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DIFFERENTIAL GEOMETRY IN ARRAY PROCESSING

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To my wife, Eleni

Preface

During the past few decades, there has been significant research into sensor array signal processing, culminating in the development of super-resolution array processing, which asymptotically exhibits infinite resolution capabilities.

Array processing has an enormous set of applications and has recently experienced an explosive interest due to the realization that arrays have a major role to play in the development of future communication systems, wireless computing, biomedicine (bio-array processing) and environmental monitoring.

However, the "heart" of any application is the structure of the employed array of sensors and this is completely characterized [1] by the *array manifold*. The *array manifold* is a fundamental concept and is defined as the locus of all the response vectors of the array over the feasible set of source/signal parameters. In view of the nature of the array manifold and its significance in the area of array processing and array communications, the role of differential geometry as the most particularly appropriate analysis tool, cannot be over-emphasized.

Differential geometry is a branch of mathematics concerned with the application of differential calculus for the investigation of the properties of geometric objects (curves, surfaces, etc.) referred to, collectively, as "manifolds". This is a vast subject area with numerous abstract definitions, theorems, notations and rigorous formal proofs [2,3] and is mainly confined to the investigation of the geometrical properties of manifolds in three-dimensional Euclidean space $\mathcal{R}^3$ and in *real* spaces of higher dimension.

However, the array manifolds are embedded not in real, but in N-dimensional complex space (where N is the number of sensors). Therefore, by extending the theoretical framework of $\mathcal{R}^3$ to complex spaces, the underlying and under-pinning objective of this book is to present a summary of those results of differential geometry which are exploitable and of practical interest in the study of linear, planar and three-dimensional array geometries.

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Keywords

linear arrays, non-linear arrays, planar arrays, array manifolds, differential geometry, array design, array ambiguities, array bounds, resolution and detection.

Contents

Chapter 1

Introduction

An array system is a collection of sensors (transducers) which are spatially distributed at judicious locations in the 3-dimensional real space, with a common reference point. How the sensors are spatially distributed (array geometry) is influential not only on the overall array capabilities but also on its "abnormalities." The type of the sensors varies with the application and sensors can take a wide variety of forms. Some common examples of sensors include electromagnetic devices (such as RF antennas, optical receivers, etc.) and acoustic transducers (such as hydrophones, geophones, ultrasound probes, etc.).

The signals at the array elements contain both temporal and spatial information about the array signal environment which is usually contaminated by background and sensor noise. Thus, the main aim of array processing is to extract and then exploit this spatio-temporal information to the fullest extent possible in order to provide estimates of the parameters of interest of the array signal environment. Depending on the application, typical parameters of interest associated with emitting sources (i.e. signals that use the same frequency and/or time-slot and/or code) can be the number of incident signals, Directions-of-Arrival (DOAs), Times-of-Arrival (TOAs), ranges, velocities etc. Indeed, with an array system operating in the presence of a number of emitting sources, and by observing the received array signal-vectors $\underline{x}(t)$, the following four general problems are of great interest:

(1) Detection problem — concerned with the determination/estimation of the number of incident signals. This problem is essentially the spatial analogue of model order selection in time-series analysis. Thus, the most popular methods for the solution of this problem are based on "Akaike Information Criterion" (AIC) [4] and the "Minimum Description Length" (MDL) criterion [5, 6]. Both methods involve the

minimization of a function of the noise eigenvalues of the array output covariance matrix.

(2) Parameter estimation problem — where various signal and channel parameters are estimated. One important problem of this type is the "Direction Finding" (DF). In this case the parameters of interest are the bearings of emitters/targets (e.g. [7]). This problem is essentially the spatial analogue of the frequency estimation problem in time-series analysis.

(3) Interference cancellation (or reception problem) — the acquisition of one (desired) signal from a particular direction and the cancellation of unwanted co-channel interfering signals (or jammers), from all other directions. When the desired signal and the interference occupy the same frequency band, temporal filtering is inappropriate. However, the spatial separation of the sources can be exploited using an array of sensors (e.g. an antenna array). This operation falls, in array processing terms, under the general heading of "beamforming" while, in communication systems terms, a beamformer is a "linear receiver" (e.g. [8]).

(4) Imaging — here the parameters of interest are the shapes and sizes of various objects in the environment. These are typically determined by the generation of two- or three-dimensional maps depicting some feature of the received signals (e.g. intensity) as a function of their spatial coordinates (e.g. [9]).

The four types of problem described above are inter-related and the solution to one problem may result in a partial or complete solution to another. For example, the successful operation of all parametric parameter estimation algorithms requires solving firstly the detection problem (i.e. *a priori* knowledge of the number of emitters present). Furthermore, once the number and directions-of-arrival (DOAs) of signals received at the array site are estimated by solving the detection and direction-finding problems, nulls may be readily placed along the directions of the unwanted signals, hence achieving interference cancellation.

The applications of arrays in various scientific disciplines (such as the ones already mentioned) are extensive and suffice to reveal the multi-dimensional significance of the array concept. For instance, although array processing has been extensively used in high frequency communications in the past, the explosive growth in demand for cellular services in recent years has placed it at the centre of interest. Spatial diversity is considered to be one of the most promising solutions for increasing capacity and spectral efficiency. Indeed, the integration of array processing and communications techniques, exploiting the structure of antenna-array systems, has evolved into a well-established technology. This technology is moving from the conventional direction nulling and phase-arrays to advanced *superresolution spatiotemporal-arrays*, MIMO array systems and arrayed wireless sensor networks, which exploit the spatial and temporal properties of the channel in their quest to handle multipaths, and to increase capacity and spectral efficiency. Using these properties, an extra layer of co-channel interference (CCI) and inter-symbol-interference (ISI) cancellation is achieved — asymptotically providing complete interference cancellation.

The performance of array systems, especially the ones with superresolution capabilities is, in general, limited by three main factors:

- The presence of inherent background and sensor noise.
- The limited amount of information the sensors can measure due to finite observation interval (number of snapshots) and array geometry.
- The lack of calibration, modelling errors and system uncertainties that are embedded in the received array signal-vector $\underline{x}(t)$, which are not accounted for. Examples include uncertainties in mutual coupling between sensors, perturbations in the geometrical and electrical characteristics of the array, the presence of moving emitters, nonplanar wavefronts, source angular/temporal spread, etc.

However, the overall quality of the system's performance is naturally a function of the array structure in conjunction with the geometrical characteristics of the signal environment, as well as the algorithms employed. An algorithm would behave differently when used with different array structures and, vice-versa, a certain array would generate different results when its output is applied to different algorithms.

1.1 Nomenclature

It is assumed that the reader is familiar with the fundamentals of vector and matrix algebra. In this book, for typographical convenience, matrices will be denoted by blackboard bold symbols (e.g. $\mathbb{A}, \mathbb{T}, \mathbb{I}$) or, in the absence of a corresponding blackboard bold symbol, by boldface (e.g. $\mathbf{r}$, $\mathbf{k}$, $\mathbf{\Gamma}$). Any underlined symbol will represent a column vector, e.g. $\underline{A}$, $\underline{a}$, $\underline{\mathbf{a}}$. Derivatives with respect to a general parameter p will be denoted with a "dot" (e.g. $\dot{\underline{\mathbf{a}}}$), while the "prime" symbol (e.g. $\underline{\mathbf{a}}'$) will be reserved for differentiation with respect to certain "invariant" parameters. The overall notation to be employed in this book is as follows:

A, a	Scalar
$\underline{A}, \underline{a}$	Column vector
$\mathbb{A}, \mathbf{A}$	Matrix
$(\cdot)^T$	Transpose
$(\cdot)^H$	Hermitian transpose
$(\cdot)^\dagger$	Pseudoinverse
$\|\cdot\|_F$	Frobenius norm of a matrix
$\|\cdot\|$	Norm of a vector
$\lvert \cdot \rvert$	Magnitude
$\odot, \oslash$	Hadamard (Schur) product and division respectively
$\otimes$	Kronecker product
$\exp(\underline{A} \text{ or } \mathbb{A})$	Elementwise exponential of vector $\underline{A}$ or matrix $\mathbb{A}$
$\text{expm}(\mathbb{A})$	Matrix exponential
$\text{Tr}(\mathbb{A})$	Trace of matrix $\mathbb{A}$
$\det(\mathbb{A})$	Determinant of $\mathbb{A}$
$\text{diag}(\underline{A})$	Diagonal matrix formed from the elements of $\underline{A}$
$\underline{\text{diag}}(\mathbb{A})$	Column vector consisting of the diagonal elements of $\mathbb{A}$
$\text{row}_i(\mathbb{A})$	i^{th} row of $\mathbb{A}$
$\text{ele}_{ij}(\mathbb{A})$	(i^{th}, j^{th}) element of $\mathbb{A}$
$\text{fix}(A)$	Round down to integer
$\mathcal{E}\{\cdot\}$	Expectation operator
$\underline{A}^b$	Element by element power
$\underline{0}_N$	Zero vector of N elements
$\underline{1}_N$	Column vector of N ones
$\mathbb{I}_N$	$N \times N$ Identity matrix
$\mathbb{O}_{N\times d}$	$N \times d$ Zero matrix
$\mathcal{R}$	Set of real numbers
$\mathcal{N}$	Set of natural numbers
$\mathcal{Z}$	Set of integer numbers
$\mathfrak{C}$	Field of complex numbers

1.2 Main Abbreviations

AGS	Ambiguous Generator Set
ELA	Equivalent Linear Array
CRB	Cramer Rao Bound
DF	Direction Finding
DOA	Directions of Arrival

FOV	Field-of-View
SNR	Signal-to-Noise Ratio
RMS	Root Mean Square
ULA	Uniform Linear Array
UCA	Uniform Circular Array

1.3 Array of Sensors — Environment

By distributing, in the 3-dimensional Cartesian space, a number $N \geqslant 2$ of sensors (transducing elements, antennas, receivers, etc.) with a common reference point, an array is formed. In general, the positions of the sensors are given by the matrix $\mathbf{r} \in \mathcal{R}^{3\times N}$

$$\mathbf{r} = \left[\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_N\right] = \left[\underline{r}_x, \underline{r}_y, \underline{r}_z\right]^T \tag{1.1}$$

with $\underline{r}_k = [\mathrm{x}_k, \mathrm{y}_k, \mathrm{z}_k]^T \in \mathcal{R}^{3\times 1}$ denoting the Cartesian coordinates (location) of the kth sensor of the array $\forall k = 1, 2, \ldots, N$.

It is common practice to express the direction of a wave impinging on the array in terms of the azimuth angle θ, measured anticlockwise from the positive x-axis, and the elevation angle ϕ, measured anticlockwise from the x-y plane, as illustrated in Fig. 1.1. Then, the (3×1) real unit-norm vector

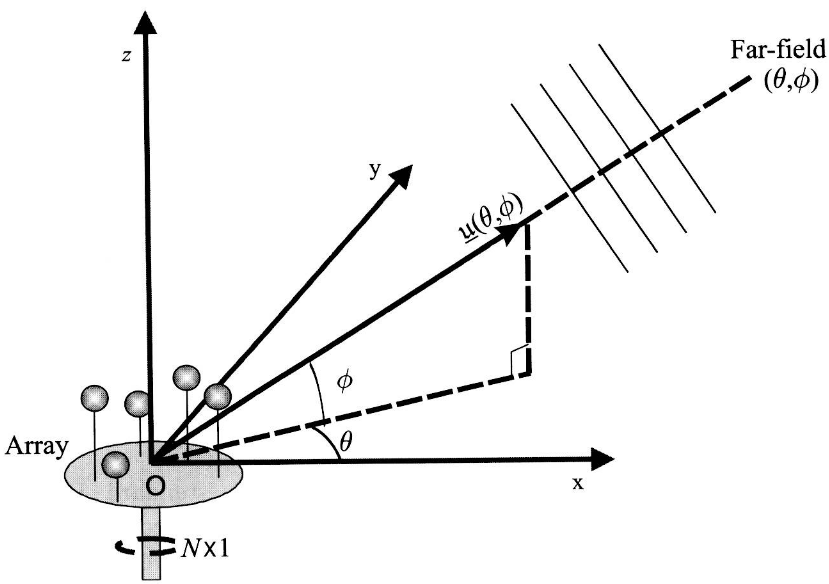

Fig. 1.1 Relative geometry between a far-field emitting source and an array of sensors.

pointing towards the direction (θ, ϕ) is

$$\underline{u} \triangleq \underline{u}(\theta, \phi) = \left[\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi\right]^T \tag{1.2}$$

Note that $\|\underline{u}\| = 1$. If the velocity, wavelength and frequency of propagation of the incident wave is denoted by c, λ and F_c, respectively, then the wavenumber vector in the direction (θ, ϕ) is defined as

$$\underline{k} = \underline{k}(\theta, \phi) = \begin{cases} \dfrac{2\pi F_c}{c} \cdot \underline{u} = \dfrac{2\pi}{\lambda} \cdot \underline{u} & \text{in meters} \\ \pi \cdot \underline{u} & \text{in } \lambda/2 \end{cases} \tag{1.3}$$

In the most general case, the parameter space is

$$\Omega = \{(\theta, \phi) : \theta \in [0^\circ, 360^\circ) \quad \text{and} \quad \phi \in (-90^\circ, 90^\circ)\} \tag{1.4}$$

but in most applications, Ω is restricted to only a sector of interest or, in other words, field-of-view (FOV). For instance, in the case of ground surveillance radars, only signals in the plane of the array are of interest — i.e. the system is azimuth-only.

The array configuration is, to a large extent, dictated by the application of interest. One obvious restriction is the shape and size of the available site, which might be, to cite just a few examples, an aircraft's wing, a ship's hull, a building rooftop, or simply a terrain. In addition, if the signals to be intercepted are known to be coplanar and within a 180° field-of-view, as in ground and marine navigation applications, then a linear or 1-dimensional (1D) array of sensors may be sufficient.

(1) *Linear or 1-dimensional (1D) Array.*
The linear or 1D array consists of a one-dimensional distribution of sensors along a line conventionally taken as the x-axis (Fig. 1.2(a)), with sensor positions in units of half-wavelengths given by the matrix

$$\mathbf{r} = [\underline{r}_x, \underline{0}_N, \underline{0}_N]^T \in \mathcal{R}^{3\times N} \tag{1.5}$$

where

$$\underline{r}_x = [r_1, r_2, \ldots, r_N]^T$$

The most popular array of this type is the standard Uniform Linear Array (ULA) whose sensors are uniformly spaced at one half-wavelength apart along the x-axis. For example, a 5-sensor standard