taking $r, s, t, \cdots$, in (1.1) as given, the Economic Welfare Function.[7]

6. I am assuming that an individual's labor time may be divided among the different types of work in any desired proportions.

7. It should be emphasized that in (1.2) other factors affecting wel-

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Let us write the amounts of X and Y produced respectively by the X and Y production units as functions,

(1.3) $\quad X = X(A^x, B^x, C^x, D^x); \quad Y = Y(A^y, B^y, C^y, D^y),$

where A^x and B^x are the amounts of the two kinds of labor and C^x and D^x are the amounts of the other two factors of production employed in the X production unit; and A^y, B^y, C^y, D^y are defined similarly for the Y production unit.

If we assume that E varies continuously with $x_1, y_1, \cdots$, we may write as a general condition for a position of maximum economic welfare that, subject to the limitations of the given technique of production and the given amounts of resources,

(1.4) $\qquad\qquad\qquad dE = 0.$

Equation (1.4) requires that in the neighborhood of the maximum position any small adjustment will leave the welfare of the community unchanged. By use of (1.3) and (1.4) it is possible immediately to state in general terms the conditions for a maximum welfare.[8]

One group of maximum conditions relates to the consumption and supply of services by each individual in the community. They require that the marginal economic welfare of each commodity and the marginal economic diswelfare of each type of work be the same with respect to each individual in the community.[9] If we denote the marginal economic welfare of commodity X with respect to the

i^{th} individual, $\dfrac{\partial E}{\partial x_i}$, and of Y, $\dfrac{\partial E}{\partial y_i}$, the first group of these con-

fare are taken as given. I do *not* assume that economic welfare is an independent element which may be added to other welfare to get total welfare.

8. The conditions I shall develop in this section are a group of necessary conditions for a maximum. They are also the conditions for any critical point, and are sufficient in number to determine the location of such a point (or points) if there is one. In section IV below I shall consider the problem of determining whether a given critical point is a maximum or not.

9. This rather awkward terminology is adopted instead of, say, the phrase marginal economic welfare *of* the ith individual in order to include the possibility that an increment of X or Y given to the ith individual will affect the welfare of others.

ditions requires that, for all i, and for some p, q, and ω,

$$(1.5) \qquad \frac{\partial E}{\partial x_i} = \omega p$$

and

$$(1.6) \qquad \frac{\partial E}{\partial y_i} = \omega q.$$

Similarly if we denote the marginal economic diswelfare of the various types of work with respect to the ith individual $\dfrac{\partial E}{\partial a_i^x}, \dfrac{\partial E}{\partial b_i^x}, \dfrac{\partial E}{\partial a_i^y}, \dfrac{\partial E}{\partial b_i^y}$, the second group of these conditions requires that, for all i and for some g^x, h^x, g^y, h^y, and for the ω already chosen,

$$(1.7) \quad -\frac{\partial E}{\partial a_i^x} = \omega g^x, \qquad\qquad (1.8) \quad -\frac{\partial E}{\partial b_i^x} = \omega h^x,$$

$$(1.9) \quad -\frac{\partial E}{\partial a_i^y} = \omega g^y, \qquad\qquad (1.10) \quad -\frac{\partial E}{\partial b_i^y} = \omega h^y.$$

The minus signs and the multiplicative factor ω are inserted in these equations for convenience.

The remaining maximum conditions relate to production. They require that the economic welfare of the consumers' goods produced by a marginal increment of each type of work should equal the negative of the diswelfare of that increment of work, and that the increment of economic welfare due to the shift of a marginal unit of factors C and D from one production unit to another should equal the negative of the diswelfare caused by this adjustment. Using the notation $\dfrac{\partial X}{\partial A^x}$ for the marginal productivity of A^x, and a similar notation for the other marginal productivities, we may write these conditions in the form,

$$(1.11) \quad p\,\frac{\partial X}{\partial A^x} = g^x, \qquad\qquad (1.12) \quad p\,\frac{\partial X}{\partial B^x} = h^x,$$

$$(1.13) \quad q\,\frac{\partial Y}{\partial A^y} = g^y, \qquad\qquad (1.14) \quad q\,\frac{\partial Y}{\partial B^y} = h^y,$$

and,[1]

$$(1.15) \quad \omega\left(p\frac{\partial X}{\partial C^x} - q\frac{\partial Y}{\partial C^y}\right) = -\left(\frac{\partial E}{\partial C^x} - \frac{\partial E}{\partial C^y}\right),$$

$$(1.16) \quad \omega\left(p\frac{\partial X}{\partial D^x} - q\frac{\partial Y}{\partial D^y}\right) = -\left(\frac{\partial E}{\partial D^x} - \frac{\partial E}{\partial D^y}\right).$$

In equations (1.11) through (1.14), ω, which was present in all terms, has been divided out.[2]

It will be convenient to designate p the *price* of X, q the *price* of Y, and g^x, g^y, h^x, h^y, the *wage* of the types of work A^x, A^y, B^x, B^y. Equations (1.5) and (1.6) thus require that the marginal economic welfare per "dollar's worth" of each

commodity, $\dfrac{\partial E}{\partial x_i} \cdot \dfrac{1}{p}$ and $\dfrac{\partial E}{\partial y_i} \cdot \dfrac{1}{q}$, be the same for each com-

modity and for all individuals in the community. Similarly equations (1.7) through (1.10) require that the marginal economic diswelfare per "dollar's worth" of each kind of work be the same with respect to each kind of work and each individual in the community; equations (1.11) through (1.14) require that the wages of each type of labor should equal the marginal value productivity of that type of labor;[3] and with an analogous interpretation, equations (1.15) and (1.16) require that the marginal value productivity equal the cost due to a shift in C or D from one use to another.

II

The maximum conditions presented in section I are the general conditions for a position of maximum economic

1. The derivatives on the right hand sides of (1.15) and (1.16) indicate the effect on welfare of an adjustment in C or D for which all other elements — x^i, y^i, etc.— in welfare are constant. Such an effect would arise, for example, through a positive or negative evaluation of the relative amounts and kinds of "factory smoke" emitted in the two production units for varying amounts of one or the other factors employed in each unit.

2. Strictly speaking this procedure assumes a value proposition, which we shall introduce later, to the effect that ω is unequal to zero.

3. In the present essay it will be understood that all value productivities are *social* value productivities. Compare footnote 5, p. 311, supra.

welfare for any Economic Welfare Function. The maximum conditions presented in the welfare studies relate to a particular family of welfare functions. Their derivation thus requires the introduction of restrictions on the shape of the Economic Welfare Function I have presented. Three groups of value propositions suffice for this purpose.

I shall designate the various maximum conditions derived by the names of those writers, or groups of writers, who have been especially responsible for their elucidation. For reasons which will appear I have altered somewhat the content of the conditions, and there are differences in the analyses of the various writers which must also be noted. The latter differences will be pointed out in this section and in the one following.

THE LERNER CONDITIONS

The First Group of Value Propositions: *a shift in a unit of any factor of production, other than labor, from one production unit to another would leave economic welfare unchanged, provided the amounts of all the other elements in welfare were constant.*

The First Group of Value Propositions enables us to state certain of the maximum conditions in terms of the production functions alone. From these evaluations the right hand side of (1.15) and of (1.16) must equal zero.[4] The two equations thus may be written,

$$(2.1) \qquad p\frac{\partial X}{\partial C^x} = q\frac{\partial Y}{\partial C^y},$$

$$(2.2) \qquad p\frac{\partial X}{\partial D^x} = q\frac{\partial Y}{\partial D^y},$$

and they now impose the condition that the marginal value productivity of factors other than labor be the same in every use.

Equations (2.1) and (2.2) still contain the variables p

4. The net effect on the community's welfare of the "factory smoke" arising from a shift of the non-labor factors from one use to another is zero. (Cf. footnote 1, p. 315.)

and q, which involve derivatives of the Economic Welfare Function. If we combine (2.1) and (2.2), however, we have two equations,

$$(2.3) \qquad \frac{q}{p} = \frac{\partial X}{\partial C^x} \bigg/ \frac{\partial Y}{\partial C^y} = \frac{\partial X}{\partial D^x} \bigg/ \frac{\partial Y}{\partial D^y} \,,$$

the second of which involves only the derivatives of the production functions. It requires that in the maximum position the ratio of the marginal productivity of a factor in one use to its marginal productivity in any other use be the same for all factors of production, other than labor. The first equation of (2.3) requires that all these ratios equal the price ratio.

The significance of (2.3) for the determination of maximum welfare may be expressed in the following manner: whatever the relative evaluations of commodity X and commodity Y, that is, in Barone's terminology, whatever their ratio of equivalence, (2.3) requires that in the maximum position given that one factor C is so distributed that a small shift from one production unit to another would alter the amounts of X and Y in such a manner as to leave welfare unchanged, i.e., given that C is so distributed that $\dfrac{\partial X}{\partial C^x} \bigg/ \dfrac{\partial Y}{\partial C^y}$ equals the ratio of equivalence of the two commodities, then the other factors in order to be so distributed must have a ratio of marginal productivities equal to $\dfrac{\partial X}{\partial C^x} \bigg/ \dfrac{\partial Y}{\partial C^y}.$

The condition (2.3) can be interpreted in another manner, which however does not bring out as directly the significance of the condition for a position of maximum *welfare*. The equality of the marginal productivity ratios implies that there is no possible further adjustment for which the amount of one commodity will be increased without that of another being reduced. A shift in one factor from X to Y can at best be just compensated by a shift of another from Y to X, if (2.3) is satisfied.[5]

5. Mr. Lerner, as far as I am aware, is the only economist to present

THE PARETO-BARONE-CAMBRIDGE CONDITIONS

The Fundamental Value Propositions of Individual Preference: *if the amounts of the various commodities and types of work were constant for all individuals in the community except any i^{th} individual, and if the ith individual consumed the various commodities and performed the various types of work in combinations which were indifferent to him, economic welfare would be constant.*

The First Group of Value Propositions implies that under the assumption that the amounts of the factors of production other than labor are constant, the Economic Welfare Function may be written as

$$(2.4) \quad E = E(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y, \cdots, \\ x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y).$$

For from these propositions a shift in C or D from one production unit to another would have no effect on welfare, if all the other elements were constant. The Fundamental Value Propositions require that E be some function of the form,

$$(2.5) \quad E = E[S^1(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y), \cdots, \\ S^n(x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y)],$$

where the function,

$$(2.6) \qquad S^i = S^i(x_i, y_i, a_i^x, b_i^x, a_i^y, b_i^y),$$

and interpret (2.1) and (2.2) in the form of (2.3), his interpretation being the second of the two alternatives I have noted. In the studies of Pareto, Barone, and Marshall the conditions (2.1) and (2.2) are presented with the price ratios already equated to the individual marginal rates of substitution (cf. infra). In the studies of Professor Pigou and Mr. Kahn the procedure is the same as that of Pareto, Barone, and Marshall except that these two writers include in their analysis the possibility of departures from (2.1) and (2.2) due to such effects as are discussed above in footnote 1, p. 315.

Mr. Lerner advances the conditions (2.3) for all factors of production, labor as well as non-labor (Review of Economic Studies, October, 1934, p. 57). On the face of the matter this formulation is inconsistent with Mr. Lerner's own advocacy of the supremacy of individual tastes in the sphere of consumption, and I have therefore taken the liberty to modify his conditions accordingly. The other economists also do not allow for individual preferences as between production units in their analysis.