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chapter I Friction in Machinery

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The science of lubrication grew out of the need for reducing the friction in machinery. The detrimental effects of friction are chiefly:

1. The resisting force opposing the motion of bodies.
2. Loss of power owing to work done against friction.
3. Temperature rise and consequent surface damage.

These effects are discussed below, together with useful applications of friction.

1. FRICTIONAL RESISTANCE

Friction is the name given to the force resisting the relative motion of two bodies that are initially at rest or moving without acceleration. The frictional resistance is called *starting* or *static* friction while the bodies are at rest. After motion has begun it is called *kinetic* friction. The two bodies may be in direct contact, or in indirect contact through the medium of a lubricant.

Frictional resistance must be distinguished from inertia. If we pull harder than necessary to get a stationary body moving, it starts with a noticeably accelerated motion. The force applied in the direction of motion is equal to the sum of the frictional resistance and the product of mass by acceleration. The torque applied to a rotating body is equal to the sum of the frictional torque and the product of moment of inertia by angular acceleration.

Static friction on a horizontal surface can be measured by the minimum pull required for barely perceptible motion; or better, by the mean between that value and the maximum pull failing to start motion. Static friction may be measured either by a direct pull or by an adjustable inclined plane (Fig. 1). Precise values of static friction are hard to obtain since static friction increases with the duration of the load, as discovered by Coulomb, and even depends on the rate of application of the pull. Incipient sliding can be difficult to recognize because of elastic deformation and creep effects.

The measurement of kinetic friction offers no such difficulty except under low-speed conditions where "stick-slip" may occur. This action is familiar in the lubricated ways or guides of grinding machines, for example. It depends upon elasticity and inertia factors and is met in the speed range where friction diminishes with increasing speed (Chapter XIII). Kinetic friction in machinery has been measured by various methods, including direct observation of torque applied to the rotor; deceleration of the rotor; and measurement of frictional heat carried off in the oil.

Frictional resistance is of special interest when starting heavy machinery from rest and moving it slowly. Good examples are the starting of hydroelectric generators supported by large thrust bearings; cold starting of diesel engines; and assembling of freight cars on level track to make up a train. Our mention of the grinding machine, with its horizontal ways, illustrates a requirement for low friction combined with smooth motion. Another such example would be the steady rotation of a telescope structure in following the motion of the stars, as described by Professor Fuller (Chapter 1 of 1956). He showed how starting friction can be avoided by hydrostatic support, using externally pressurized bearings.

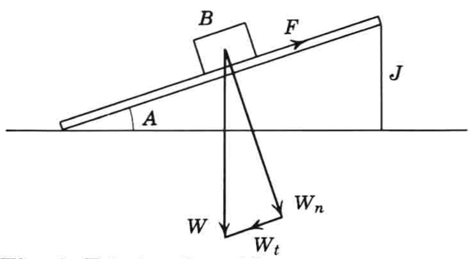

Fig. 1. Friction F on block B sliding down inclined plane (W_n, W_t: normal and tangential components of weight W).

That part of a rotating shaft supported in a bearing is called the *journal*, and the combination is called a *journal bearing*. The journal transmits a radial load to the bearing. Axial movement of the shaft is prevented by a *thrust bearing*. Either the end of the shaft or a collar on the shaft exerts an axial load on the thrust bearing.

Better control of frictional resistance in the future should facilitate plans for harnessing the power of the winds and the tides, or of solar energy, or any other source where friction losses are likely to represent a large part of the power available. Frictional resistance is discussed at length in books by Stanton (1923), Gümbel (1925), and Gemant (1950); also by Kragel'skii & co-authors (1955–56–62); and in a continued article by Palmer (1945). Vibration helps to offset the effects of static friction, as shown by Goodier (1945) in his theory of nuts and bolts. Michell notes that the size of a railway locomotive is fixed by the amount of starting friction, since that determines the size of the propelling units. Thus in the traction industry, capital cost—not just operating cost—depends on bearing friction (page 188 of 1950). The kinetic friction met by projectiles penetrating metal was determined by Krafft (1955). On friction of rigid bodies see Drescher (1959).

Historical Notes. Leonardo da Vinci discovered that the static and low-speed frictional resistances of ordinary bodies were proportional to their weights and independent of the nominal area of contact. A solid block had the same friction whether sliding on a broad flat face or on a narrow side. Smoothing or lubricating the surfaces reduced their friction. Leonardo's experiments are described by Beck (1906), Hart (1925), and Bowden & Tabor (1950). After two hundred years, Amontons (1699) rediscovered the same facts. When Coulomb (1785) learned of Amontons' observations nearly a century later, he extended them systematically. Coulomb found friction independent of speed over the short range investigated, except that kinetic friction was less than static. Hence the Amontons-Coulomb law, or "Coulomb's law" for short, namely that frictional resistance is proportional to the load and unaffected by area or speed. Coulomb set up an exponential equation for static friction as a function of the time elasped under load.

Rennie (1829) noticed that friction was greater for soft than for hard substances but depended more on the lubricant than upon the solid materials. General Morin (1832) confirmed Coulomb's law as an approximation for journal bearings. He observed that static and kinetic friction differed most for compressible materials, and discovered that starting friction could be reduced by vibration. Hirn (1854) discovered the effect of "running-in." He concluded that the friction of lubricated surfaces might be taken as roughly proportional to the square roots of the load and speed.

To anyone familiar with machinery and having a normal degree of scientific curiosity, there can hardly be a more fascinating study than the history of friction and lubrication. The subject has been outlined from several points of view by Benton (1926); the present writer (1933); Vogelpohl (1940); Fuller (1954); and by Courtel & Tichvinsky (1963).

Useful Friction. Without friction solid objects would not stay put on sloping surfaces. Wedges would slide out, corks pop out of bottles, screws and bolts would lose their hold. We should hear no more violin music; even walking would be difficult or impossible. When chromium plate became popular, one of our railroads installed shiny horizontal rods for footrests in passenger cars. There was nothing restful about those slippery rods! Foot pressure had to be precisely at right angles, for want of frictional resistance.

Friction is usefully applied in forced fits and in brakes, in belt drives and friction transmissions, and in providing locomotives and automobiles with the necessary traction to permit acceleration. See, for example, Swanger on shrink fits (1934); McCune on braking high-speed trains (1939); Thomas (1954) and Hewko (X: 1962) on friction drives with rolling contact. Friction sawing is described by Chamberland (1946); friction welding, a more recent development, by Vill' and others (1959).

2. SOME FRICTION DEMONSTRATIONS

The block sliding down an inclined plane is a classical lecture-table demonstration. In Fig. 1 a wooden plank, or metal plate, is raised to an angle A by jack J, provided with graduated drum, or other means for reading elevation. At starting or constant speed, the friction F is given by W_t or $W_n \tan A$, where W_n is the load or the normal component of the weight of the block. A striking example of very low friction is seen in the Crookes radiometer. Here the rotating arms, tipped with mica vanes, are carried by an inverted glass cup or thrust bearing resting on the point of a needle. Molecular bombardment in a high vacuum exerts greater force on the blackened side of each vane. This slight differential force is enough to overcome friction. A flash of light sets the vanes into rapid rotation. This demonstration goes far toward answering the ancient question, "how many angels can dance on the point of a needle?"

A transparent journal bearing, hand operated, oil lubricated, was devised by John Boyd (1948) to show the sudden drop in friction that accompanies oil-film formation. Pointer and scale indicate static friction coefficients of 0.1, 0.2, or more when the crank is first gently turned. The pointer drops back practically to zero as soon as a thick oil film is dragged into the clearance space. Capillary-size manometer tubes drilled in the bearing member register the film pressure. A transparent slider-block exhibits similar results when pushed along an oiled surface, all because of a taper concealed in the base. Both experiments demonstrate the principle of the convergent film, due to Reynolds (1886), discussed in the next chapter. These demonstrations and others have been described in recent articles (Tichvinsky, 1954, Hersey, 1964).

Frictional effects are suprisingly shown in the "tippe-top" or "toupie magique," which has a globular form with tall stem at upper end. When rapidly spun and then left to itself, the top soon turns a half somersault and continues to spin upside down. Explanations were given by Braams and Hugenholtz (1952) and confirmed by Pliskin (1954).

The Friction Pendulum. Instructive experiments can be made with the friction pendulum in various forms. It is commonly used for determining the average friction of an oscillating bearing. The pendulum is hung from the test shaft instead of from a knife-edge. The bearing is firmly inserted in the top end of the pendulum, which may be simply a flat stick of wood having a weight fixed to the lower end.

1. When the shaft is stationary, the oscillations of the pendulum are rapidly damped by friction in the bearing, whether it be of the plain or rolling type. The drop in height of the center of gravity of the pendulum from its initial amplitude until it comes to rest, multiplied by its weight, gives the potential energy lost. This equals the work done against friction, which is the product of the unknown frictional resistance times the sum of the distances moved over by the bearing member.

2. If the shaft is rotated at a steady speed while the pendulum is restrained from oscillating, it will stand out at a small angle. When two opposed partial bearings are pressed against the journal by a loading spring in the pendulum, the frictional torque on the journal will be measured, and the journal axis may be taken as the point of support. This method was used by Thurston and others in their early measurements on the friction of plain bearings with different lubricants and with different bearing metals at varying speeds, loads, and temperatures.

3. If the pendulum is now allowed to oscillate while supported on a rotating shaft, we have a combination known as the *Froude pendulum.* It was discovered by William Froude that such a pendulum can oscillate indefinitely without damping! Under some conditions the amplitude of the oscillations will increase. A demonstration can be made by forcing a smooth wooden sleeve onto the end of a motor shaft and letting the bearing consist in a smooth hole drilled in the top of the wooden stick. The undamped action is startling and suggests perpetual motion. It was explained qualitatively by Lord Rayleigh in his *Theory of Sound.* He likened it to bowing a violin string, which depends on the fact that the friction of rubbing surfaces decreases with increase of speed when the speed is not too high. Mathematical solutions would be desirable in order to find the limiting and the optimum conditions for a successful demonstration. See Fig. 4 in (Hersey, 1964).

The Froude pendulum was cited independently by two investigators in explaining the failure of a bascule bridge over the Hackensack River. Each leaf in such a bridge is counterweighted so that it can be raised slowly while the whole weight is supported on a trunnion bearing. While the trunnion or journal rotates steadily during the lift-up, the bearing member, nominally stationary, oscillates somewhat owing to the elasticity of the structure and the mass of the counterweight. It seems that the designers were counting on bearing friction to damp the oscillations, if anything; but instead, friction had just the opposite effect, whereupon the structure collapsed, causing one leaf of the bridge to fall into the river. The Froude pendulum, with references to the bridge investigation, was brought to our attention by L. B. Tuckerman (1938).

3. COEFFICIENTS OF FRICTION

Frictional resistance in machine elements is commonly expressed by the *coefficient of friction* f, or ratio of the frictional force F to the load, or force normal to the surface, W_n. Thus in Figs. 1 and 2, $f = F/W_n = \tan A$. In applying this ratio to fluid film bearings, F is defined as the equivalent tangential friction, and W_n may be taken as the resultant load W on the bearing. To determine F experimentally we measure the friction torque, or moment M on the member in question, say the rotor; and then write $F = M/r$, where r is the radius of the journal, or the mean radius in a thrust bearing. In either case the coefficient is given by $f = M/rW$. In fluid film bearings the journal displacement is in the direction of motion U, opposite to that for a dry bearing, as explained in Chapter II.

Frictional forces are of such a nature that the formula $F = fW$ gives only a maximum or limiting value. The frictional resistance may be anywhere from zero to fW, depending on conditions. The story is told of a civil engineering instructor who drew a bridge truss on the blackboard. An arrow pointing to the right was marked "friction." Another pointing to the left was marked "wind." The sketch was to illustrate equilibrium. "Then if the wind stops blowing," asked a student, "why doesn't the friction push the truss off the piers?" Paradoxically, it doesn't because friction never exceeds the opposing force. It builds up so as to just balance that force unless the latter exceeds fW, in which event the motion will be accelerated.

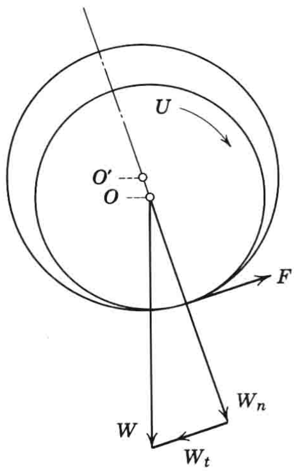

Fig. 2. Friction on the journal in a dry bearing (O, O', journal and bearing centers).

Leonardo da Vinci found $f = \frac{1}{4}$ for polished surfaces, while Amontons reported an average value of 0.3. Four-hundred years after Leonardo, his value was closely confirmed by Douglas Galton (1878), who found coefficients from 0.24 to 0.29 for the adhesion of railway wheels to the track.

During most of the nineteenth century it was customary to treat engineering problems on the assumption of a constant coefficient of friction. Great collections of examples have been worked out mathematically by Jellet and others (1872) on that assumption. Even today it is not uncommon to find calculations for elaborate mechanisms like gear wheels based on the slender assumption of Coulomb's law.

Coulomb had accepted Amontons' explanation attributing solid friction to the interlocking of asperities. It remained for the twentieth century to come up with new concepts like that of Tomlinson on the action of molecular forces (XIII: 1929), or those of Bowden and coworkers on the shearing of welded junctions (Chapter XIII). Although these investigations into the cause of friction tend to confirm the Amontons-Coulomb law, it is known that the friction coefficient ranges from practically zero to infinity under various conditions.

The lowest values are found in heavily loaded hydrostatic bearings; the highest between clean, dry solids or in fluid films at high speeds under light loads, as in vertical shaft guide bearings. These are journal bearings provided only to position the shaft and protect it from accidental radial loads. The coefficient of friction of lubricated surfaces necessarily approaches infinity as the load approaches zero, since the quotient of any finite quantity, such as the shearing resistance of an oil film, divided by zero is infinite. Values found for the coefficient with different solids and lubricants under nonhydrodynamic conditions are set forth by Dudley Fuller (1957) in his compilation for the American Institute of Physics. See also the coefficients found by Boyd & Robertson (1945) under high contact pressures, including data on solid lubricants. The variability of friction values with duration of loading and other test conditions is convincingly shown by Schmidt & Weiter (1957).

The conventional coefficient of friction is most useful when frictional resistance is nearly proportional to the load, as in dry or heavily loaded bearings. It is least useful in *concentric*, or lightly loaded bearings, where the journal axis practically coincides with the bearing axis. Here, friction is nearly independent of load. These limitations were emphasized by Dennison in his study of engine-bearing design (1936). He proposed using a different characteristic. Dennison's coefficient is proportional to the quotient of the frictional torque divided by the product of viscosity and speed. Both coefficients are dimensionless. The conventional coefficient approaches constancy when Coulomb's law is followed. Dennison's approaches constancy when Petroff's equation can be applied, that is, under hydrodynamic conditions with a lightly loaded bearing, as described in Chapter II. Styri preferred torque to coefficient in reporting friction of ball and roller bearings (Chapter X).

Another variant is the "incremental" coefficient of friction introduced by Burwell & Strang (1949). This coefficient is defined by the slope, at any point, of a graph showing frictional resistance against load. It would reduce to the conventional coefficient if the graph happened to be a straight line passing through the origin. The incremental coefficient has been useful in analyzing data on imperfect lubrication (Chapter XIII).

A distinction must be made between the coefficient of friction on the journal and that on the bearing member. There is no difference in a dry bearing, since the surfaces are in direct contact there. But in a fluid film bearing with journal appreciably eccentric, the friction torque on the journal is greater than that on the bearing (Chapter II). For coefficients less than 0.001 the difference can be as great as 50 per cent. When not otherwise specified, it will be understood that f refers to the journal coefficient. Since forces acting on a stationary body do no work, the bearing coefficient is not needed for computing power loss. Experimenters often measure the friction moment on the bearing member only; hence the need for a correction.

4. FRICTIONAL POWER LOSS

Robert H. Thurston, who became the first President of the American Society of Mechanical Engineers, was impressed by the need for greater economy in the use of power. His first book on friction and lubrication (1879) was republished under the title *Friction and Lost Work in Machinery and Millwork* (1885). In this form the book went through seven editions, stimulating widespread interest among engineers in the understanding and reduction of friction losses.

Petroff, in the meantime, was conducting a similar campaign of education in Russia. His long paper on "Friction in Machines and the Effect of the Lubricant" (1883) was awarded the Lomonosoff Prize of the Imperial Russian Academy of Sciences. It showed how the Russian supplies of petroleum could be made into suitable lubricants for reducing the waste of mechanical power, and contained formulas needed for estimating power loss.

An experienced mechanical engineer in Pittsburgh was asked by the management of a steel mill to make a complete analysis of their friction losses (page 196 of Hersey, 1936). His report was an eye-opener: total power loss 90 per cent for all machinery in the mill. From 40 to 50 per cent of the power delivered to the roll stands was consumed in the roll-neck bearings alone—a figure that has been lowered by hydrodynamic bearing design. Dr. Georg Vogelpohl (1951), a European authority, presented a survey of lubrication problems before the Third World Petroleum Congress at the Hague. Vogelpohl estimated that from one-third to one-half of the world's energy production is consumed in friction. See also the opening pages in Professor Fuller's book (1956). More study of power loss might well be included in engineering education (*Mech. Eng.* 1934; Hersey & Hopkins, 1949).

Several investigators, beginning with Thurston, who made a special study of the steam engine, have tried to pinpoint these friction losses. Sparrow & Thorne (1927) did it with some success for piston-type aviation engines; Lichty & Carson (1933) for the automobile engine, operating different parts separately; and Dutcher (1938) for various reciprocating engines. Professor Marks, on a lecture tour, compared the efficiencies of a large power windmill, a new water turbine, high-temperature steam turbines, the mercury-steam turbine, diesel engine, and gas turbine (1942). See also Takahasi & co-author (1951).

It seems that the electric utilities charge higher rates during peak load periods. Accordingly, the Baltimore Street Railways, we are told, effected a saving in power cost by reducing the viscosity grade of the lubricants used. The practice spread to other cities. Following up this lead, the American Electric Railway Engineering Association instituted experiments on friction loss in street railway reduction gears. The results made it possible to level off some of the peak-load power demands. These tests were conducted by S. A. McKee at the National Bureau of Standards, as described in Chapter XI.

The question of power loss and its distribution formed the subject of a Symposium in the Society of Automotive Engineers (1956). It was concluded that some 30 per cent of the engine power goes into friction in the normal operation of a modern automobile. Now according to Newton's first law of motion, if it were not for frictional resistance, no power would be needed to maintain a uniform speed on the road. Fuel would be required only to bring the car up to speed, and the kinetic energy of the car could be recovered and stored in some other form when stopping. A large part of the resistance at medium and high speeds is, of course, due to windage rather than friction in the mechanism.

An appraisal of the foregoing studies, aimed at greater experimental accuracy, was offered by R. E. Gish and co-authors (1958). It was followed up by Vasilica & Nica (VIII: 1963). In the course of determining power loss in engines these authors found that the pumping cycles account for about half the total loss.

Fuel Economy. Since fuel consumption parallels friction loss, it is of interest to note several investigations aimed expressly at fuel economy. Recommendations by W. H. Graves (1933) for reducing crankcase viscosity have been widely adopted with good results; and are justified by the improved construction standards, closer fits, and lower wear rates of later model automobiles.

Dr. William H. Kenerson gave us a first-hand account of a nationwide economy contest in which he and Professor Lockwood of Yale were picked to run the tests with an air-cooled engine, in their respective states. They achieved 30 to 35 miles per gallon, aided by the following procedure:

1. Choose level country and good roads.
2. Pump tires up "hard as rocks."
3. Lower the top and remove windshield.
4. Use very low viscosity oil in crankcase.
5. Accelerate at full throttle to 25 mph, then coast to a walk, and repeat.
6. Minimize air cooling, as engine is more efficient when hot.

Lockwood won the competition with Kenerson second, but the winner may have had fewer hills to climb. Another trick sometimes used, according to N. MacCoull, is to drain the transmission. These procedures reflect some of the factors responsible for power loss.

More detail is given by Greenshields (1950) in reporting a later economy contest. We are not surprised that it was won by a Studebaker, owing to the prior experience gained by leading engineers of the Company in related research at the National Bureau of Standards. Actually a lighter car went far beyond 150 mpg, but was penalized under the rules because of its light weight. New data on power loss in automotive engines will be found in papers by Wilford (1957) and by Clayton (1960). See also the references on piston and ring lubrication in Chapter VIII.

Railroad fuel consumption is discussed by W. M. Keller in a conference arranged by the National Research Council of Canada (1962). He informs us that "the railroads of North America have a fuel bill of over a million dollars a day" (p. 180). Since most of the fuel is needed for overcoming friction, he suggests that if the loss could be reduced only 1 per cent, it might save $10,000 a day.

5. FRICTIONAL HEATING

Without friction, who could strike a match? Or even start fire by twirling a pointed stick? Friction sawing and welding, previously mentioned, are useful effects of the temperature rise caused by friction. Yet, we are more frequently aware of detrimental effects. For example, we often read of people severely burned by sliding down ropes. Describing frictional heat, Edward Turner, author of an early chemistry textbook (1832), states that "The axle-tree of carriages has been burned from this cause, and the sides of ships have taken fire by the rapid descent of the cable."

The man on the street tries to keep well informed on the "re-entry" problem as affected by atmospheric friction, but he may not know how much of the temperature rise is due to sudden compression and how much to shearing the air film. He probably does not know that every 778 ft-lb of work done against friction generates nearly 1 Btu of heat. Where does each Btu go? If absorbed by a pound of water, it makes the temperature rise 1 deg F. Absorbed by an equal weight of oil, the rise will be over 2 deg; of metal, from 5 to 12 deg, the latter figure for steel.

When a lightly loaded full journal bearing 3 in. long by 3 in. in diameter and of customary clearance operates at 1800 rpm with a medium viscosity oil, the frictional power loss will come to about $\frac{2}{3}$

hp, or $\frac{1}{2}$ kw. If the heat generated thereby were to remain in the oil, where it originates, and if the oil were to stay put in the clearance space, the film temperature would rise initially at the rate of 750 deg F/sec! In practice the heat is removed by oil flow, by conduction into the shaft and bearing, and otherwise—a very central problem in the theory of lubrication (Chapter III).

Extremely high temperatures can be reached in dry friction at high speeds. Railway brake shoes were tested up to surface speeds of a mile a minute, or 88 ft/sec by Galton in his historic investigation. The decrease in friction with increasing speed found in those tests may have been due in part to air-film lubrication. Bowden & Ridler (1936) reported early experiments on surface temperatures in which surface melting was observed at speeds of the order of 50 ft/sec. Many investigations to 110 ft/sec were conducted by the National Advisory Committee for Aeronautics, and described by E. E. Bisson and co-authors (1957, 1964). Friction was measured at speeds to 2000 ft/sec by W. W. Shugarts, Jr., and others (1953) at the Franklin Institute incident to interior ballistics. Speeds of 1000 ft/sec were reached by J. M. Kraft (cited previously) in his experiments on ballistic penetration. A new type of friction apparatus was described by Bowden & Freitag (1955), by which the friction of metals was investigated in a vacuum at speeds approaching 3000 ft/sec. The characteristic drop in friction with increasing speed was confirmed without the complication of an air film. The drop was attributed to lack of time for propagation of plastic deformation, together with the high temperatures developed.

Hotboxes. Whenever the frictional heat in a bearing accumulates faster than it can be removed at a maximum safe temperature, an unstable condition is reached known as a "hotbox." Axles have been broken and railroad cars set afire, thus leading to many investigations. The wrecking of the Congressional Express with 79 lives lost is an extreme example (Associated Press, 1943). Whether caused by lack of oil or mechanical faults, hotboxes came to be recognized as a definite problem. See Andersen (1953) and Hawthorne (1953). Automatic detectors have been invented and usefully applied in freight and passenger service, although Downes (1947) advocated roller bearings as the only certain remedy.

Surface Damage. The temperature rise caused by friction aggravates wear and surface failure in machine elements. These conditions are well described by Barwell (1956), Bisson and co-authors (1957, 1964), and Wilcock (1957). Goodzeit (1956), Roach (1956), and others experimented on the friction and surface damage of a large number of different metals in sliding contact with steel. Friction and wear tests have been carried to 1000 C (over 1800 F) in a study by E. P. Kingsbury & Ernest Rabinowicz (1959). Further references to frictional damage are given in Chapters X–XIII.

6. ROLLING FRICTION

Starting friction can often be reduced by substituting rolling in place of sliding contact; hence the use of wheels, rollers, and rolling bearings. According to Coulomb's law of rolling friction, the resistance to rolling will be proportional to the load, independent of the speed, and inversely proportional to the radius of the rolling body. This approximate relation remains in common use even today, with the aid of handbook data, because of its convenient form. In symbols, $F = \text{const}\ (W/r)$, where F is the force needed to overcome rolling resistance when applied in the direction of motion at the axis of the roller, W is the total load, and r the radius.

But consider the meaning of the constant of proportionality. In order that a body shall experience rolling friction, it must either be deformable, or traveling over a deformable surface, or both. The resultant upward force W' that supports a deformable body moving in response to horizontal forces will act off center. It will be displaced *forward* of the load line by some small distance e that may be called the eccentricity (Fig. 3). That distance is the moment arm of a couple formed by the load W and the equal and parallel supporting force

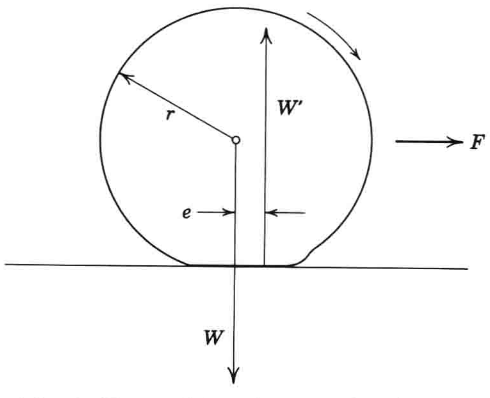

Fig. 3. Eccentricity of supporting force on deformable roller, on rigid surface.

W'. The couple is the source of the rolling resistance. It is balanced by the moment of the pull F acting at a height r above the contact area. Under equilibrium conditions, $F = e(W/r)$.

The constant in Coulomb's law will therefore be recognized as the eccentricity of the supporting force, or moment arm of rolling friction. Some textbooks call this length e the "coefficient of rolling friction." It would seem more logical to define the coefficient of rolling friction by the ratio F/W. To avoid confusion we shall not use the term "coefficient of rolling friction" for the time being; speaking, instead, of the *rolling friction ratio* for F/W, and the *eccentricity* for the moment arm e. Handbooks, in the light of Coulomb's law, usually give values of e on the assumption that they are constant for any given pair of materials, regardless of the roller radius r. The fallacy of that assumption can readily be seen from the fact that the moment arm cannot possibly exceed a fraction of the radius; hence with diminishing values of the radius, the eccentricity must continually decrease. It is easy to see that e must approach zero as r approaches zero.

Dupuit (1842) was apparently the first to offer a rational formula in place of Coulomb's. He confirmed the proportionality of rolling friction to load, but found it inversely proportional to the square root of the radius. This is equivalent to finding e directly proportional to the square root of the radius. Experiments conducted with cast-iron car wheels on steel rails by H. E. Wetzel (1924), M. S. Downes (1925), and the writer tend to support Dupuit's law with respect to the influence of the radius.

The rolling friction ratio F/W diminished slightly with increasing load, and increased appreciably with increasing speed over the limited range of our tests. A value of about 0.004 was found by interpolation for 10-in. diameter wheels with machined treads, running from 3 to 4 mph under a load of 750 lb per wheel (Wetzel, 1924). This comes to nearly $\frac{1}{3}$ of the total friction in a roller-bearing mine car truck, under the conditions stated. Further data are given by the Engineering Foundation (1946), and a more complete report is in preparation. The tests were run at the Pittsburgh Experiment Station of the U. S. Bureau of Mines as part of a cooperative program on mine car friction (Hersey, et al., 1925). See also Chapter X.

An amusing experience in this connection illustrates the significance of rolling friction (Hersey, 1936, p. 194). A ball-bearing car was submitted by the bearing manufacturer in competition with three types of roller-bearing cars. To the surprise of everyone, the ball-bearing car, No. 1, showed the highest friction of the lot. One of the roller-bearing cars, No. 2, gave almost double the friction expected. It was