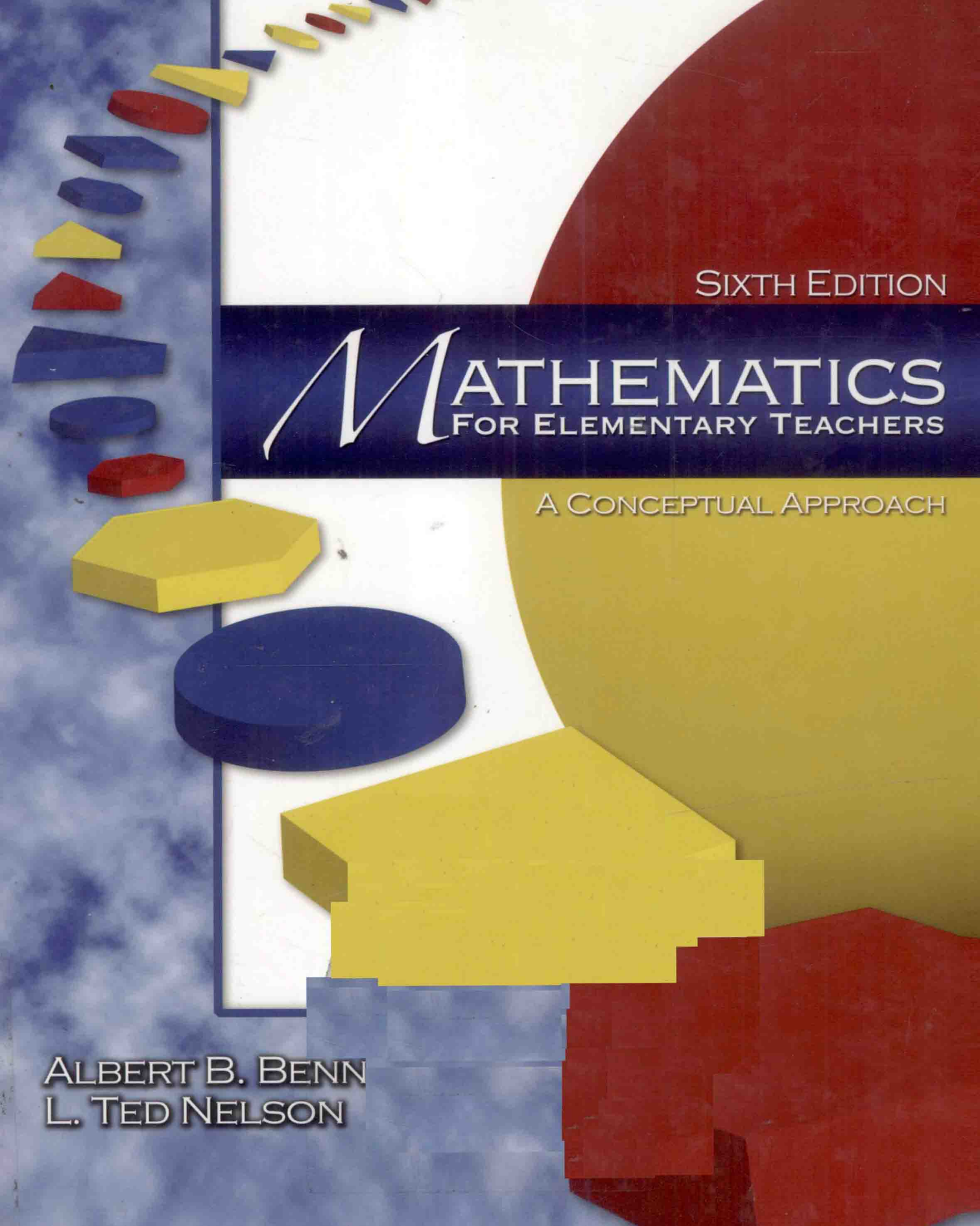
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FOR ELEMENTARY TEACHERS
A CONCEPTUAL APPROACH
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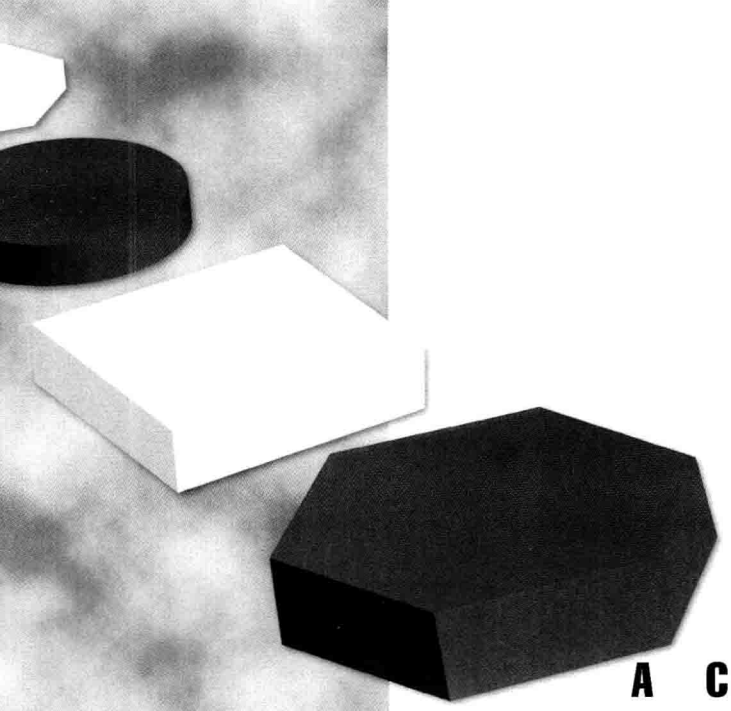

Sixth Edition

MATHEMATICS FOR ELEMENTARY TEACHERS

A CONCEPTUAL APPROACH

Albert B. Bennett, Jr.
University of New Hampshire

L. Ted Nelson
Portland State University

Mc Graw Hill Higher Education

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MATHEMATICS FOR ELEMENTARY TEACHERS: A CONCEPTUAL APPROACH
SIXTH EDITION

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TO THE STUDENT

You are preparing to enter an exciting profession, one in which there are many changes taking place in content, methodology, and technology. In the past, elementary school mathematics programs stressed computational rules and speed and accuracy in computing. The focus is now changing, and today's children need to develop conceptual understanding, the ability to reason and communicate through mathematics, and the ability to solve problems. When estimations and approximations are not sufficient, computations can be done with calculators and computers. Children who are able to see mathematics conceptually have a better chance of solving problems and acquiring confidence in their ability to reason.

During your career as a teacher, you will have far-reaching influence on the lives of your students; a huge responsibility! As a teacher you will be among energetic and enthusiastic young people who will need your support and understanding to build their confidence.

Over the years that the authors of this book taught mathematics courses to prospective elementary school teachers, they asked their students to write about their mathematical background, including special experiences and influences. It was not uncommon to receive statements such as, "My math teacher was awesome and changed my attitude toward mathematics. Because of this influence I decided to become a teacher." Unfortunately, negative comments about teachers and teaching practices were also common, and students discussed the fears and anxieties resulting from such experiences. If you have a school teacher that you thought was outstanding, you might want to write a note of thanks. If you do enter the teaching profession, you may find that such notes and words of appreciation are among your most prized possessions.

The opening paragraph in NCTM's *Principles and Standards for School Mathematics* (Standards 2000) states that its recommendations are grounded in the belief that all students should learn important mathematical concepts and processes with understanding. This belief has guided and influenced the writing of the first six editions of *Mathematics for Elementary Teachers: A Conceptual Approach,* which continues to place emphasis on the use of models and processes for providing insights into mathematical concepts before generalizations.

Standards 2000 contains frequent references to the dual need for acquiring *conceptual understanding* and *procedural fluency.* It states that conceptual understanding is essential in learning to solve new kinds of problems. *Mathematics for Elementary Teachers: A Conceptual Approach* develops conceptual understanding and models mathematical procedures through its extensive use of diagrams, applications, and problem solving.

The primary objective of *Mathematics for Elementary Teachers: A Conceptual Approach* is to present mathematics in a format that prepares teachers to teach elementary school mathematics. Teachers need a firm foundation in the theory of mathematics as it pertains to the elementary school curriculum. They also need ideas and methods for teaching mathematics to elementary school children in a way that will provide an understanding of concepts and generate interest and enthusiasm.

This edition of *Mathematics for Elementary Teachers: A Conceptual Approach* contains several features to help future teachers see connections between their college mathematics courses and the mathematics they envision teaching to elementary school students. There is a one-page **Math Activity** at the beginning of each section which involves materials and ideas that can be adapted to teaching elementary school mathematics. New to this edition are **sample pages from elementary school texts** that show the close relationship between some of the mathematical content and processes in *Mathematics for Elementary Teachers: A Conceptual Approach* and the elementary school curriculum. There are also **statements from research** about learning and teaching mathematics in elementary and middle schools that suggest good practices and show some of the common learning difficulties of students. Finally, numerous **statements from Standards 2000** have been added to highlight recommendations for teaching mathematics with conceptual understanding.

NCTM's Process Standards

Standards 2000 has five Content Standards: **Number and Operations, Algebra, Geometry, Measurement,** and **Data Analysis and Probability.** It also has five Process Standards: **Problem Solving, Reasoning and Proof, Communication, Connections,** and **Representation.** The Process Standards discuss ways of acquiring and using content knowledge, and the following outline shows how this is accomplished in *Mathematics for Elementary Teachers: A Conceptual Approach.*

PROBLEM SOLVING

Several problem-solving strategies are illustrated in Chapter 1 and additional strategies are introduced in each chapter of the text.

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Problem Openers at the beginning of each section provide opportunities for using the problem-solving strategies.
Problem-Solving Applications in each section illustrate specific strategies using Polya's four-step approach.
Reasoning and Problem Solving in each exercise set require explanations and justifications.

REASONING AND PROOF

Inductive reasoning and conjecture forming are introduced in Chapter 1 for extending patterns.
Deductive reasoning and Venn diagrams are introduced in Chapter 2, and both inductive and deductive reasoning are used throughout the text.
Puzzlers occur in each section and provide opportunities to grapple with problems that require extra effort.
Reasoning and Problem Solving in the exercise sets require both inductive and deductive reasoning.

COMMUNICATION

One-page *Math Activities* at the beginning of each section provide problems employing manipulative materials for small group discussions of strategies and solutions.
Research Statements in the margins of the text relate the mathematical content to school-student performance.
Problems for discussion and writing in mathematics for each section of the text are on the website for the text (see following pages).
Problem Openers at the beginning of each section provide opportunities for class or small group discussions of strategies and solutions.
Statements from Standards 2000 have been placed in the margins to relate the content, models, and pedagogy of the text to the proposals in the standards.

CONNECTIONS

Spotlights on Teaching at the beginning of each chapter contain examples from NCTM's standards which show connections to school mathematics.
Elementary School Text Pages relate the content and models of elementary school mathematics to key topics from the text.
Numerous photographs throughout the text of crystals, buildings, and many other types of objects illustrate connections to the real world.
An abundance of models and visuals illustrate mathematical concepts and provide insights and connections across number systems. For example, models help to show that the underlying concepts of the basic operations on numbers remain the same regardless of the type of numbers being used.
Techniques for estimation and mental calculation provide another common thread across the number system chapters.
Historical Highlights show the evolution of key ideas and provide background on some of the world's outstanding mathematicians.

REPRESENTATION

One-page *Math Activities* at the beginning of each section represent concepts with models and diagrams.

Mathematical concepts throughout the text are illustrated with models and diagrams before introducing definitions and generalizations.
Mathematical statements and relationships are represented throughout the text by tables, graphs, equations, and algebraic expressions.
Instruction and exercises for using calculators occur throughout the text and are marked by icons.
Graphing Calculator and Computer Investigations for specific topics are contained on the website. The *Mathematics Investigator* software on the website illustrates the power and convenience of computers in quickly generating large amounts of data.
Models and diagrams provide opportunities for representing relationships at the concrete level, then describing relationships verbally, and finally expressing them by algebraic statements.

Suggestions for Active Student Participation

NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) and Standards 2000 recommend that students develop mathematics by looking for patterns, making conjectures, and verifying hypotheses. Many instructors have been influenced by such recommendations and are using instructional methods which involve more active student participation and less time in the lecture format. Following are a few suggestions involving the special features of the text which encourage active student participation.

MATH ACTIVITIES

The one-page Math Activities preceding each section of the text are augmented by the Manipulative Kit of colored, perforated cardstock materials which can be packaged with the text, if requested by the instructor (ISBN 0-07-287392-2). These Math Activities serve as: homework assignments to provide background for a section of the text; ideas for projects or papers involving the design of an elementary school activity; small group instruction for introducing sections of the text. They are designed to: introduce students to activities that develop concepts and mathematical reasoning; provide opportunities for students to look for patterns, form conjectures, and express their thinking; familiarize students with activities that can be adapted to the elementary school curriculum.

PROBLEM OPENERS

Each section of the text begins with a Problem Opener related to the content of that section. Problem Openers may be used for small, group problem solving and class discussions. They can be used to open a lesson before the content of the section is introduced and to motivate interest in the topic. The solution to each Problem Opener and the problem-solving strategies required are contained in the Instructor's Resource Manual. The manual also includes one or more ideas for looking back and extending each Problem Opener for additional problem solving in class or on assignments and tests.

INVESTIGATIONS

The National Council of Supervisors of Mathematics has stated, "Students need to explore mathematics using manipulatives, measuring devices, models, calculators,

and computers."* At the end of each section of the text there is a website reference for a mathematics investigation that is designed specifically for the mathematical content of the section. These investigations pose open-ended questions that require collecting data, looking for patterns, and forming and verifying conjectures. The investigations can be used for student papers or class reports. There are three types of investigations: computer, calculator, and laboratory. Some of the computer investigations are designed for discovering relationships in geometry with one of several interactive geometry software packages; and some are designed for numerical discoveries with the software Mathematics Investigator which is on the text's Online Learning Center located at www.mhhe.com/bennett-nelson. This software is described on the following pages under Supplements.

PROBLEM-SOLVING APPLICATIONS

Each section of the text contains one or more Problem-Solving Application, which applies the subject matter of the section and is analyzed with Polya's four-step plan. These problems can be posed to the class for small group problem solving. A follow-up discussion can involve comparing students' plans for solving a problem and their solutions with those suggested in the text.

Special Approaches and Features

Reasoning and **Problem Solving** exercises marked by PS icon
Math Activities at the beginning of each section that employ the Manipulative Kit of colored, perforated materials
Parity of Exercises for all skills and concepts
Problem-solving Strategies introduced throughout the text
Statements from Standards 2000 in margins of text
Elementary School Text Pages set within the text to illustrate practical application of concepts
Problem Openers at the beginning of each section
Research Statements throughout the margins of the text
Calculator Paragraphs and Exercises marked with an icon
Problem-solving Applications developed by Polya's four-step plan
Spotlights on Teaching from NCTM's Standards at the beginnings of chapters
Puzzlers for challenges and reasoning
Historical Highlights for origins of important ideas
Mental Calculating and Estimating are required in number systems chapters
Chapter Reviews and Chapter Tests at the end of each chapter
Numerous Photographs to illustrate connections and applications
Boxed Features for key definitions, rules, properties
Answer Secton with selected answers for the section-opening Math Activities (marked with ★), odd-numbered exercises, puzzlers, and chapter tests

*National Council of Supervisors of Mathematics, "Essential Mathematics for the 21st Century" (Minneapolis, MN: NCSNI Essential Mathematics Task Force, 1988), 3–4.

Supplements

ONLINE LEARNING CENTER

The Bennett/Nelson Online Learning Center for the sixth edition of *Mathematics for Elementary Teachers*, located at www.mhhe.com/bennett-nelson includes improved and updated versions of the Bennett/Nelson online material, in addition to many NEW features developed especially for the sixth edition. Online Learning Centers are also compatible with a number of full-service online course delivery systems or outside educational service providers.

Digital Manipulative Kit A new, digitally interactive version of the Bennett/Nelson manipulative kit is now provided on the website for carrying out activities.

Interactive Mathematics Applets Content-specific interactive applets demonstrate key mathematical concepts.

Writing/Discussion Problems Correlated directly to the 34 sections of the text, these exercises raise classroom teaching issues and require explanations of mathematical concepts.

Color Transparencies Color copies of the materials from the Manipulative Kit may be downloaded and used for printing color transparencies.

Grids and Dot paper Black and white masters of geoboards, regular polygons, blank Decimal Squares, base-ten grid, coordinate system, random number chart, and a wide variety of grids, dot paper, and spinners are available to students and instructors.

Math Investigations 34 investigations that are classified as computer, calculator, and laboratory. Data for 14 of these investigations may be generated by the user-friendly, browser-based Mathematics Investigator software. Updated with a new design especially for this edition.

Bibliography An expanded set of bibliographies and Internet links for each of the 34 sections of the text.

Logo Instruction Instruction in Logo, including special commands, worked examples, and exercises. Answers for the odd-numbered exercises are included in the website and answers for the even-numbered exercises are included in the Instructor's Manual.

Network Graphs Instruction Instruction in Network Graphs, including worked examples and exercises. Answers for the odd-numbered exercises are included here on the Online Learning Center and answers for the even-numbered exercises are included in the Instructor's Resource Manual.

NetTutor NetTutor is a revolutionary system that enables students to interact with a live tutor over the World Wide Web. Students can receive instruction from live tutors using NetTutor's Web-based, graphical chat capabilities. They can also submit questions and receive answers, browse previously answered questions, and view previous live chat sessions.

INSTRUCTOR'S RESOURCE MANUAL

The *Instructor's Resource Manual* to accompany *Mathematics for Elementary Teachers: A Conceptual Approach* (ISBN 0-07-253298-X) contains extensions for all problem openers and answers for the problem openers and extensions; answers for all even-numbered Exercises and Problems; answers for the Online Mathematics Investigations, chapter tests with answers (two tests for

each chapter); transparency masters (various grids and dot paper); and a description of the *Mathematics Investigator* website software.

STUDENT'S SOLUTION MANUAL

(ISBN 0-07-253297-1) Newly revised for this edition by its author, Joseph Ediger of Portland State University, this manual contains detailed solutions to the even-numbered exercises and problems and the chapter tests.

MATHEMATICS INVESTIGATOR

The *Mathematics Investigator* is software containing 14 programs (see list below) designed to demonstrate the computer investigations on the website. Students may use this software to gather data and run simulations for the investigations. These investigations pose questions to generate interest in various mathematical topics and encourage students to formulate and investigate their own conjectures. Instructors may use this software to demonstrate computer simulations and the process of forming conjectures and looking for counterexamples. This newly-updated, browser-based software is compatible with both Macintosh and PC platforms. Functionality such as editing, cutting, pasting, copying text to other files, and printing can be used with the programs on this software.

MATHEMATICS INVESTIGATOR

Triangular Numbers
Palindromic Sums
Palindromic Differences
Palindromic Decimals
Consecutive Numbers
Differences of Squares
Repeating Decimals
Factorizations
Frequency of Primes
Number Chains
Integer Differences
Standard Deviations
Dice Roll Simulations
Coin Toss Simulation

ACTIVITY BOOK

(ISBN 0-07-253307-2) *Mathematics for Elementary Teachers: An Activity Approach,* Sixth Edition, contains an activity set corresponding to each section of the text. Each activity set is a sequence of inductive activities and experiments that enable the student to build an understanding of mathematical ideas through the use of models and the discovery of patterns. The activity sets augment the ideas presented in the corresponding sections of the text. Over 50 Material Cards, some with colored manipulatives are packaged with *Mathematics for Elementary Teachers: An Activity Approach.* A section on *Ideas for the Elementary Classroom* at the end of each chapter includes a suggested elementary school activity and a list of selected sources. There are puzzlers throughout the book and the activity sets are followed by *Just for Fun* enrichment activities. The text, *Mathematics for Elementary Teachers: A Conceptual Approach,* may be packaged with the activity book, *Mathematics for Elementary Teachers: An Activity Approach,* and the Manipulative Kit (ISBN 0-07-287392-2).

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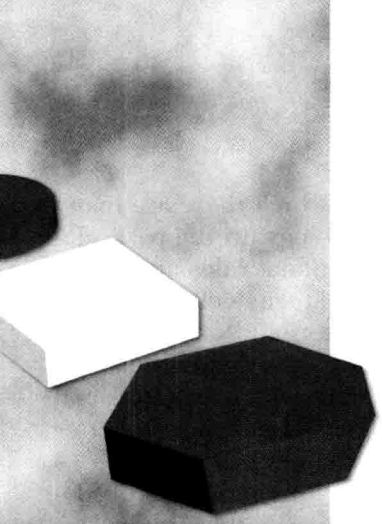

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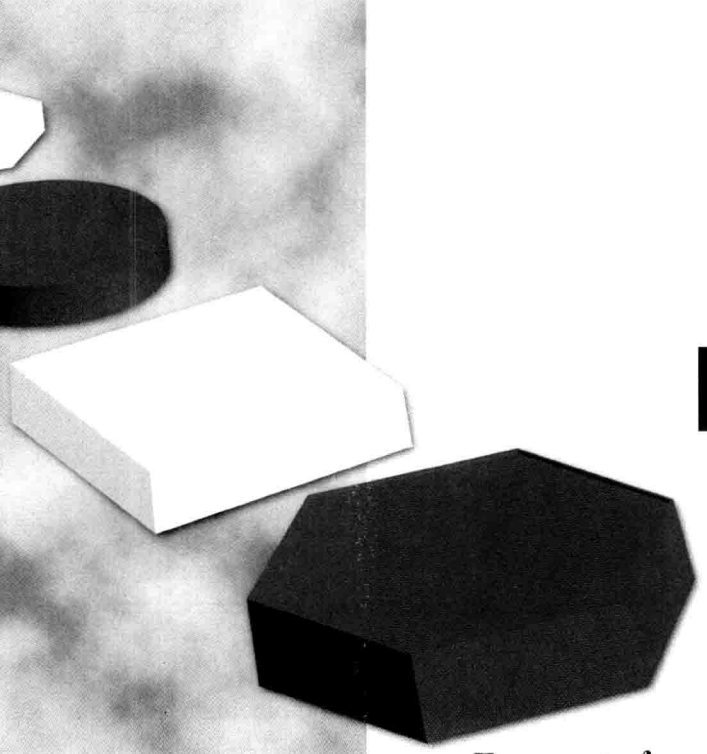


CHAPTER 1

PROBLEM SOLVING

SPOTLIGHT ON TEACHING

Excerpts from NCTM's Standards for School Mathematics Prekindergarten through Grade 12*

Problem solving can and should be used to help students develop fluency with specific skills. For example, consider the following problem, which is adapted from the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989, p. 24):

I have pennies, nickels, and dimes in my pocket. If I take three coins out of my pocket, how much money could I have taken?

This problem leads children to adopt a trial-and-error strategy. They can also act out the problem by using real coins. Children verify that their answers meet the problem conditions. Follow-up questions can also be posed: "Is it possible for me to have 4 cents? 11 cents? Can you list all the possible amounts I can have when I pick three coins?" The last question provides a challenge for older or more mathematically sophisticated children and requires them to make an organized list, perhaps like the one below.

PENNIES	NICKELS	DIMES	TOTAL VALUE
0	0	3	30
0	1	2	25
0	2	1	20
0	3	0	15
1	0	2	21
⋮	⋮	⋮	⋮

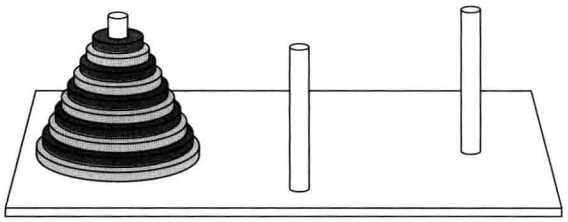
Working on this problem offers good practice in addition skills. But the important mathematical goal of this problem—helping students to think systematically about possibilities and to organize and record their thinking—need not wait until students can add fluently.

**Principles and Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 2000), p. 52.

MATH ACTIVITY 1.1

TOWER PUZZLE PATTERNS

Puzzle: One of the three towers shown here has 10 disks of increasing size. What is the least number of moves needed to transfer these 10 disks from one tower to a different tower if only one disk can be moved at a time and a disk cannot be placed on top of a smaller one?



NO. OF DISKS	NO. OF MOVES
1	1
2	—
3	—
4	—
5	—

NO. OF DISKS	NO. OF MOVES
6	1
7	—
8	—
9	—
10	—

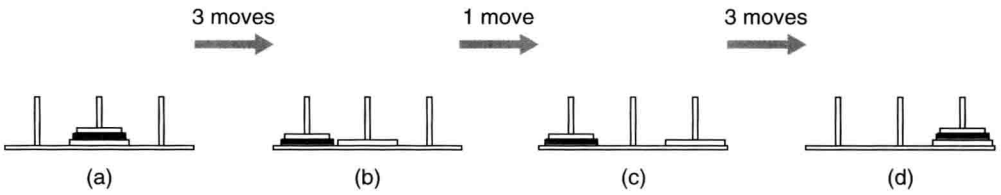
1. **Use a model** by drawing three towers on a sheet of paper and placing a quarter, nickel, penny, and dime on one of the towers (or disks can be cut from paper). Experiment.

*2. **Solve a simpler problem** by using fewer disks. What is the smallest number of moves needed to transfer 2 disks? Then try 3 disks.

*3. **Make a table** and record the smallest number of moves for 2, 3, and 4 disks. Try to predict the number of moves for 5 disks.

*4. **Find a pattern** in the table and extend the table for up to 10 disks.

5. You may have noticed a pattern in transferring the disks in the first three activities. The sequence of four figures below shows 3 disks being transferred from one tower to another. Three moves are needed to go from *(a)* to *(b)* because it takes 3 moves to transfer the top 2 disks from one tower to another. Then 1 move is used from *(b)* to *(c)* to transfer the third disk, and 3 more moves are needed from *(c)* to *(d)* to place the 2 smaller disks on top of the third disk.



Notice how these figures show that the number of moves for transferring 2 disks can be used to determine the number of moves for transferring 3 disks. Draw a similar sketch, and explain how the number of moves for transferring 3 disks can be used to determine the number of moves for transferring 4 disks.

*A star indicates an activity is answered or suggestions are given in the Answer Section.

SECTION 1.1

INTRODUCTION TO PROBLEM SOLVING

There is no more significant privilege than to release the creative power of a child's mind.
Franz F. Hohn
Courtesy of International Business Machines Corporation

PROBLEM OPENER

Alice counted 7 cycle riders and 19 cycle wheels going past her house. How many tricycles were there?

"Learning to solve problems is the principal reason for studying mathematics."* This statement by the National Council of Supervisors of Mathematics represents a widespread opinion that problem solving should be the central focus of the mathematics curriculum.

Problem solving is the hallmark of mathematical activity and a major means of developing mathematical knowledge.

Standards 2000, p. 116.

A **problem** exists when there is a situation you want to resolve but no solution is readily apparent. **Problem solving** is the process by which the unfamiliar situation is resolved. A situation that is a problem to one person may not be a problem to someone else. For example, determining the number of people in 3 cars when each car contains 5 people may be a problem to some elementary school students. They might solve this problem by placing chips in boxes or by making a drawing to represent each car and each person (Figure 1.1) and then counting to determine the total number of people.

Figure 1.1

You may be surprised to know that some problems in mathematics are unsolved and have resisted the efforts of some of the best mathematicians to solve

*National Council of Supervisors of Mathematics, *Essential Mathematics for the 21st Century* (Minneapolis, MN: Essential Mathematics Task Force, 1988).

them. One such problem was discovered by Arthur Hamann, a seventh-grade student. He noticed that every even number could be written as the difference of two primes.* For example,

$$2 = 5 - 3 \qquad 4 = 11 - 7 \qquad 6 = 11 - 5 \qquad 8 = 13 - 5 \qquad 10 = 13 - 3$$

After showing that this was true for all even numbers less than 250, he predicted that every even number could be written as the difference of two primes. No one has been able to prove or disprove this statement. When a statement is thought to be true but remains unproved, it is called a **conjecture.**

Doing mathematics involves discoverey. Conjecture—that is, informed guessing—is a major pathway to discovery. Teachers and researchers agree that students can learn to make, refine, and test conjectures in elementary school.

Standards 2000, p. 57

Problem solving is the subject of a major portion of research and publishing in mathematics education. Much of this research is founded on the problem-solving writings of George Polya, one of the foremost twentieth-century mathematicians. Polya devoted much of his teaching to helping students become better problem solvers. His book *How to Solve It* has been translated into 18 languages. In this book, he outlines the following four-step process for solving problems.

Understanding the Problem Polya suggests that a problem solver needs to become better acquainted with a problem and work toward a clearer understanding of it before progressing toward a solution. Increased understanding can come from rereading the statement of the problem, drawing a sketch or diagram to show connections and relationships, restating the problem in your own words, or making a reasonable guess at the solution to help become acquainted with the details.

Devising a Plan The path from understanding a problem to devising a plan may sometimes be long. Most interesting problems do not have obvious solutions. Experience and practice are the best teachers for devising plans. Throughout the text you will be introduced to strategies for devising plans to solve problems.

Carrying Out the Plan The plan gives a general outline of direction. Write down your thinking so your steps can be retraced. Is it clear that each step has been done correctly? Also, it's all right to be stuck, and if this happens, it is sometimes better to put aside the problem and return to it later.

Looking Back When a result has been reached, verify or check it by referring to the original problem. In the process of reaching a solution, other ways of looking at the problem may become apparent. Quite often after you become familiar with a problem, new or perhaps more novel approaches may occur to you. Also, while solving a problem, you may find other interesting questions or variations that are worth exploring.

Polya's problem-solving steps will be used throughout the text. The purpose of this section is to help you become familiar with the four-step process and to acquaint you with some of the common strategies for solving problems: *making a drawing, guessing and checking, making a table, using a model,* and *working backward.* Additional strategies will be introduced throughout the text.

*M. R. Frame, "Hamann's Conjecture," *Arithmetic Teacher* 23, no. 1 (January 1976): 34–35.

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