Brief Calculus with Applications

Alternate Third Edition

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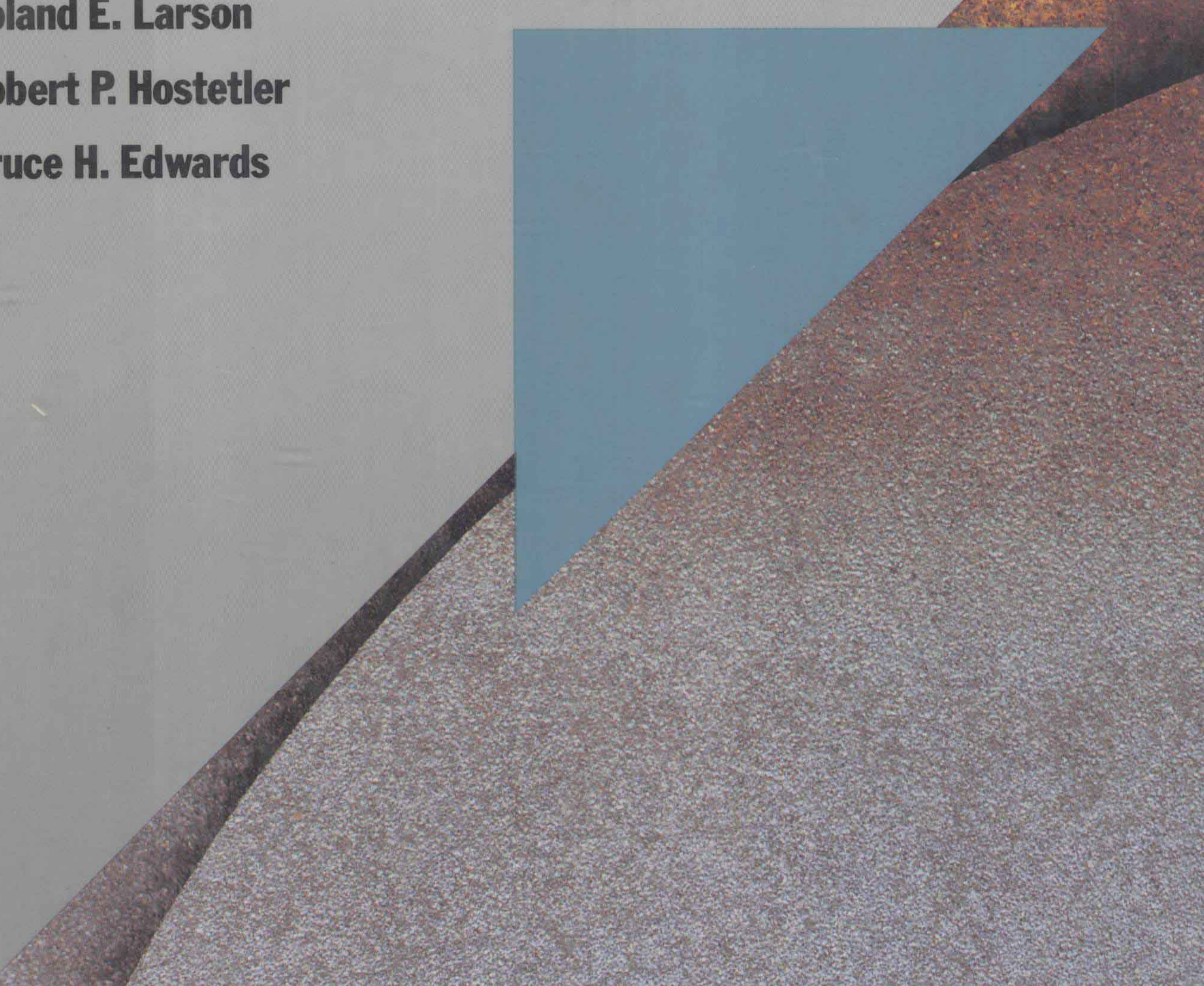
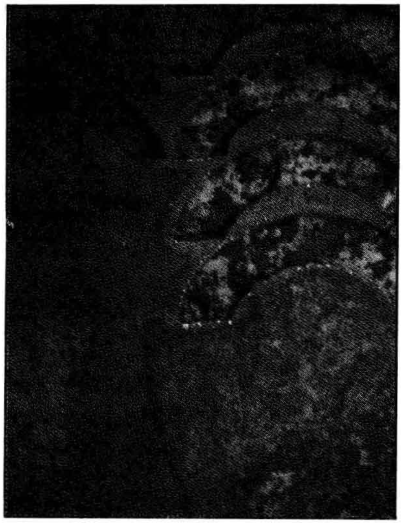
Robert P. Hostetler

Bruce H. Edwards





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Preface

Brief Calculus with Applications, Alternate Third Edition, is designed for use in a beginning calculus course for students in business, economics, management, and the social and life sciences. In writing this edition, we were guided by two primary objectives formed over many years of teaching calculus. For students, our objective was to write in a precise and readable manner, with the basic concepts, techniques, and applications of calculus clearly defined and demonstrated. For instructors, our objective was to create a comprehensive teaching instrument that uses proven pedagogical techniques, thus freeing instructors to make the most efficient use of classroom time.

Changes in the Third Edition

Chapter Overviews We added an overview to the beginning of each chapter of the Alternate Third Edition. These overviews provide a survey of the contents of each chapter *and* show students how the topics fit into the overall development of calculus.

Questions for Discussion Each chapter in the text now ends with a set of discussion questions. These questions are designed to help students synthesize the mathematics of the chapter. The questions can be used by instructors to stimulate class discussion or as topics for individual or group assignments. The concept of "writing across the curriculum" has been attracting interest in the past few years, and the discussion questions were designed in part to meet this need.

Computer/Graphing-Calculator Exercises We have added new computer/graphing-calculator exercises to most of the exercise sets. These optional exercises usually occur near the end of an exercise set and are designated with the computer symbol .

Business Applications Many new business applications were added to the text. For instance, new material on marginal analysis was added to Sections 3.8 and 4.3, new material on consumer and producer surplus was added to Section 4.4, and new material on present value was added to Section 6.2.

Probability and Calculus The Second Edition of the text had only two sections on probability and calculus. Those sections have been expanded to form the new Chapter 7.

Features of the Third Edition

Introductory Examples Each section in the text begins with a one-page motivational example designed to show the applicability of the material. Students should not expect to understand an introductory example completely when reading it for the first time. After finishing the section, a student will be able to return to the Introductory Example to see how the theory and techniques of calculus are applied in business, economics, and the life and social sciences. As a new feature in the Alternate Third Edition, each Introductory Example is tied to one or more exercises in the section.

Prerequisites Chapter 0 is intended as a quick review of the algebra needed to study calculus. An instructor may elect to skip Chapter 0 and begin the course with Chapter 1. In such cases, Chapter 0 can serve as an algebraic reference for students.

Order of Topics The nine chapters can be adapted to either the semester or quarter system. There is some flexibility in the order and depth in which the chapters can be covered. Chapter 0 and the first part of Chapter 1 can be omitted. Chapters 7 (probability) is optional. Sections toward the end of each chapter tend to be optional as well. For instance, Section 2.8 on related rates and Section 4.6 on volumes of solids of revolution are optional.

Examples The text contains over 500 examples that illustrate specific concepts or problem-solving techniques. Each example is titled for easy reference. Many of the examples include side comments (printed in color) that clarify the steps of the solution.

Exercises Over 375 new exercises have been added, so the text now contains nearly 3500 exercises. The exercises are graded, progressing from skill-development problems to more challenging problems involving applications and proofs. Many exercise sets begin with a group of exercises that provide the graphs of the functions involved. Review exercises are included at the end of each chapter. The answers to the odd-numbered exercises are given in the back of the text.

Calculators Special emphasis is given to the use of hand calculators in sections dealing with limits and numerical integration. In addition, many of the exercise sets contain problems identified by the symbol as calculator exercises.

Applications The text has approximately 450 applications taken from a variety of fields, with an emphasis on applications in business and economics. An index of applications is given on the inside covers of the text.

Graphics The Alternate Third Edition has over 1100 figures. Of these, over 450 are in the examples and exposition, nearly 375 are in the exercise sets, and over 225 are in the odd-numbered answers. The new art program in the Alternate Third Edition was computer generated for accuracy.

Theorems and Definitions Special care has been taken to state the theorems and definitions simply, without sacrificing accuracy. The theorems and definitions are set off by colored boxes to emphasize their importance. Guidelines and summaries, on the other hand, are set off by gray boxes.

Section Topics Each section begins with a list of the major topics to be covered.

Remarks The text contains special instructional notes in the form of Remarks. These appear after definitions, theorems, or examples and are designed to give additional insight, help avoid common errors, or describe generalizations.

Reference Tables An extensive set of reference tables is included in Appendix B. These tables summarize important formulas from calculus, algebra, geometry, business, and other fields.

Chapter Summaries Near the end of each chapter (other than Chapter 0) is a summary that lists important terms, techniques, and formulas from the chapter.

Supplements

FOR STUDENTS

The *Student Solutions Guide* by Dianna L. Zook of Indiana University, Purdue University at Fort Wayne contains step-by-step solutions to the odd-numbered exercises. This supplement also contains a practice test for each chapter.

The *Study Guide and Workbook* by Ronnie Khuri of the University of Florida contains additional worked examples and questions.

The *Brief Calculus TUTOR* by Timothy R. Larson is an "electronic study guide" for the text. This interactive software contains examples with step-by-step solutions, exercises from the text with diagnostic feedback, chapter summaries, a glossary, and warm-ups and post-tests with diagnostics.

The Algebra of Calculus by Eric Braude offers a review of calculus topics, examples with solutions, related problems and their answers, and exercises with answers in the back of the book.

FOR INSTRUCTORS

The *Complete Solutions Guide* by Dianna L. Zook of Indiana University, Purdue University at Fort Wayne contains brief solutions to every exercise in the text, including exercises requiring proofs.

The *Instructor's Guide* by Ann R. Kraus of The Pennsylvania State University, the Behrend College, contains sample tests for each chapter, suggestions for classroom instruction, and the answers to the even-numbered exercises.

The *Test Item File* by Meredith M. Burrows contains over 1500 sample test questions. Some of the test items are multiple-choice, and others are open-ended. This supplement accompanies *HeathTest Plus for Brief Calculus*, a computerized testing program.

A package of color transparencies is available.

Acknowledgments

We would like to thank the many people who have helped us at various stages of this project during the past ten years. Their encouragement, criticisms, and suggestions have been invaluable to us.

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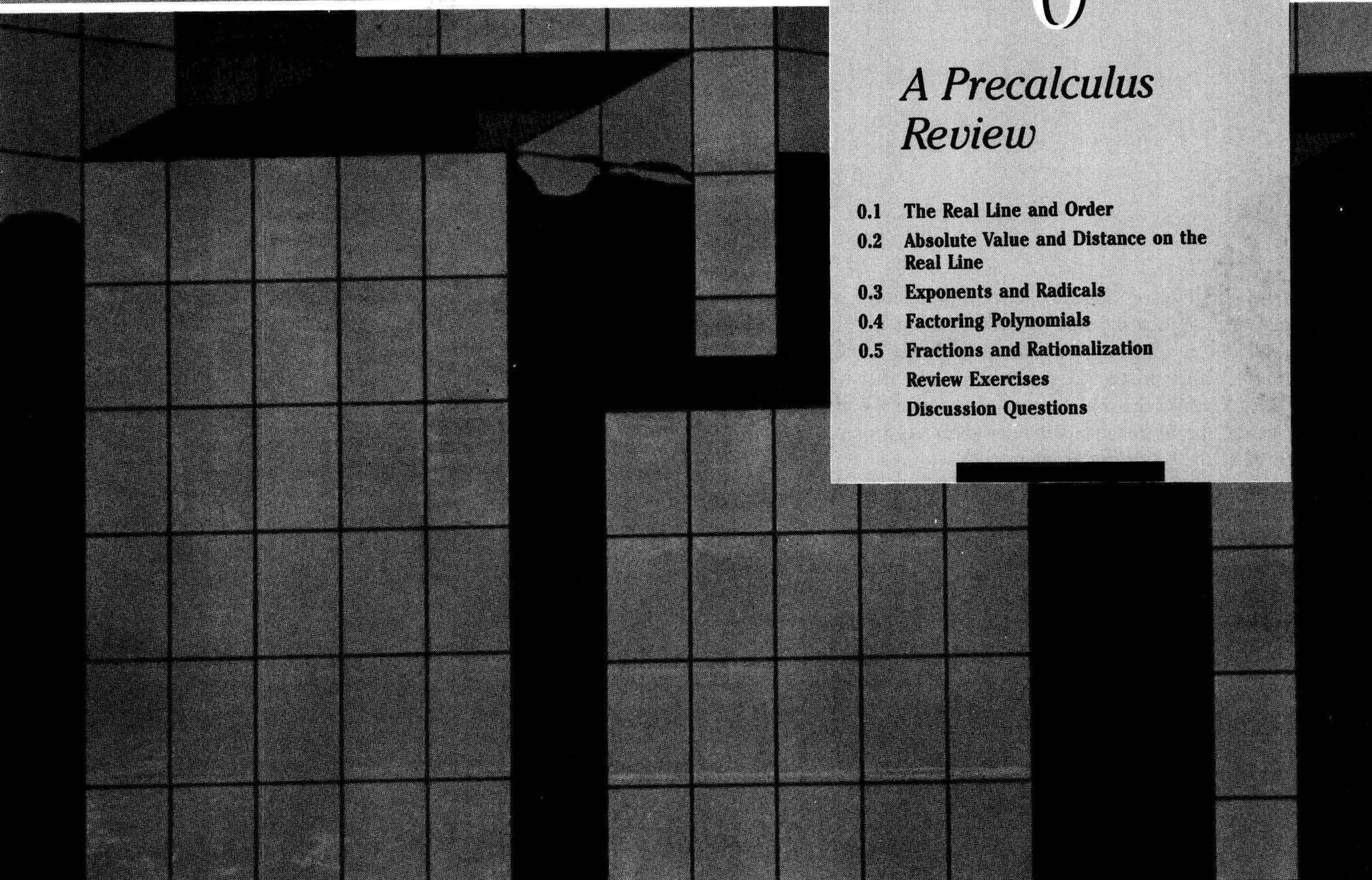
Several other people worked on this project. David E. Heyd assisted us in writing the text and solved the exercises; Dianna L. Zook solved the exercises and wrote the *Student Solutions Guide* and the *Complete Solutions Guide*; Ann R. Kraus wrote the *Instructor's Guide*; Ronnie Khuri wrote the *Study Guide and Workbook*; Timothy R. Larson prepared the art; Linda L. Kifer proofread the galleys; Linda M. Bollinger proofread the galleys and typed the supplements; Helen Medley solved the exercises and performed an accuracy check for the text; Paula M. Sibeto solved the exercises and worked on the tutorial software; Richard Bambauer prepared the art for the supplements; Lisa Edwards assisted in proofing the exercises.

On a personal level, we are grateful to our wives, Deanna Gilbert Larson, Eloise Hostetler, and Consuelo Edwards, for their love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving the text, please feel free to write to us. Over the past two decades we have received many useful comments from both instructors and students, and we value these very much.

Roland E. Larson
Robert P. Hostetler
Bruce H. Edwards

Contents



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A Precalculus Review

CHAPTER OVERVIEW This preliminary chapter reviews much of the algebra you will need to begin your study of calculus. Even if your instructor does not cover this chapter in class, we strongly encourage you to review the material presented here.

The first section deals with real numbers, the real line, order on the real line, intervals, and solving linear and quadratic inequalities. The second section continues to review other properties of real numbers and the real line, including absolute value, distance on the real line, intervals defined by absolute value, and the midpoint of an interval.

In the next three sections, we review the basic operations of algebra. Section 0.3 discusses exponents, radicals, simplification techniques, and the domain of an algebraic expression.

Section 0.4 is especially important because it reviews techniques for factoring polynomials. This skill is necessary for solving many of the equations that arise later in the text.

Finally, Section 0.5 reviews operations with fractional expressions, additional simplification techniques, and rationalization techniques.

Sufficient Sales to Obtain a Profit

Many problems in business concern the number of units of a particular item that must be sold in order to "break even." Typically, a company is required to invest some amount of money before production can begin. Only after sales are sufficient to offset the initial investment does the company begin making a profit (for that particular item). As a simple example, consider the following problem.

A small business sells wooden patio chairs for \$39.50 each. The initial investment required to obtain the tools and space to produce the chairs is \$12,685. The cost of material and labor for each chair is \$24.75. How many chairs must be sold before the business realizes a profit?

Because each chair sells for \$39.50, it follows that **revenue** for selling x chairs is given by the equation

$$R = 39.5x. \qquad \textit{Revenue}$$

On the other hand, the **cost** of producing x chairs is given by the equation

$$C = 24.75x + 12{,}685. \qquad \textit{Cost}$$

Finally, the **profit** for selling x chairs is given by the equation

$$\begin{aligned} P &= R - C \qquad \textit{Profit} \\ &= 39.5x - (24.75x + 12{,}685) \\ &= 14.75x - 12{,}685. \end{aligned}$$

To determine the number of units that must be sold to obtain a profit, we solve the inequality $P \geq 0$ as follows.

$$\begin{aligned} 14.75x - 12{,}685 &\geq 0 \\ 14.75x &\geq 12{,}685 \\ x &\geq \frac{12{,}685}{14.75} \\ x &\geq 860 \end{aligned}$$

Thus, the business must sell 860 chairs before it will begin to make a profit. Table 0.1 compares the revenue, cost, and profit for selling various numbers of chairs.

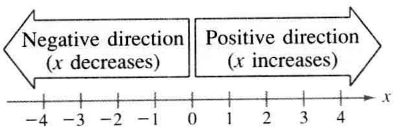
(In Exercise 37 you are asked to solve a similar problem.)

TABLE 0.1

x (number of units)	0	500	860	1,000	2,000
R (revenue)	\$0	\$19,750	\$33,970	\$39,500	\$79,000
C (cost)	\$12,685	\$25,060	\$33,970	\$37,435	\$62,185
P (profit)	−\$12,685	−\$5,310	\$0	\$2,065	\$16,815

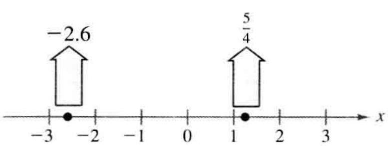
0.1 *The Real Line and Order*

The Real Line* ▪ *Order and Intervals on the Real Line* ▪ *Solving Inequalities* ▪ *Applications

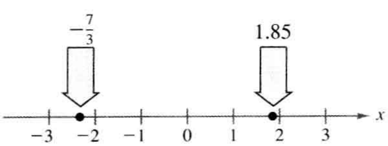


The Real Line

FIGURE 0.1



Every point on the real line corresponds to a real number.



Every real number corresponds to a point on the real line.

FIGURE 0.2

The Real Line

To represent the real numbers, we use a coordinate system called the **real line** (or x-axis), as shown in Figure 0.1. The **positive direction** (to the right) is denoted by an arrowhead and indicates the direction of increasing values of x. The real number corresponding to a particular point on the real line is called the **coordinate** of the point. As shown in Figure 0.1, it is customary to label those points whose coordinates are integers.

The point on the real line corresponding to zero is called the **origin.** Numbers to the right of the origin are **positive,** and numbers to the left of the origin are **negative.** We use the term **nonnegative** to describe a number that is either positive or zero.

The importance of the real line is that it provides us with a conceptually perfect picture of the real numbers. That is, each point on the real line corresponds to one and only one real number, and each real number corresponds to one and only one point on the real line. This type of relationship is called a **one-to-one correspondence** and is illustrated in Figure 0.2.

Each of the four points in Figure 0.2 corresponds to a real number that can be expressed as the ratio of two integers. (Note that $1.85 = \frac{37}{20}$ and $-2.6 = -\frac{13}{5}$.) We call such numbers **rational.** Rational numbers have either terminating or infinitely repeating decimal representations.

Terminating decimals	*Infinitely repeating decimals*
$\frac{2}{5} = 0.4$	$\frac{1}{3} = 0.333\ldots = 0.\overline{3}$*
$\frac{7}{8} = 0.875$	$\frac{12}{7} = 1.714285714285\ldots = 1.\overline{714285}$

Real numbers that are not rational are called **irrational,** and they cannot be represented as the ratio of two integers (or as terminating or infinitely repeating decimals). To represent an irrational number, we usually resort to a decimal approximation. Some irrational numbers occur so frequently in applications that mathematicians have invented special symbols to represent them. For example, the symbols $\sqrt{2}$, π, and e represent irrational numbers whose decimal approximations are as follows.

$$\sqrt{2} \approx 1.4142135623$$
$$\pi \approx 3.1415926535$$
$$e \approx 2.7182818284$$

*The bar indicates which digits repeat.

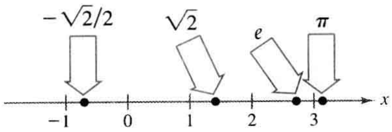


FIGURE 0.3

REMARK We use $\approx$ to mean *approximately equal to*. Remember that even though we cannot represent irrational numbers *exactly* as terminating decimals, we can represent them *exactly* with points on the real line, as shown in Figure 0.3.

Order and Intervals on the Real Line

One important property of the real numbers is that they are **ordered:** 0 is less than 1, -3 is less than -2.5, π is less than $\frac{22}{7}$, and so on. We can visualize this property on the real line by observing that a is less than b if and only if a lies to the left of b. Symbolically, we denote "a is less than b" by the inequality

$$a < b.$$

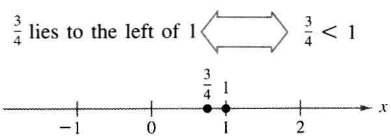


FIGURE 0.4

For example, the inequality $\frac{3}{4} < 1$ follows from the fact that $\frac{3}{4}$ lies to the left of 1 on the real line, as shown in Figure 0.4.

When three real numbers, a, x, and b are ordered such that $a < x$ and $x < b$, we say that x is **between** a and b and write

$$a < x < b. \qquad \textit{x is between a and b}$$

The set of *all* real numbers between a and b is called the **open interval** between a and b and is denoted by (a, b). An interval of the form (a, b) does not contain the "endpoints" a and b. Intervals that include their endpoints are called **closed** and are denoted by $[a, b]$. Intervals of the form $[a, b)$ and $(a, b]$ are called **half-open intervals.** Table 0.2 shows the nine types of intervals on the real line.

TABLE 0.2 Intervals on the Real Line

Open interval	Half-open intervals	Infinite intervals
(a, b): $a < x < b$	$(a, b]$: $a < x \le b$	$(-\infty, a)$: $x < a$
		(b, ∞): $b < x$
Closed interval	$[a, b)$: $a \le x < b$	$(-\infty, a]$: $x \le a$
$[a, b]$: $a \le x \le b$		$[b, \infty)$: $b \le x$
		$(-\infty, \infty)$