FRACTURE TOUGHNESS TESTING AND ITS APPLICATIONS

Published by the
AMERICAN SOCIETY FOR TESTING AND MATERIALS
in cooperation with the
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

FRACTURE TOUGHNESS TESTING AND ITS APPLICATIONS

A symposium presented at the

SIXTY-SEVENTH ANNUAL MEETING

AMERICAN SOCIETY FOR TESTING AND MATERIALS

Chicago, Ill., June 21–26, 1964

ASTM Special Technical Publication No. 381

Published by the
AMERICAN SOCIETY FOR TESTING AND MATERIALS
1916 Race St., Philadelphia 3, Pa.
in cooperation with the
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Library of Congress Catalog Card Number: 65-16811

Printed in Baltimore, Md.
April, 1965
Second Printing, May 1970
Third Printing, January 1975
Fourth Printing, October 1981

FOREWORD

The development of various new high-strength alloys and the broadening range of their applications, particularly in aerospace and in cryogenics, has brought about increased emphasis on the study of fracture characteristics. As a result, the technology of testing for fracture toughness and crack propagation has grown rapidly in recent years. So, too, has understanding of how to apply this testing technology to design problems such as selection of materials, heat treatment, welding procedures, structural shape and size, and effects of environment.

This collection of papers constitutes an authoritative and reasonably complete statement of the current procedure and concepts in the field of fracture mechanics. It should thus be of primary value to those concerned with fracture testing and with applications of test data.

This publication is a cooperative effort of the American Society for Testing and Materials and the National Aeronautics and Space Administration. It helps to fulfill the obligation of the ASTM to provide the technical community with test methods, and with a sound understanding of their usefulness and their limitations. Through its Special Committee on Fracture Testing of High-Strength Materials (now ASTM Committee E-24 on Fracture Testing of Metallic Materials), ASTM has provided important technical leadership. This volume is the latest in a series of valuable publications on fracture testing and its application sponsored by this committee.

By cooperation with the ASTM, NASA is helping to fulfill its obligation to provide for the widest practicable and appropriate dissemination of results from its activities. Not only have aerospace problems directly furthered activity on fracture mechanics, but NASA scientists and engineers have directly contributed much to this new technology. It is the purpose of this publication to make the information in this important field as widely available as possible.

The Symposium on Fracture Toughness Testing and Its Applications was held at the Sixty-seventh ASTM Annual Meeting, in Chicago, Ill., June 21–26, 1964. It was sponsored by the ASTM Special Committee on Fracture Testing of High-Strength Materials. Chairman of the committee is J. R. Low, General Electric Co. Symposium chairman was W. F. Brown, Jr., National Aeronautics and Space Administration.

The symposium comprised three papers sessions and a panel discussion. Co-chairmen of the first session, on basic aspects of fracture mechanics, were T. J. Dolan, University of Illinois, and Harold Liebowitz, Office of Naval Research. Co-chairmen of the second session, on test methods, were Edward Steigerwald, Thompson Ramo Wooldridge, and Z. P. Saperstein, Douglas Aircraft Co. Co-chairmen of the third session, on practical applications, were B. M. Wundt, General Electric Co., and C. M. Carman, U. S. Army Ordnance. Mr. Brown was chairman of the panel discussion, and the other panelists were V. Weiss, S. Yukawa, P. Paris, J. E. Srawley, C. F. Tiffany, G. R. Irwin, T. J. Dolan, J. A. Kies, and W. F. Payne.

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FRACTURE TOUGHNESS TESTING AND ITS APPLICATION

INTRODUCTION

By W. F. Brown, Jr.[1]

The phenomenon of structural failure by catastrophic crack propagation at average stresses well below the yield strength has been known for many years. Rashes of such brittle failures have occurred with increasing frequency as the strength and size of our engineering structures have increased. In the past, each series of failures has given rise to a set of empirical tests and procedures that sometimes provided a solution to the specific problem at hand but did not result in a generally useful approach that would permit avoiding future failures.

Recent military and aerospace requirements for very-high-strength, lightweight hardware have given added importance to the problem of brittle fracture and greatly emphasized the need for a quantitative approach to the general problem of crack tolerance in structures. This need was dramatically highlighted several years ago by the repeated failures of early Polaris rocket motor cases at stresses well below the design value. The ASTM Special Committee on Fracture Testing of High Strength Materials was formed at the request of the Office of the Secretary of Defense to assist in providing a solution to this and related problems.

Over a period of the last five years this committee has been concerned with the question of how to evaluate the strength of metals in the presence of cracks or crack-like defects. The goal has been to provide laboratory tests and analytical techniques which will permit a quantitative measure of crack tolerance useful not only in evaluating materials for a given application but also in development of rational procedure for design against fracture. To achieve this goal requires the development of an essentially new branch of engineering science, and this, of course, is an evolutionary process which will take considerable time to complete. However, with the Irwin linear elastic fracture mechanics as a basis, considerable progess has been made in the desired direction, and today there are available reliable if somewhat overconservative procedures for avoiding failure by fracture in a new structure.

The primary purpose of this symposium was to review the methods for fracture toughness testing as proposed by the ASTM Special Committee on Fracture Testing of High Strength Materials, with a view toward defining their limitations and the extent to which they can be applied in structural design and alloy development. With this in mind the authors were asked to direct attention more toward clarification of concepts and procedures rather than toward presentation of new information. In order to further assist in this review function, the last session of the symposium consisted of a panel discussion

[1] Chairman of the symposium committee, NASA-Lewis Research Center, Cleveland, Ohio.

which gave those concerned with fracture testing an opportunity to put questions to a group of persons who have been active in the work of the ASTM Fracture Testing Committee.

There are, of course, many fracture test methods other than those discussed in this volume. Some of these often provide useful information regarding the fracture behavior of metallic materials. The pre-cracked Charpy impact test is a recent example of such a test which is easy to perform and uses only small specimens. Some efforts have been made to demonstrate a correlation between the results of pre-cracked Charpy tests and fracture toughness tests on larger specimens. A paper by G. M. Orner and C. E. Hartbower on this topic was presented at the symposium meeting, but because of space limitations does not appear in this volume. However, the reader should note that the panel discussion contains a considerable amount of information regarding the use of the pre-cracked Charpy test and references to investigations in this area.

Basic Aspects of Fracture Mechanics

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CRITICAL APPRAISAL OF FRACTURE MECHANICS

By V. Weiss[1] and S. Yukawa[2]

Synopsis

A critical review of the basic premises of fracture mechanics is presented. The applicability of the theoretical concepts developed by Griffith and considerably expanded by Irwin and co-workers to materials testing and the determination of a unique and characteristic value of "fracture toughness" is examined. Finally, the usefulness and limitations of sharp crack fracture mechanics to the solution of engineering design problems are discussed.

The present symposium is devoted to an evaluation of fracture testing and its applications. It is devoted to a discussion of the question concerning the condition under which a sharp crack propagates to failure in a cataclysmic fashion, in terms of what is now referred to as sharp crack fracture mechanics or fracture mechanics. It is not a symposium devoted to a discussion of fracture per se, ductile or brittle, but a symposium on the engineering aspects of fracture, fracture testing, and utilization of results from fracture testing in design applications for avoiding fracture.

Sharp crack fracture mechanics originated from a crack-propagation concept proposed some 44 years ago by A. A. Griffith (**1**)[3] which states that an existing crack will propagate in a cataclysmic fashion if the available elastic strain energy release rate exceeds the increase in surface energy of the crack. The reaction to this concept has ranged from complete acceptance to total rejection over the past 44 years. The proponents of the concept have endorsed it primarily because: (1) it yields the correct functional relationship between stress at fracture and flaw size as evidenced by many results on brittle-behaving materials including those obtained originally by Griffith (**2,3**); and (2) because it predicts a theoretical cohesive strength of the defect-free material of the right order of magnitude (0.1 E) which has also been verified approximately on single-crystal whiskers (**4**).

The principal argument against accepting the Griffith concept is the elusiveness of the value for surface tension which figures so dominantly in the concept (**5,6**). Others object to it on experimental grounds, mostly on the basis of data obtained with ductile materials where no appreciable crack-length effect, as predicted by the Griffith concept, was observed (**7**); or on the grounds that in addition to surface energy and elastic strain energy, the possibility of an energy barrier to crack initiation must be admitted. One last

[1] Associate professor of metallurgy, Syracuse University, Syracuse, N. Y.

[2] Manager, Metallurgy, Materials and Processes Laboratory, Large Steam Turbine-Generator Department, General Electric Co., Schenectady, N. Y.

[3] The boldface numbers in parentheses refer to the list of references appended to this paper.

and perhaps most serious objection to the application of the Griffith concept to structural materials may be that it represents an oversimplification (8) of a series of much more complicated phenomena in an age where there is no need to resort to such gross oversimplification, because of the development of science and the availability of computers, etc. Yet, the very simplicity of the fracture-mechanics approach, a one-parameter design concept of great potential, is to a large extent responsible for the recent progress in design against brittle fracture.

To adopt either of these two extreme positions would be unrealistic; to ignore the arguments would be folly. As engineers we must attempt to solve the problems put before us. The wealth of experimental data on sharp crack fracture mechanics in itself attests to serious consideration or acceptance of the proposed analysis by a good portion of the engineering community. The present appraisal should, therefore, be aimed at inspiring the necessary caution in applying the recommended concepts by delineating the limitations of sharp crack fracture mechanics on the basis of the applicability of the fundamental premises utilized. The emphasis has to be placed on the engineering usefulness of the approach rather than on its scientific and philosophical accuracy.

The symposium reflects this orientation towards the use of sharp crack fracture mechanics for the solution of engineering problems. The basic mathematical model, its physical implications, and limitations are discussed in the first section; in the second section, test methods to obtain the "design numbers" suggested by the mathematical model are discussed; the third section is devoted to a discussion of the use of the results of these tests and the mathematical analysis of sharp crack fracture mechanics for the solution of actual design problems. In this fashion, the symposium hopes to show that the engineering approach to the solution of problems—the theoretical (mathematical) model → testing → design-application sequence—is also applicable toward a solution of the problem of designing against fracture. The final section is a panel discussion. In addition to providing an over-all summary, the panel discussion provides for further clarification of the various problem areas, for the establishment of various interdisciplinary connections that have not already been clearly established during the first three sections, and for extended discussion of the current status and urgent research requirements.

This introductory paper has the same, if somewhat more mixed, organization and is, therefore, a broad preview of what is to follow. After a brief historical review of the developments of fracture mechanics since Griffith, the surface-energy – plastic-work analogy and its consequences will be discussed. This will be followed by comments on the aspects of initiation, propagation, and reinitiation of cracks which are intimately related to plasticity and the various plasticity-correction procedures. An attempt will also be made to relate the observed section-size effects to the stress-concentration effects as predicted by fracture mechanics, taking into consideration the influence of inhomogeneities on the mechanical behavior of the material. Finally, an outlook is given on the potential of the fracture-mechanics analysis to fatigue, stress-corrosion cracking, liquid-metal embrittlement, and fracture of nonmetals.

HISTORICAL REVIEW

Our present view of fracture certainly started with the Griffith concept of crack propagation which was presented on February 26, 1920 (1). The now well-known concept essentially states that an existing crack will propagate if thereby the total energy of the system is lowered. The stress analysis used to calculate the stored elastic energy was taken from Inglis's work (9) published in 1913 and was also based on the work of Taylor and Griffith (10) dated 1917. In his paper Griffith states that "the general conclusion may be drawn that the weakness of isotropic solids, as ordinarily met with, is due to the presence of discontinuities, or flaws, as they may be more correctly called, whose ruling dimensions are large compared with molecular distances. The effective strength of technical materials might increase ten or twenty times at least if these flaws can be eliminated." His theory provides a means of estimating the theoretical strength of solids. It also gives, for brittle materials, the correct relationship between fracture strength and defect size. There is no evidence that the advent of dislocation theory in 1934 has influenced fracture research along the lines proposed by Griffith or stimulated the application of Griffith's concept to solids other than glasses. Smekal has published a number of papers (11–17) on the brittle fracture of glasses in which he recognizes the need to consider other material inhomogeneities in addition to the starting cracks. This concern was shared by Weibull who in 1939 published his statistical theory of fracture (18). In 1944, Zener and Hollomon (19) connected the Griffith crack-propagation concept with the brittle fracture of metallic materials for the first time. Orowan referred to X ray work in 1945 (20) which showed extensive plastic deformation on the fracture surfaces of materials which had failed in a "brittle" fashion. In 1948, Irwin (21) pointed out that the Griffith-type energy balance must be between the strain energy stored in the specimen and the surface energy plus the work done in plastic deformation. He also recognized that for relatively ductile materials the work done against surface tension is generally not significant in comparison with the work done against plastic deformation. The same arguments were also stated independently at that time by Orowan (22) who in 1955 demonstrated that the modified Griffith condition for brittle fracture is not only a necessary but also a sufficient condition for crack propagation. In 1955, Irwin indicated (23) and in 1957, showed (24) that the energy approach is equivalent to a stress-intensity approach according to which fracture occurs when a critical stress distribution, characteristic of the material, is reached. In 1959, the ASTM Special Committee on Fracture Testing of High-Strength Metallic Materials was formed to launch a broad assault on fracture, based on the by-then called Griffith-Irwin concept or sharp crack fracture mechanics. The need to design specimens with a most severe artificial flaw and to test these specimens under the most severe condition was recognized and advocated by 1959 (25,26). Subsequently, the demand for plane-strain fracture toughness values was voiced and pop-in reactions were observed (27). Recent work at the Lewis Research Center of NASA with highly sensitive acoustical devices (28) indicates the need to study plane-strain crack extension instability in greater detail.

Plasticity treatments of the stress and strain fields of notches were given by Hill (29), Allen and Southwell (30), Lee (31), and Neuber (32,33). In 1956, Hult and McClintock (34) presented, for the first time, a plasticity analysis of the stress and strain fields of sharp cracks in shear; McClintock subsequently applied this analysis to ductile fracture (35). A nonlinear solution for loading without growth was presented by Neuber in 1961 (33). The conditions for the dynamics of a propagating crack were first formulated by Mott (36) in 1948 and a specific aspect of it was treated later by Yoffe (37). A good review is given by Schardin (38). Dynamic loading problems are now being studied by Krafft et al (39) in relation to strain rate sensitive materials.

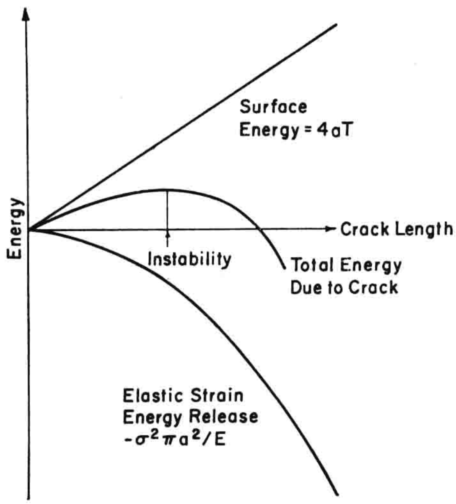

FIG. 1—Energy Balance of Crack in Infinite Plate.

There have been a number of important symposia which devoted major attention to this approach starting with an ASM symposium in 1947 (40), an MIT symposium on fatigue and fracture of metals (41), the First (42) and Second (43) Symposium (1958 and 1960) on Naval Structural Mechanics, the 1959 International Conference on the Atomic Mechanism of Fracture held in Swampscott (44) and, most recently, the 1962 AIME conference held at Maple Valley, Washington (45). The present symposium is perhaps unique in its relation to the symposia mentioned, in that it is the first symposium devoted solely to sharp crack fracture mechanics in relation to engineering and design applications.

THE SURFACE-ENERGY – PLASTIC-WORK ANALOGY

According to Griffith (1) crack growth under plane-stress conditions will occur if

$$\frac{d}{da}\left(-\frac{\sigma^2\pi a^2}{E} + 4aT\right) = 0 \ldots\ldots (1)$$

where the first term inside parentheses represents the elastic energy loss of a plate of unit thickness under a stress, σ, measured far away from the crack, if a crack of length $2a$ were suddenly cut into the plate at right angles to the direction of σ. The second term represents the energy gain of the plate due to the creation of the new surface having a surface tension, T. This is illustrated in Fig. 1 which is a schematic representation of the two energy terms and their sum as a function of crack length. When the elastic energy release due to an increment of crack growth, da, outweighs the demand for surface energy for the same crack growth, the crack will become unstable. One can define a gross fracture stress from this instability condition as

$$\sigma = (2ET/\pi a)^{1/2} \ldots\ldots\ldots\ldots (2)$$

which has, in the form $\sigma\sqrt{a}$ = constant, been shown to hold quite well for brittle and semibrittle metals. However, application of this analysis to such brittle and semibrittle metals has also shown that the data extrapolate, for $2a$ values of atomic dimensions, to T values considerably above most realistic estimates. This, together with experimental X ray evidence of cleavage facets, etc. (20,46), led to the conclusion (2,3) that in the fracture of metals the energy balance is primarily between the elastic energy release and the plastic work in crack propagation, which overshadows the energy requirements for the creation of new surfaces. Since the predicted functional relationship between the stress and the crack length was in good agreement with experimental evidence, it was suggested (2,20) simply to add a plastic-work factor, P, to the surface tension, T, in Eq 2.

The implications of this assumption together with the fact that $\sigma\sqrt{a}$ = constant holds for a great variety of

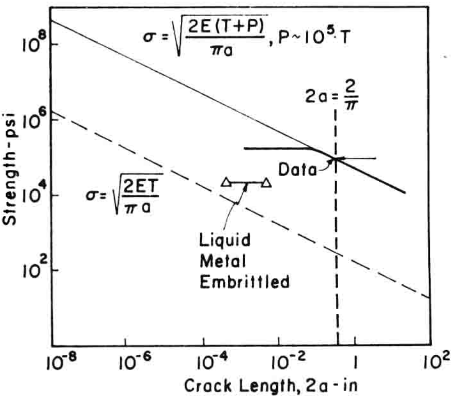

FIG. 2—Schematic Illustration of Observed and Predicted Strength–Crack Length Relationship, the Plastic Work Term, and the Effect of Liquid Metal Embrittlement.

test conditions (plane strain, plane stress, circumferential cracks, etc.) are quite astonishing. If the elastic energy release due to the crack has the form, $A\sigma^m a^n$, then the plastic-work term must have the form, $B\sigma^{m-2}a^{n-1}$. Since theory of elasticity dictates $m = 2$, the plastic-work term must be independent of stress. The elastic strain energy of a cracked plate per unit thickness is proportional to a^2 (that is, $n = 2$) and, therefore, the plastic-work term should be proportional to a. However, one might expect it to depend on the plastic volume per unit thickness which is proportional to a^2.[4] The calculations of Goodier and Field (47), which are based on Dugdale's hypothesis (48), confirm this. Other calculations show at least terms of the type, log a, to be present after differentiation.

The inadequacy of the energy, and in particular surface-energy, approach is further illuminated by a consideration of fracture results obtained under conditions of liquid-metal embrittlement (49), or other environmental effects which affect the crack-fracture strength. At first glance these effects would tend to confirm the predicted influence of surface energy on fracture strength. As a matter of fact, the Griffith-type fracture analysis is unique in this respect as it is the only crack- or notch-fracture analysis of the many proposed which seems to provide an understanding of environmental effects. However, Fig. 2 and Eq 2 clearly show the inapplicability of the type of reasoning whereby the loss in fracture strength in the presence of liquid metals is due to a reduced surface energy. If surface energy alone were responsible for fracture, the fracture toughness, K_c, would be somewhere around 10^{-5} E psi $\times$ in.$^{1/2}$, where E is Young's modulus. Even quite brittle materials have K_c values near 10^{-3} to 10^{-2} E psi $\times$ in.$^{1/2}$. Thus, the plastic-work factor, P, is 10^4 to 10^6 times the surface energy and any change in T due to environmental effects, even if T is reduced to zero, would have negligible effect on the fracture strength. The experimental results in this area must, therefore, lead to the conclusion that the influence of the environment, if it affects the fracture strength and the fracture toughness, is on the material's ability to deform plastically rather than on a change in surface energy. This may indeed be accomplished by such phenomena as slow crack growth (50) or

[4] See also: H. W. Liu, "Fracture Criterion of a Cracked Plate," *GALCIT SM63-29*, July, 1963.