

SPRINGER TRACTS IN MODERN PHYSICS

Volume 89

F. Cannata H. Überall

Giant Resonance Phenomena in Intermediate-Energy Nuclear Reactions



Springer-Verlag Berlin Heidelberg New York

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With 43 Figures



Springer-Verlag
Berlin Heidelberg New York 1980

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ISBN 3-540-10105-5 Springer-Verlag Berlin Heidelberg New York

ISBN 0-387-10105-5 Springer-Verlag New York Heidelberg Berlin

Library of Congress Cataloging in Publication Data. Cannata, F. 1945-. Giant resonance phenomena in intermediate-energy nuclear reactions. (Springer tracts in modern physics ; 89). Includes bibliographical references and index.
1. Nuclear magnetic resonance, Giant. 2. Nuclear reactions. I. Überall, Herbert, 1931- joint author. II. Title. III. Series.
QC1.S797 vol. 89 [QC762] 530s [538'.362] 80-14938

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Printed in Germany

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Offset printing and bookbinding: Brühlsche Universitätsdruckerei, Giessen
2153/3130 — 5 4 3 2 1 0

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Preface

In 1947, Baldwin and Klaiber carried out experiments bombarding atomic nuclei with gamma quanta, and observed that at certain high (15-20 MeV) excitation energies, the nucleus began to act as a strong absorber of the incident photons. This phenomenon was termed the "nuclear giant resonance", and was quickly interpreted by Goldhaber and Teller (1948) as the excitation of a collective mode of nuclear vibrations in which all the protons in the nucleus moved together against all the neutrons, thus performing a "giant" dipole vibration. A universal nuclear feature, giant dipole resonances were found to exist in all nuclei throughout the periodic table, originally mainly via photonuclear experiments.

A decisive step forward occurred with the hypothesis (Foldy and Walecka, 1964) that the giant resonances formed isomultiplets, and hence would also be present in neighboring isobars where they could be excited in charge-exchange reactions such as muon capture. This approach succeeded in explaining muon capture rates in terms of photonuclear cross sections.

With the advent of the inelastic electron scattering technique, the nuclear giant resonances were shown to be excited in this type of experiment also. In addition, a new mode of dipole vibrations of nuclear matter was discovered, one in which protons with spin up move collectively, together with neutrons with spin down, against the remaining protons with spin down and neutrons with spin up (Oberall, 1965). These spin and isospin extensions of the giant resonances permit their description in terms of an SU4 vector supermultiplet in the framework of Wigner's nuclear supermultiplet theory.

The most recent advance in the field occurred with the discovery of giant resonance vibrations of higher multipolarity, using proton scattering (Lewis and Bertrand, 1972) and electron scattering (Pitthan and Walcher, 1971; Fukuda and Torizuka, 1972). Such higher-order collective multipole vibrations had been theoretically predicted long before (Danos, 1952), and were classified in terms of SU4 multiplets (Raphael, Oberall and Wernitz, 1966).

The foregoing has already implied that giant resonances, as a property of the nuclear spectrum, may be excited in a variety of nuclear reactions, weak, electromagnetic and strong, employing different types of projectiles and reaction mechanisms. We are thus offered the advantage that this interesting nuclear feature

can be investigated from many different viewpoints, so that taking all these approaches together, we may obtain a quite complete description of it. The present monograph capitalizes on this fact, and discusses our progress in giant resonance studies as obtained through photonuclear and electron scattering reactions, muon capture and neutrino excitations, pion photoproduction and radiative capture, and finally through strong interactions of protons, alpha particles, and pions. We hope that in this way the reader has been presented a fairly complete (although perhaps not exhaustive) picture of our present status of knowledge on the giant nuclear resonances, as well as of the way in which this knowledge has been obtained. We wish to acknowledge the support of the National Science Foundation in the preparation of this report. One of us (F.C.) is particularly indebted to the late Prof. J.I. Fujita for many enlightening discussions.

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April 1980

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Introduction

Giant resonance excitation in nuclei, first observed in 1947 in photonuclear absorption experiments /1/, have been shown to constitute a universal nuclear phenomenon, upon which much subsequent interest has been focused. Previous studies have for a long time been restricted first to the electric dipole components of the resonances (see, e.g., /2, 3/), with later extension to giant magnetic dipole states /4/; but the subject has received renewed interest through the more recent discovery of higher multipole giant resonances (/5/, see also /6-11/), whose existence had been discussed /12/ several years before their actual discovery. In the mentioned theoretical study predicting these resonances /12/, attention had been given to the effects they would produce in inelastic electron scattering experiments /13/; and in fact, giant electric quadrupole resonances were indeed observed in such reactions also /14/, but independently in inelastic proton and alpha scattering experiments as well.

This example illustrates an additional feature of the giant resonances in general, which puts them among the most interesting objects of study for intermediate-energy nuclear experiments: namely, the fact that they can be excited in a variety of nuclear reactions, involving electromagnetic, weak, as well as strong interactions. The advantages of a multi-faceted study of this single nuclear feature which thereby offers itself, are considerable. The information thus gained regarding the nuclear transition densities of the giant resonance states, overlaps in the various reaction channels of the intermediate energy regime, hence providing corroborating information on the level structure. Selection rules in different reactions (such as spin, isospin, etc.) will emphasize one or the other feature and hence provide complementary information on the transition densities not obtainable with a single probe.

In the present review we shall adopt the viewpoint, just outlined, of a multi-reaction approach for the study of giant resonance phenomena. A unified treatment is possible for the latter, because although excitation processes such as neutrino-induced reactions and muon capture have a different SU4 geometry, the amplitudes are the same as, e.g., for electroexcitation processes and photo-reactions. The forms of the nuclear interactions for these various processes are discussed in Chap. 1 as well as the classification of the various giant

resonance components based on certain nuclear models. This section discusses also the dynamics of the giant resonance based on hydrodynamical models /12/. Their interpretation in terms of Regge poles /15/ is given in Chap. 2. That section also deals with the giant resonance form factors that are studied in inelastic electron scattering. We emphasize the role of spin-flip contributions which are important in electronuclear processes, not only for the well-established spin-isospin giant resonance excitation, but also for their interference with the isospin excitation which is sensitive to the ground state SU4 impurities of the target. There exists a similar situation in muon capture, the subject of Chap. 3, for which a microscopic theory for SU4 breaking will be discussed taking into account such ground state impurities. This discussion will also be extended to non-doubly closed shell nuclei.

In Chap. 4, high-energy neutrino-induced reactions will be considered, and their relation to inelastic electron scattering discussed. The small cross sections of this process seem to preclude any detailed experimental study of giant resonance excitation here, but the predicted rates may serve as background estimates for a variety of elementary neutrino reactions /16-18/.

In Chap. 5, we discuss photopion reactions (γ, π) and (π, γ) on nuclei, which mainly proceed via spin-flip transitions, and hence are capable of isolating the spin-flip component of the giant resonance. Giant resonances have been observed experimentally in the radiative capture of stopped pions.

In Chap. 6, purely hadronic interactions with the nucleus will be discussed, namely (π, π') , (p, p') and (α, α') . The latter two reactions have been used in the original discovery of higher-multipole giant resonances. In these reactions, there also exists the possibility of the giant resonance entering as an intermediate state in two-step processes which manifest themselves in the backward angular distribution. When analyzed in this way /19/, additional information on the higher giant resonances may be gained. The interaction operator in the theory of hadronic interactions is not as well understood as for weak and electromagnetic interactions, so that the theoretical interpretation may be less reliable here.

In the appendices, several relevant topics involving giant resonances will be discussed such as sum rules, SU4 properties of interactions, and current algebra.

1. The Interaction Between the Nucleus and an External Probe

1.1 Nuclear Probes

The study of nuclear systems generally involves the use of external probes, i.e., the interaction with other (simpler) physical systems, the dynamics of which is better understood. A classical example of such a probe is the electromagnetic field, which intervenes, e.g., in photoreactions, or in electron scattering /13, 20, 21/. In addition, weak interactions have been successfully employed for nuclear structure studies, such as beta decay /22/ or muon capture /23/. Strongly interacting particles do not offer in general the advantages of photons, electrons, muons and neutrinos for a use as nuclear probes since they entail the problem of distinguishing between effects of nuclear structure itself and of the reaction mechanism. However, as noted by ERICSON /24/, there is an exception among these, namely, slow pions. Indeed, the scattering lengths for pion-nucleon scattering are exceptionally small: $a(\pi N) = 0.1 \text{ fm}$ ($\text{fm} = \text{fermi} = 10^{-13} \text{ cm}$) in contrast to the scattering lengths for K meson- and nucleon-nucleon interactions whose values are $a(KN) = 1 \text{ fm}$, $a(NN) = 10 \text{ fm}$. Therefore, the interaction of slow pions with the nucleus is sufficiently weak so as not to disturb the nucleus in any violent fashion (unlike the interaction of other hadrons), which is desirable for a good probe.

In the remainder of this section, we shall consider primarily the electromagnetic and weak interactions as probes of nuclear structure, while, however, also devoting some attention to hadron interactions in the later sections of this review.

1.2 Electromagnetic and Weak Interactions of Nuclei

The usual starting point for the calculation of nuclear electromagnetic transitions /25/ is the expression for the electromagnetic nuclear 4-current density operator in the Schrödinger representation (we employ units in which $\hbar = c = 1$)

$$j_{em, \mu}^{nuc}(\underline{r}) = (\underline{j}, \rho) \quad (1a)$$

with

$$\begin{aligned}
 j_{em}^{nuc}(\underline{r}) = e \sum_{j=1}^A & \left[\frac{d}{dt} \left(\frac{1+\tau_{3j}}{2} \frac{\underline{r}_j}{2} \right) \delta(\underline{r}-\underline{r}_j) + \delta(\underline{r}-\underline{r}_j) \frac{d}{dt} \left(\frac{1+\tau_{3j}}{2} \cdot \frac{\underline{r}_j}{2} \right) \right. \\
 & \left. + \frac{1}{2m} \left(\mu_p \frac{1+\tau_{3j}}{2} + \mu_n \frac{1-\tau_{3j}}{2} \right) \delta(\underline{r}-\underline{r}_j) \underline{\sigma}_j \times \underline{\nabla} \right]
 \end{aligned} \quad (1b)$$

(the $\underline{\nabla}$ is acting on the leptonic variables),

$$\rho_{em}^{nuc}(\underline{r}) = e \sum_{j=1}^A \frac{1+\tau_{3j}}{2} \delta(\underline{r}-\underline{r}_j). \quad (1c)$$

We note here that the expression of the 4-current, written above in local form, is not in general completely satisfactory, since this current is conserved only in the case of ordinary nonexchange potentials. Otherwise, it satisfies the continuity equation only in the dipole approximation, i.e.,

$$\frac{d}{dt} \int \rho(\underline{r}) \, x_k \, d^3r = - \int j_k(\underline{r}) \, d^3r. \quad (2)$$

The symbols in (1) are as follows:

e : proton charge

μ_p (μ_n): proton (neutron) magnetic moment in units of nuclear magnetons ($e/2m$)

$\underline{\sigma}_j$: spin operator of the j^{th} nucleon

$\underline{\tau}_j$: isospin operator of the j^{th} nucleon

$(d/dt)\underline{r}_j = \underline{v}_j$: velocity operator of the j^{th} nucleon

m : nucleon mass.

The above expression for the 4-current density is approximate in the following sense:

- 1) the nucleons are treated as nonrelativistic,
- 2) they are considered to be point-like (a nucleon form factor may be inserted for high-momentum transfer reactions),
- 3) each nucleon is assumed to interact independently with the electromagnetic field, so that no explicit two-body currents are present, i.e., the impulse approximation is valid /26/,
- 4) terms of the order of q^2 have been neglected (\underline{q} = momentum transfer, and $q^2 \equiv |\underline{q}|^2$).

We shall not discuss these approximations in detail here because they are standard. The matrix element between nuclear states of the Fourier transform of the electromagnetic 3-current density gives the amplitude for photon absorption

by the nuclear target. In a similar way, one may describe the scattering of electrons (with initial and final wave numbers k_i and k_f) from a nucleus which simultaneously undergoes a transition to an excited state /13, 20/. Indeed, one can imagine that the electron is scattered in the electromagnetic field of the nucleus A_μ^{nuc} , where $\square A_\mu^{\text{nuc}} = j_{\text{em},\mu}^{\text{nuc}}$.

The discussion of inelastic electron-nucleus reactions is carried out in great detail in the mentioned references. The main difference with photoreactions consists in the fact that in electron scattering, the photon absorbed by the nucleus is virtual; therefore, the transition charge density of the target also enters in the process, and $q^2 \geq \omega^2$ where q and ω are the momentum and energy of the photon, respectively. Since ω is also the nuclear excitation energy and q the momentum transfer, this means that in photoreactions where $q = \omega$, the momentum transfer is rather small (≈ 25 MeV) but in electron scattering, it can be made much larger. It is apparent from (1) that the weight of the magnetic term grows as q increases, so that electron scattering constitutes a better means than photoreactions of observing specific effects caused by this spin-dependent operator.

We shall quote here the multipole expressions for the electromagnetic transition operators /13, 20/, and shall also specialize them to the dipole operators whose matrix elements will be of separate interest to us. The charge density may be expanded in Coulomb multipoles, while the 3-current density is expanded in transverse multipoles (both electric and magnetic). With the notations of /13, 20/ the transverse electric multipole operator is written as

$$\mathcal{T}_{LM}^{el}(q) \equiv \frac{i^L}{qe} \int d^3r j_{\text{em}}^{\text{nuc}}(\underline{r}) \cdot \underline{\nabla} \times [j_L(qr) \underline{Y}_{LL}^M(\hat{r})], \quad (3a)$$

where $j_L(qr)$ is the spherical Bessel function, and \underline{Y}_{LL}^M , the vector spherical harmonic. The Coulomb multipole operator is

$$\mathcal{M}_{LM}^{\text{Coul}}(q) \equiv \frac{i^L}{e} \int d^3r \rho_{\text{em}}^{\text{nuc}}(\underline{r}) j_L(qr) Y_{LM}(\hat{r}). \quad (3b)$$

We note that in the long-wavelength approximation, where

$$j_L(qr) \approx (qr)^L / (2L+1)!! \quad (4)$$

the matrix elements of the isoscalar dipole operator ($L = 1$) vanish in the non-relativistic limit /25/ for the operator $\rho_{\text{em}}^{\text{nuc}}(\underline{r})$, see (1c). In this approximation ($q \rightarrow 0$), one has further

$$\mathcal{T}_{1M}^{el} \rightarrow \frac{\sqrt{2}}{3!!} \int d^3r r Y_{1M}(\hat{r}) \underline{\nabla} \cdot j_{\text{em}}^{\text{nuc}}(\underline{r}). \quad (5)$$

Using the continuity equation

$$\nabla \cdot \underline{j} = -i [\mathcal{H}, \rho], \quad (6)$$

where \mathcal{H} is the nuclear Hamiltonian, we obtain the familiar result known as SIEGERT's theorem /27/, generalized to the L^{th} multipole

$$\langle f | \mathcal{T}_{LM}^{e1}(q) | i \rangle_{q \rightarrow 0} \rightarrow \frac{\omega}{q} \left(\frac{L+1}{L} \right)^{1/2} \langle f | \mathcal{M}_{LM}^{\text{Coul}}(q) | i \rangle. \quad (7)$$

Sometimes, the magnetic term in the current may not be negligible, e.g., for predominantly spin-flip transitions. In this case, the approximation of (7) has to be improved /28/.

Developing $\mathcal{T}_{10}^{e1}(q)$ and using (1), one obtains after neglecting j_2 , and approximating $j_0(qr)$ by unity and $j_1(qr)$ by $qr/3$ (unretarded dipole approximation), to first order in q^2

$$\begin{aligned} \left[\mathcal{T}_{10}^{e1}(q) \right]_{q \rightarrow 0} &= \frac{-1}{(4\pi)^{1/2}} \left(\frac{2}{3} \right)^{1/2} \sum_j \frac{d}{dt} \left(\frac{1+\tau_{3j}}{2} z_j \right) \\ &- \frac{q^2}{(24\pi)^{1/2}} \frac{1}{2m} \sum_j \left(\mu_p \frac{1+\tau_{3j}}{2} + \mu_n \frac{1-\tau_{3j}}{2} \right) (\underline{r}_j \times \underline{\sigma}_j)_z. \end{aligned} \quad (8)$$

Strictly speaking, a correction term of order q^2 should have been retained in the first term of this equation. If used for the evaluation of isovector transitions, however, the magnetic term is enhanced by the factor $(\mu_p - \mu_n) = 4.7$, which gives justification for the form of (8).

In Chap. 2 we shall study matrix elements of the operator of (8) in inelastic electron scattering. It is worth noting that in the limit $q^2 \rightarrow 0$, the nucleons may be assumed to be point-like; however, for large values of q , the expression (1) for the 4-current should not contain the static values of the charge and the magnetic moments, but instead, the Dirac and Pauli form factors of the nucleons evaluated at the specific 4-momentum transfer of the reaction under consideration.

The transverse magnetic multipole operator is written as (/13, 20/)

$$\mathcal{T}_{LM}^{\text{mag}}(q) \equiv \frac{iL}{e} \int d^3r \underline{j}_{\text{em}}^{\text{nuc}}(\underline{r}) \cdot \underline{j}_L(qr) \underline{Y}_{LL}^M(\hat{r}). \quad (9)$$

In the long-wavelength limit, one obtains from this for the magnetic dipole operator ($L = 1$)

$$\begin{aligned} \left[\mathcal{T}_{10}^{\text{mag}}(q) \right]_{q \rightarrow 0} &= - \frac{1}{(6\pi)^{1/2}} \frac{q}{2m} \left[\sum_{j=1}^A \frac{1+\tau_{3j}}{2} l_j + \right. \\ &\quad \left. + \sum_j \left(\mu_p \frac{1+\tau_{3j}}{2} + \mu_n \frac{1-\tau_{3j}}{2} \right) \underline{\sigma}_j \right] \end{aligned} \quad (10)$$

where $l_j = (\underline{r}_j \times \underline{v}_j) \cdot m$ is the nucleon orbital angular momentum operator.

Next, we shall consider the weak interactions of nuclei which are semileptonic interactions, described by the coupling of the weak nuclear current to the weak leptonic current of the muon or electron, and their neutrinos. Using the same approximations as for the electromagnetic current [stated after (2)], the weak nuclear current (disregarding a pseudoscalar term) is given by /29/

$$J_{\text{weak},\mu}^{\text{nuc}\pm}(\underline{r}) = J_{\text{ax},\mu}^{\text{nuc}\pm}(\underline{r}) + J_{\text{vec},\mu}^{\text{nuc}\pm}(\underline{r}) \quad (11a)$$

(Neutral currents will be introduced in Chap. 4.),

where with the convention $\tau^{\pm} = \frac{1}{2} (\tau_1 \pm \tau_2)$

$$J_{\text{ax}}^{\text{nuc}\pm}(\underline{r}) = \sum_{j=1}^A \tau_j^{\pm} g_A \underline{\sigma}_j \delta(\underline{r}-\underline{r}_j) \quad (11b)$$

$$\rho_{\text{ax}}^{\text{nuc}\pm}(\underline{r}) = \frac{1}{2} \sum_{j=1}^A \left\{ g_A \underline{\sigma}_j \cdot \underline{v}_j - \frac{i}{2m} g_A \underline{\sigma}_j \cdot \underline{\nabla} \delta(\underline{r}-\underline{r}_j) \right\} \tau_j^{\pm} \quad (11c)$$

$$J_{\text{vec}}^{\text{nuc}\pm}(\underline{r}) = \sum_{j=1}^A \left\{ \tau_j^{\pm} g_V \delta(\underline{r}-\underline{r}_j) \left[\frac{\mu_p - \mu_n}{2m} \underline{\sigma}_j \times \underline{\nabla} \right] \right. \\ \left. + \frac{g_V}{2} \left[\frac{d}{dt} (\tau_j^{\pm} \underline{r}_j) \delta(\underline{r}-\underline{r}_j) + \delta(\underline{r}-\underline{r}_j) \frac{d}{dt} (\tau_j^{\pm} \underline{r}_j) \right] \right\} \quad (11d)$$

$$\rho_{\text{vec}}^{\text{nuc}\pm}(\underline{r}) = \sum_{j=1}^A g_V \tau_j^{\pm} \delta(\underline{r}-\underline{r}_j). \quad (11e)$$

Here, $\underline{\nabla}$ acts on the leptonic current. One should note, however, that the approximations mentioned in obtaining (11) [stated after (2)], in particular the assumed nucleon point structure, are not strictly valid for the axial current. Its nucleon matrix element is known to have a pseudoscalar form factor g_p , essentially due to one-pion exchange with the leptonic current. (A similar term, the photoelectric term, appears in pion photoproduction, to be discussed below). The pseudoscalar term has a strongly momentum-transfer dependent matrix element; and the finite nucleon size even causes momentum-transfer dependent form factors to accompany g_V and g_A which may, however, be taken constant for moderately low values of $q \leq 100 \text{ MeV}/c$ (in contrast to g_p). In (11), the assumptions of the V-A interaction, the conserved vector current hypothesis, and muon-electron universality have been used (see, e.g. /22/).

In the weak interaction, a multipole expansion may be performed also. In Chaps. 3 and 4, we shall be mainly interested in the Fermi and Gamow-Teller dipole operators, given by the expressions $\sum_j \tau_j^{\pm} \underline{r}_j$ and $\sum_j (\underline{\sigma}_j \otimes \underline{r}_j) \tau_j^{\pm}$, respectively.

In the expansion in powers of q of the Fourier transform of the weak current, the dipole terms are those which contain the first order in q . Therefore, they contribute to 1st forbidden transitions, while the zeroth-order term is called the allowed contribution.

1.3 Excitation Operators and Nuclear Structure

The experimental study of nuclear structure with electromagnetic and weak probes consists essentially of two kinds of experiments:

- 1) Elastic scattering, in which the final nuclear state is again the ground state.
- 2) Inelastic scattering (or photoabsorption processes) where the nuclear final state is an excited state.

Among other processes, we shall study the weak and electromagnetic excitation of doubly-closed-shell nuclei ${}^4\text{He}$, ${}^{16}\text{O}$ and ${}^{40}\text{Ca}$ to $T = 1$ negative parity levels. From experimental data /30/ the squares of the allowed weak matrix elements for these nuclei turn out to be $\sim 1/100$ of the squared weak matrix elements of other nuclei in general, therefore the 1st forbidden weak operators play a dominant role /31/. All these operators, i.e., $\sum_j \tau_j^\pm r_j$, $\sum_j \tau_j^\pm (r_j \otimes \sigma_j)$ together with their electromagnetic analogs $\sum_j \tau_{3j} r_j$ and $\sum_j \tau_{3j} (r_j \otimes \sigma_j)$ which are dipole operators and dipole-spin-flip operators, respectively, give rise to giant resonance phenomena, i.e., to large peaks in the differential cross section $d\sigma(\omega)/d\omega$ (where ω : energy loss of the probe = nuclear excitation energy) at excitation energies ~ 20 MeV /32/. The corresponding excited states (giant resonance states: giant resonance and spin-flip giant resonance) are usefully classified in the framework of the SU4 approximate symmetry, meaning approximate spin and isospin independence of the nuclear Hamiltonian /33/. This classification is possible due to the fact that:

- 1) these operators have a definite tensor character (vector) under this group, the generators of which are

$$\begin{aligned} T_\alpha &= \frac{1}{2} \sum_{j=1}^A \tau_{\alpha j} \\ S_\lambda &= \frac{1}{2} \sum_{j=1}^A \sigma_{\lambda j} \\ Y_{\alpha\lambda} &= \frac{1}{2} \sum_{j=1}^A \tau_{\alpha j} \sigma_{\lambda j} \end{aligned} \quad (12)$$

(the operators commute with a spin and isospin independent Hamiltonian) /34/,

- 2) the ground states of doubly closed shell nuclei are approximately scalar supermultiplets as a consequence of the short-range attractive character of nuclear