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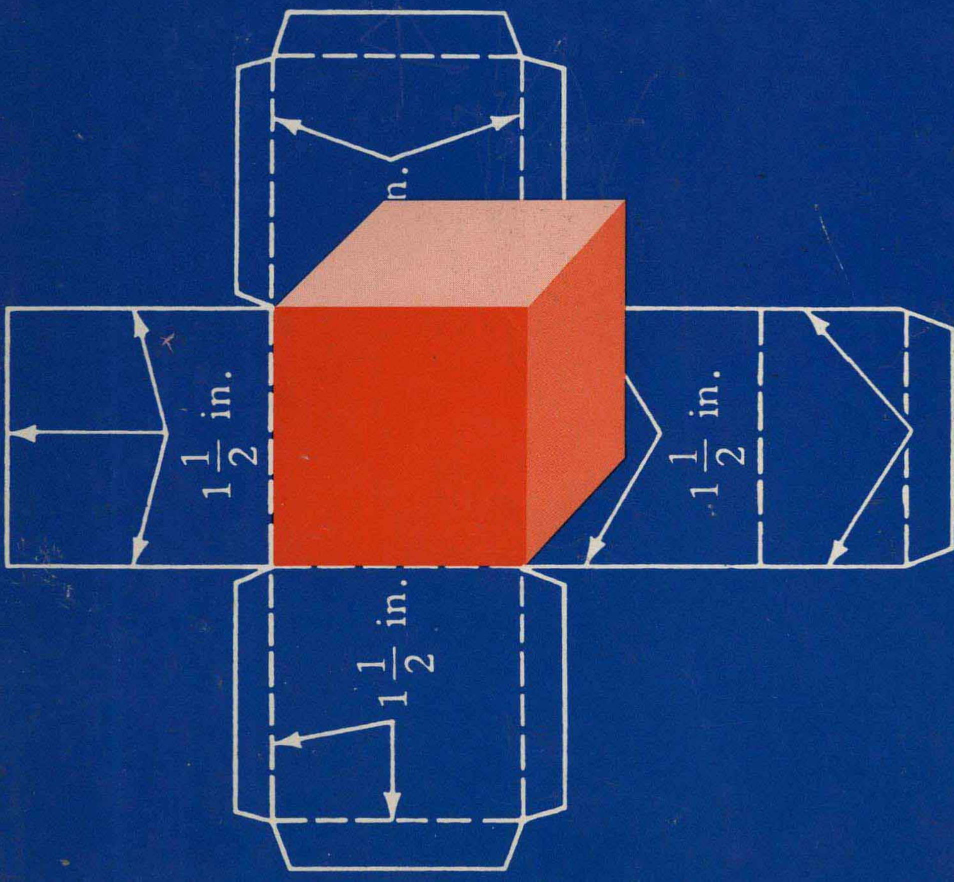
Geometry for Teachers



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GEOMETRY FOR TEACHERS

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Geometry
for
Teachers

To our families

PREFACE TO THE INSTRUCTOR

THE INTUITIVE APPROACH

Geometry for Teachers is an intentionally intuitive approach to the discipline, based on the conviction that students must develop geometric intuition before any true appreciation of a rigorous development can be attained. Our intended audience is prospective teachers of mathematics, both in elementary and middle schools. Our goal is to foster among these undergraduates a more mature understanding of geometric concepts and their interrelationships. We wish to both heighten their appreciation of mathematics in general and improve their understanding of geometric content.

Geometry for Teachers is based on material we used at Southern Connecticut State College over a four-year period. The approach embodied in this text was developed because we felt that schoolteachers' understanding of geometry was usually based on misconceptions arrived at when they were high school sophomores. The motivation for this text also reflects the fact that our discipline is seeing a reaffirmation of the need to stress intuition and analysis in the study of all mathematics. An examination of some of the newer elementary school mathematics series shows that geometry is being allotted an integral and important place in the curriculum.

Usually any book intended for prospective schoolteachers begins either with a study of sets or logic. Mercifully (for the student, at any rate), *we have decided to do neither.* We find that the majority of students entering college these days have already been exposed, at various points in their education, to concepts of set theory quite sufficient for our purposes. Perhaps the reader will find this omission at the beginning of the study a gesture of good faith.

As for logic, its inclusion at the beginning of a mathematical study may seem to be mathematically correct, *but we question its pedagogical correctness.* Just as we wish to build understanding and appreciation for proof throughout this book, we wish to polish the logical development *as we proceed.* This seems much more appropriate to natural development. Indeed, we find that our own students are not necessarily illogical, merely disorganized.

More specific special features of *Geometry for Teachers* are listed below.

1. Material is developed in a spiral fashion—key geometrical concepts are encountered in various places throughout the text. Each new encounter tends to extend and broaden the students' understanding of the concept by progressing from the concrete to the abstract.
2. Each chapter begins with a prologue, the purpose of which is not only to be informative and motivational, but also to remind students of precisely what point they have reached in the spiral, that is, what they have already learned about a particular concept and the direction they will head in their further development of the concept.
3. All chapters include one or more experiments. These are included to provide students with new and relevant concrete experiences with acquired concepts by identifying problems and questions that need to be solved. An atmosphere is created in which the readers are encouraged to discover solutions to these problems. Some of the experiments have predictable outcomes; others are open-ended, allowing different students to draw different conclusions. Some of the experiments serve as models for experiments that teachers can use in a mathematics laboratory. All have been selected with an eye toward setting a good example for our prospective teachers.
4. Congruence and similarity are treated through a transformation approach. It appears that transformation geometry is an idea whose time has finally arrived. The approach is not only more "modern," but it also seems more mathematically and pedagogically satisfactory.
5. Coordinate geometry is developed in three chapters, each separated by chapters that are largely synthetic in content and development. Our decision to depart from the usual practice of developing coordinate geometry in its entirety in a single section of the book is based on our firm conviction that a truly modern approach to geometry should be a blend of synthetic *and* analytic ideas. This approach also seems more consistent with the concept of spiral development.
6. The body of each chapter is mostly informative—developing answers to questions that have been raised in the prologue, establishing the need for good definitions, and providing procedures for problem solving. The chapter summary, of course, provides students with a quick review. The exercises,

found at the end of each section, not only pose questions that test students' understanding of the important ideas, but are designed to provide motivation for and transition into the next section.

7. Throughout the book, inductive reasoning is stressed. We find that students are more successful with deductive reasoning if *they* have played some role in the formulation of the relationships to be proven. The difficulty of the material is meant to increase slowly; the earlier chapters are fairly elementary, whereas later chapters include considerably more sophisticated ideas.
8. A key element of the spiral development is that *formal* deductive proof is not required until Chapter 15. Students are encouraged from the first page onward to provide explanations, justifications, and convincing arguments to support certain conclusions. As students progress through the text, their ability to *communicate* such arguments is steadily cultivated until it culminates with the treatment of formal proof in Chapter 15.
9. Throughout the manuscript, we have included certain asides entitled "In our opinion" and "Comment." The former include comments of a pedagogical and philosophical nature that we have found useful to share with our students. Frequently, these opinions have inspired interesting and, we believe, productive discussions. Their purpose is to help us reach our goal of affecting students' attitudes toward mathematics itself. The latter type of aside consists of short observations that should aid students in understanding and appreciating the topic under discussion.

Although this text was designed for use in a two-semester course, it is possible to leave out certain sections and teach a logically sound single semester or two-quarter course while retaining the features listed above. This can be done by omitting substantial portions that relate to coordinate and solid geometry. More specifically, a single semester course can be obtained by following this sequence of sections: 1.1–2.4, 2.6–4.3, 4.7–5.1, 5.3, 6.1–6.2, 6.4–8.5, 9.1–9.4, 10.2–10.3, 13.1–13.6, 15.1–15.8, 15.9.

We wish to express special gratitude to our typist, Mrs. Maryann Franco; to our colleagues Dorothy Schrader, Helen Bass, Leo Kuczynski, Bert Latil, Michael Meck, and Philip Smith for their encouragement and criticism of the manuscript; to our students at Southern Connecticut State College, particularly Cecilia Lowe, Mary Dereshiwsky, and Patricia Consoli; and to the editors and staff of Harper & Row for their many hours of assistance. We extend particular thanks to our families, whose faith in our ability to write this book and unfailing encouragement has allowed us to complete this project.

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PREFACE TO THE STUDENT

AN OPPORTUNITY TO THINK DEEPLY ABOUT SIMPLE, FAMILIAR THINGS

This book was written so that you might teach yourself geometry in the most rewarding and, we hope, enjoyable way. There are various levels of abstraction at which geometry can be appreciated and we expect that you have already experienced several in your previous mathematical studies. Our purpose is not to completely repeat any of these previous approaches to geometry. We do hope to assist you in developing an intelligent understanding of geometric concepts, a more sophisticated appreciation for the organization of geometry, and a feeling for the *priorities* of the geometric ideas that you may eventually teach.

For many students, the study of geometry is made difficult by teaching strategies that foster inadequate understanding. Part of the fault is traceable to the fact that, although the "how" and even the "why" of geometry are stressed, too little emphasis is placed on helping students develop a *proper perspective.* Often questions are answered before students have even thought to raise them in the first place! Logical developments are given without sufficient *intuitive* background. Proof is stressed when there is no understanding or appreciation of exactly what constitutes a "proof."

We do not pretend to have found definitive solutions to these thorny problems. *Geometry for Teachers* has, however, been written *with them in mind*. Our intention is to give you an opportunity to think deeply about simple, familiar things, looking at geometry from a *natural* rather than a completely rigorous point of view. We do *not* wish to merely inform you, nor to make you proficient at imitation, but rather to help develop your critical and analytical senses.

Whether we succeed or not will depend mostly on you. If you are sympathetic to our goals, the material and its presentation may prove useful to you not only as a student of mathematics but eventually as a teacher of mathematics.

Throughout, it will be assumed that you have at least an intuitive familiarity with many of the concepts. Efforts will be made to *refine* this intuitive understanding by identifying subtleties, developing relationships, and determining relative significance of topics. Real-world analogs will be used to both motivate and illustrate the geometric ideas. Many terms and concepts will be used informally long before they are completely and rigorously defined.

For instance, everyone knows what a triangle or a circle is, or at least has an understanding sufficient for most purposes. We make use of this understanding to make conjectures and draw conclusions about related concepts. These terms are defined and described in more detail when the intuitive understanding later becomes insufficient for our purposes.

In a similar manner, we make use of the very familiar Pythagorean Theorem in advance of its proof because this is *convenient*. You may recall that the theorem states:

For any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs of the triangle.

You can teach yourself to utilize the results of this theorem long before you need to completely understand its meaning within a postulational system. Our approach will be to point out the various times that we can benefit from this most valuable theorem, and to "prove" the theorem only when it suits our purposes to do so later in the book. Although the concept of proof is not formally treated until Chapter 15, you are constantly encouraged to analyze, verify, justify, and explain why certain mathematical ideas are valid. The nature of validity is constantly discussed as you make conjectures and examine arguments made to support these conjectures.

Our purpose in avoiding a formal treatment of proof until late in the book derives from our conviction that it is not possible to teach the techniques, nature, and appreciation for proof by merely including a chapter on proof. Furthermore, we believe that a premature discussion of proof causes students to be *misdirected* into believing that the form and style of a proof are more important than the convincing argument that they are to communicate. Thus, the first 14 chapters lay the foundations for the chapter on proof, attempting to develop your ability *to create and communicate convincing arguments.* In numerous experiments and exercises, you are requested to make conjectures based on examination of data and observations. Further, you are asked to supply arguments that convince you of the validity of your conjectures.

Your own part in this course should be characterized by a desire to gain a more complete and mature understanding of mathematics. This will make you a better, more effective teacher. We encourage you to question, to experiment, and to ponder. Teacher, teach thyself.

R.A.N.
R.M.W.

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