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With 80 Figures

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ISBN 3-540-22247-2 Springer Berlin Heidelberg New York

Library of Congress Control Number: 2004107594

Springer. Part of Springer Science+Business Media
springeronline.com

Typesetting: data delivered by author
Cover design: deblik Berlin
Printed on acid free paper 62/3020/M - 5 4 3 2 1 0

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For Irmi, my wife

Preface

Partial differential equations are the basis for nearly all technical processes in heat transfer and fluid mechanics. In my lectures over the past seven years I became aware of the fact that a lot of the students studying mechanical or aerospace engineering and also a lot of the engineers in industry today focus more and more on numerical methods for solving these partial differential equations. Analytical methods, taught in undergraduate mathematics, in thermodynamics and fluid mechanics, are quickly discarded, because most people believe that almost all problems, appearing in *real* applications, can easily be solved by numerical methods. In addition, most of the examples shown in basic lectures are *so simple* that the students develop the impression that analytical methods are inappropriate for more complicated *realistic technical* problems.

It was exactly the above described mind set, which inspired me several years ago to give lectures on analytical methods for heat and mass transfer problems. The basic idea of these lectures is to show some selected analytical methods and to explain their application to more complicated problems, which are technically relevant. Of course, this means that some of the standard analytical methods, might not be discussed in these lectures and are also not present in this book (for example integral transforms). On the other hand, it can be shown that the analytical methods discussed here are applicable to interesting problems and the student or engineer learns how to solve useful technical problems analytically.

This means that the main intent of this book is to show the usefulness of analytical methods, in a world, which focuses more and more on numerical methods. Of course, there is no doubt that the knowledge of numerical solution methods is very important and there is a big chance in using numerical tools to gain inside into flow physics and heat transfer characteristics. However, numerical methods are always dependent on grid quality and grid size and also on a lot of implementation features. Analytical methods can be used to validate and improve numerical methods. So the engineer might simplify a problem up to the extent that he can obtain an analytical solution. This analytical solution might be used afterwards to check and to improve the numerical solution for the full problem without any simplifications.

This book has been written for graduate students and engineers. The mathematics needed to understand the solution approach is developed mostly during the actual solution of the problem under consideration. This means that the book includes only few proofs. The reader is referred in these situations to other books for these more basic mathematical considerations. This approach has been taken in order to keep the focus of the book on the solution method itself and not to disrupt the analysis of the technical problem.

The book is structured in six chapters. Chapter 1 provides a short introduction to the topic.

Chapter 2 provides an introduction to the solution of linear partial differential equations. After discussing the classification and the character of the solutions of second-order partial differential equations, the method of separation of variables is discussed in detail.

Chapter 3 is concerned with the solution of thermal entrance problems for pipe and channel flows. This means that solutions of the energy equation are considered for a hydrodynamically fully developed velocity profile. These problems lead to the solution of Sturm-Liouville eigenvalue problems, which are discussed for laminar and turbulent flows and for different wall boundary conditions. For the problems considered in Chap. 3, axial heat conduction within the flow can be ignored, because the Peclet number in the flow is sufficiently large. This means that the problems under consideration are parabolic in nature. Because this sort of eigenvalue problems normally cannot be solved analytically, a numerical procedure is discussed on how to solve them. In Appendix C, this numerical solution method is explained in detail and the reader is provided links to an internet page, containing several source codes and executables.

Chapter 4 explains analytical solution methods for Sturm-Liouville eigenvalue problems for large eigenvalues. Here the focus is to explain an asymptotic analysis for a complicated problem, which is technically relevant. This chapter also provides comparisons between numerically and analytically predicted eigenvalues and constants. These comparisons show the usefulness of the analytical solution. Furthermore, it is explained how the method can be used for related problems.

In Chapter 5 the heat transfer in pipe and channel flows for small Peclet numbers is considered. In contrast to the problems discussed in Chaps. 3-4, the axial heat conduction in the fluid cannot be ignored. This leads to elliptic problems. A method is presented, which gives rise to solutions, which are as simple as the ones presented in Chap. 3. The extension of this method to more complicated problems, for example for the heat transfer in hydrodynamically fully developed duct flows with a heated zone of finite length, is also explained.

Chapter 6 is devoted finally to the solution of nonlinear partial differential equations. The idea behind this chapter was to provide a short overview about different solution methods for nonlinear partial differential equations. However, the main focus is on the derivation of similarity solutions. Here, different solution methods are explained. These are the method of dimensional analysis, group-theory methods and the method of the free parameter. The methods are demonstrated for a simple heat conduction problem as well as for complicated boundary layer problems.

Many people helped me in all phases of the preparation of this book. I am very grateful for many helpful discussions with my colleague Prof. Jens von Wolfersdorf concerning all aspects of the analytical solution methods. I also thank very much Martin Stricker and Marco Schüler who helped me with the figures. Many thanks also go to Dr. Grazia Lamanna for the helpful discussions and her support

finishing this book. Also, I would like to thank Karl Straub very much for reading the manuscript.

I kindly acknowledge the permission of the ASME for reprinting the Figs. 1.2-1.3 and of ELSEVIER for reprinting the Figs. 3.7, 3.13-3.15, 5.1, 5.9-5.13, 5.18-5.19, 5.25 in this book. I also kindly acknowledge the permission of the Council of Mechanical Engineers (IMECHE) for reprinting Fig. 3.12, of KLUWER for reprinting the Figs. 5.3-5.4, B2-B7 and of SPRINGER for reprinting the Figs. 3.8, 3.9, 3.17-3.19 in this book. In addition, I kindly acknowledge the permission of Prof. E. Papoutsakis for reprinting the Figs. 5.3-5.4 and of Prof. Osterkamp, Dr. Zhang and Dr. Gosink for reprinting Fig. 1.4 in this book. The reference of the paper, where the figures have originally been published, is always included in the individual figure legend.

Finally, I am very grateful for the very good cooperation with Springer Press during the preparation of this manuscript. Here I would like to thank in particular Mrs. Maas, Mrs. Jantzen and Dr. Merkle for their support.

Filderstadt, April 2004 Bernhard Weigand

List of Symbols

a	[m^2/s]	heat diffusivity
a_1, a_2	[-]	functions
A	[m^2]	flow area
A_j	[-]	constants
Bi	[-]	Biot number
c	[m]	velocity of sound
c_f	[-]	friction factor
c_p	[J/(kg K)]	specific heat at constant pressure
C	[-]	Chapman-Rubesin parameter
D	[m]	hydraulic diameter
E	[-]	dimensionless energy flow
F	[-]	flow index (0 for planar channel, 1 for pipe)
F_x, F_y, F_z	[N]	forces
G	[-]	function
h	[W/(m^{2}K)]	heat transfer coefficient
h	[m]	half channel height of a planar channel
$\mathrm{J}_i(s)$	[-]	Bessel function of order i
k	[W/(m K)]	thermal conductivity
k	[-]	transformed eigenvalue
K	[W/m^3]	sink intensity
L	[m]	length scale (h for planar channel, R for pipe)
L_{th}	[m]	thermal entrance length
$\underset{\sim}{L}[\ ]$	[-]	matrix operator
l	[m]	mixing length
$M[\ ]$	[-]	operator
Ma_∞	[-]	Mach number
Nu_L	[-]	Nusselt number based on L
Nu_∞	[-]	Nusselt number for the fully developed flow
N	[-]	rotation rate
n	[m]	coordinate orthogonal to the flow direction
p	[Pa]	pressure
Pr	[-]	Prandtl number
Pr_t	[-]	turbulent Prandtl number

Pe_L	[-]	Peclet number based on L
Pe_t	[-]	turbulent Peclet number
R	[m]	pipe radius
R	[J/(kg K)]	gas constant
Re_L	[-]	Reynolds number based on L
Ri	[-]	Richardson number
t	[s]	time
T	[K]	temperature
T'	[K]	temperature fluctuation
T_W	[K]	wall temperature
T_b	[K]	bulk–temperature
u_τ	[m/s]	shear velocity
U	[m]	wetted perimeter
$\overline{u}$	[m/s]	mean velocity
u, v, w	[m/s]	velocity components
u', v', w'	[m/s]	fluctuating velocity components
V	[-]	dimensionless velocity gradient at the wall
x, y, z	[m]	coordinates
y^+	[-]	wall coordinate
Z	[-]	modified rotation parameter

Greek letter symbols

β	[1/K]	volumetric coefficient of expansion
β_1, β_2	[-]	constants
Γ	[-]	gamma function
ε_m	[m^2/s]	eddy diffusivity for momentum transfer
$\varepsilon_{hx}, \varepsilon_{hy}, \varepsilon_{hr}$	[m^2/s]	eddy diffusivity for heat transfer
ξ, η	[-]	characteristic coordinates
η	[-]	similarity variable
ϑ	[-]	transformed eigenfunction
Θ	[-]	dimensionless temperature
Θ_b	[-]	dimensionless bulk-temperature
λ_j	[-]	eigenvalue
μ	[kg/(m s)]	dynamic viscosity
ν	[m^2/s]	kinematic viscosity
ρ	[kg/m^3]	density
τ	[-]	dimensionless time
ϕ	[-]	enthalpy function

Φ_j	[-]	eigenfunctions
Φ_{Dis}	[1/s^2]	dissipation function
Φ	[m^2/s]	velocity potential
Ψ	[m^2/s]	stream function
ω	[1/s]	angular velocity

Subscripts

0	refers to inlet conditions
C	centerline of the duct
∞	free stream conditions
~, +	dimensionless quantities

Definition of non-dimensional Numbers

$\mathrm{Bi} = \dfrac{hD}{k}$	Biot number
$C = \dfrac{\rho\mu}{\rho_\infty \mu_\infty}$	Chapman-Rubesin parameter
$\mathrm{N} = \dfrac{\mathrm{Re}_\varphi}{\mathrm{Re}_D}$	Rotation rate
$\mathrm{Nu}_L = \dfrac{hL}{k} = \dfrac{-\left.\dfrac{\partial T}{\partial n}\right\vert_W L}{T_W - T_b}$	Nusselt number
$\mathrm{Ma}_\infty = \dfrac{u_\infty}{\sqrt{\kappa p / \rho}}$	Mach number
$\mathrm{Pe}_L = \dfrac{\bar{u}\, L}{a} = \mathrm{Re}_L\, \mathrm{Pr}$	Peclet number
$\mathrm{Pr} = \dfrac{\mu c_p}{k} = \dfrac{\nu}{a}$	Prandtl number
$\mathrm{Re}_L = \dfrac{\bar{u}\, L}{\nu}$	Reynolds number

$\mathrm{Re}_{\varphi} = \dfrac{w_W D}{\nu}$	Rotational Reynolds number
$\mathrm{Re}_{\tau} = \dfrac{u_{\tau} L}{\nu}$	Shear stress Reynolds number
$\mathrm{Ri} = \dfrac{2\dfrac{w}{r}\dfrac{\partial}{\partial r}(w r)}{\left(\dfrac{\partial u}{\partial r}\right)^2 + \left(r\dfrac{\partial}{\partial r}\left(\dfrac{w}{r}\right)\right)^2}$	Richardson number
$\mathrm{Z} = \mathrm{N}\dfrac{\mathrm{Re}_D}{2\,\mathrm{Re}_{\tau}} = \mathrm{N}/\sqrt{\dfrac{c_f}{8}}$	Modified rotation parameter

List of Copyrighted Figures

I kindly acknowledge the permission of the ASME for reprinting the Figs. 1.2-1.3, which have been published first in Burow P, Weigand B (1990) One-dimensional heat conduction in a semi-infinite solid with the surface temperature a harmonic function of time: A simple approximate solution for the transient behavior. Journal of Heat Transfer 112: 1076 – 1079 (Fig. 1).

I kindly acknowledge the permission of ELSEVIER for reprinting the Figs. 3.7, 5.9 and 5.18 which have been published first in Weigand B, Ferguson JR, Crawford ME (1997a) An extended Kays and Cawford turbulent Prandtl number model, Int. J. Heat Mass Transfer, 40: 4191- 4196 (Figs. 3-4). In addition, I kindly acknowledge the permission of ELSEVIER for reprinting the Figs. 3.13-3.15, which have been first published in Reich G, Beer H (1989) Fluid flow and heat transfer in an axially rotating pipe-I. Effect of rotation on turbulent pipe flow, Int. J. Heat Mass Transfer 32: 551-562 (Figs. 1, 3-4) and I kindly acknowledge the permission of ELSEVIER for reprinting the Figs. 5.1, 5.10 - 5.11, which have been first published in Weigand B (1996) An exact analytical solution for the extended turbulent Graetz problem with Dirichlet wall boundary conditions for pipe and channel flows, Int. J. Heat Mass Transfer 39: 1625-1637 (Figs. 1, 5-6). In addition, I kindly acknowledge the permission of ELSEVIER for reprinting the Figs. 5.12-5.13, 5.19 which have been first published in Weigand B, Kanzamar M, Beer H (2001) The extended Graetz problem with piecewise constant wall heat flux for pipe and channel flows, Int. J. Heat Mass Transfer 44: 3941-3952 (Figs. 1, 9).

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I kindly acknowledge the permission of KLUWER for reprinting the Figs. 5.3-5.4, which have been first published in Papoutsakis E, Ramkrishna D, Lim H (1980) The extended Graetz problem with Dirichlet wall boundary conditions, Applied Scientific Research 36: 13-34 (Figs. 1-3) and I kindly acknowledge the permission of KLUWER for reprinting Figs. B2-B7, which have been first published in Weigand B, Beer H (1994) On the universality of the velocity profiles of a turbulent flow in an axially rotating pipe, Applied Scientific Research 52: 115-132 (Figs. 5-7, 9-10, 12).

I kindly acknowledge the permission of SPRINGER for reprinting the Figs. 3.8, 3.9, 3.17-3.19 which have been first been published in Weigand B, Beer H (1989) Wärmebertragung in einem axial rotierenden, durchströmten Rohr im Bereich des thermischen Einlaufs, Wärme- und Stoffübertragung 24: 191-202

(Figs. 4 - 6, 9) and I kindly acknowledge the permission of SPRINGER for reprinting Fig. 5.25 which has been first published in Weigand B, Wrona F (2003) The extended Graetz problem with piecewise constant wall heat flux for laminar and turbulent flows inside concentric annuli, Heat and Mass Transfer 39: 313-320 (Fig. 2).

In addition, I kindly acknowledge the permission of Prof. E. Papoutsakis for reprinting the Figs. 5.3-5.4, which have been first published in Papoutsakis E, Ramkrishna D, Lim H (1980) The extended Graetz problem with Dirichlet wall boundary conditions, Applied Scientific Research 36: 13-34 (Figs. 1-3). I also kindly acknowledge the permission of Prof. Osterkamp, Dr. Zhang and Dr. Gosink for reprinting Fig. 1.4, which has been first published in Zhang T, Osterkamp TE, Gosink JP (1991) A model for the thermal regime of Permafrost within the depth of annual temperature variations. Proc. 3rd Int. Symp. on Therm. Eng. Sci. for Cold Regions, Fairbanks, Alaska, USA, pp. 341 – 347 (Fig. 3).

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