



PWS
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STATISTICS
FOR
ENGINEERING
PROBLEM
SOLVING

STEPHEN B. VARDEMAN

Statistics for Engineering Problem Solving

Stephen B. Vardeman

Iowa State University



PWS Publishing Company

Boston



PWS PUBLISHING COMPANY

20 Park Plaza, Boston, MA 02116-4324

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Library of Congress Cataloging-in-Publication Data

Vardeman, Stephen B.

Statistics for engineering problem solving / Stephen B. Vardeman.

p. cm.

Includes index.

ISBN 0-534-92871-4

1. Engineering — Statistical methods. I. Title.

TA340.V37 1993

620'.001'5195 — dc20

93-5851

CIP

Sponsoring Editor: Tom Robbins

Editorial Assistant: Cynthia Harris

Assistant Editor: Mary Thomas

Production Coordinator: Susan M.C. Caffey

Production: Hoyt Publishing Services

Interior Designer: Susan M.C. Caffey

Interior Illustrator: Hayden Graphics

Cover Designer: Books by Design, Inc./Sally Bindari

Cover Illustration: Carol Schweigert

Cover Illustration Referenced from: Stock Boston/Bill Horsman

Marketing Manager: Nathan Wilbur

Manufacturing Coordinator: Lisa M. Flanagan

Compositor: Integre Technical Publishing Co., Inc.

Cover Printer: Henry N. Sawyer Co.

Text Printer and Binder: Arcata Graphics/Hawkins

Printed and bound in the United States of America.

93 94 95 96 97 98 — 10 9 8 7 6 5 4 3 2 1

Preface

The past decade has seen widespread recognition by educators and professionals that the *methods* and *thought patterns* of applied statistics are powerful tools for engineering problem solving. The purpose of this book is to present those tools in a way that both university engineering students and working engineers will find understandable and useful.

Instructors will find this book different from existing texts in several ways. No formal background in statistics is expected of the reader, but the philosophical orientation of the book is unlike that of traditional beginning texts in engineering statistics. Rather than emphasizing mathematical theory, I have tried to stress the engineering implications of statistical inferences, and to foster the development of scientific/statistical thought processes in the reader. Mathematical precision has not been compromised. But the material in this book has been treated as a means to more effective engineering practice, not as an end in itself.

I have made some nonstandard choices of content and order of presentation. This has been done to compile into a single volume what “one-shot” engineering audiences will most typically need in their professional practice. Practical issues in engineering data collection receive early and serious consideration, as do descriptive and graphical methods and the ideas of least squares surface-fitting and factorial analyses. More emphasis is given to the making of statistical intervals (including prediction and tolerance intervals) than to significance testing. Important topics in engineering statistics — such as Shewhart control charts, 2^p factorials and 2^{p-q} fractional factorials, response surface methods, mixture analyses, and variance component estimation — are given thorough, central treatment, rather than being included as supplemental topics intended to make a general statistics book into an “engineering” statistics book. Topics that seem less relevant to common engineering practice, such as axiomatic probability and counting, have been placed in appendices. There they are available for those instructors who have class time and wish to teach them, but they do not interrupt the book’s main story line.

The approach and order of presentation found in this book have been used for a number of years in two different introductory engineering statistics courses at Iowa State University and in some industrial short-course teaching. In my experience, the route taken here has been more successful in demonstrating the usefulness of statistics to university engineering students and working engineers than the conventional approach of a little descriptive statistics, a lot of probability, and some one- and two-sample methods, followed by regression analysis.

An important feature of the book is its final chapter. Chapter 11 consists of an account of a highly successful industrial project in which engineering statistics played a key role. This case study integrates material from many different parts of the book and illustrates how effective the tools introduced in the book can be in engineering problem solving. I am grateful to Dow Chemical Company for providing the information necessary to write the case study and for permission to incorporate it in the text.

The discussions in this book are carried almost exclusively by examples involving real data and/or real scenarios. Some of these have been drawn from published engineering and engineering statistics literature, and some are from my own work with both academic and corporate engineers. A large proportion are from projects completed by students in my courses in engineering statistics and quality control at Iowa State University. The examples and exercises bearing only name citations (no article, book, or journal references) are based on student projects. I am grateful to those students for the use of their interesting data sets and scenarios.

I believe strongly that students in one-term engineering statistics courses need the experience of actually “doing statistics”—that is, carrying through a data collection and analysis project from the problem formulation stage all the way to the writing of a professional technical report based on the study. This book is full of examples of possible projects. The project descriptions that I give to students are included in the book’s Solutions Manual, which the publisher makes available to qualified adopters.

There is more material in this book than can be readily covered in a single university course. (If I had the luxury of teaching a two-course, six-semester-hour sequence, I would have no trouble making the material of this book last through such a format.) I have attempted to make accessible to students even those topics not covered in class, either as outside reading or in subsequent professional practice. I believe it is wisest to give one-term engineering statistics students a feeling for the flavor of the subject and a wide view of what is possible, and to provide them with additional, thorough reading material that is presented at their level of sophistication. If this requires trimming some standard details from lectures and examinations, and rethinking the typical order in which topics are presented in class, I personally favor doing so. Regardless, I believe that instructors will find the book to be complete and understandable in its treatment of the subjects addressed, and flexible enough to fit a variety of syllabi.

The schedule listed in Table 1 is one that I use in a three-semester-hour, junior-level course. It maps a path culminating in the final week of class with a look at the 2^{p-q} fractional factorials. (This course meets for 75 minutes twice per week for 15 weeks.) The “Q” entries listed are half-period quizzes.

Parts of the book that are completely or nearly completely omitted from lecture in this fast-paced course are Sections 4-4 and 5-3, most of Section 5-4, part of Section 7-4, Sections 7-6, 8-2, 8-4, 9-3, 9-4, 10-3, 10-4, and 10-5, Appendix A, and Appendix B. If I could squeeze another topic or two into this course, the material in Sections 9-3 and 10-4 would be my first choice for inclusion.

At ISU, we also teach a three-semester-hour, freshman-level course from this text. That course covers less ground than the one described above and is somewhat more traditional in its outlook. Covered in that course are Chapters 1, 2, 3, and 4 (except for Section 4-4); Appendix B; Sections 5-1, 5-2, 5-3, and 5-5; Chapter 6; and Sections 7-5 and 7-6. Clearly, other outlines can be constructed around the ample material in this book.

Readers familiar with the engineering statistics literature will have no trouble recognizing the influence of many other authors. I have learned much from the work of Box, Hunter, and Hunter; of Guttman, Wilks, and Hunter; of Duncan; and of Daniel. My views on the teaching and practice of engineering statistics owe much to valued interaction with David Moore, H.T. David, Bill Meeker, Gerry Hahn, Bob Kasprzyk, George Kalemkarian, Bill Fulkerson, and Harvey Arnold, to name a few who come quickly to mind. Any originality on my part can only be originality of execution (if there is such a thing), not of basic outlook.

There are many others who deserve sincere thanks as I begin to see the light at the end of the tunnel on this project. Iowa State University has been a great environment in which to work and develop this material. A one-semester Faculty Improvement Leave

TABLE 1

Session	Topic(s) and/or Quiz	Reading
1	Introduction	Chapter 1
2	Data Collection	Chapter 2
3	Data Collection	Chapter 2
4	Descriptive Statistics	Chapter 3
5	Descriptive Statistics	Chapter 3
6	Curve Fitting/ Q1	Chapter 4
7	Curve & Surface Fitting	Chapter 4
8	Surface Fitting/Factorials	Chapter 4
9	Fitted Factorial Effects	Chapter 4
10	Random Variables	Chapter 5
11	Random Variables	Chapter 5
12	Random Variables/ Q2	Chapter 5
13	Random Variables	Chapter 5
14	Random Variables/Simple Inference	Chapter 6
15	Simple Inference	Chapter 6
16	Simple Inference	Chapter 6
17	Simple Inference	Chapter 6
18	Simple Inference/ Q3	Chapter 6
19	One-Factor Analyses	Chapter 7
20	One-Factor Analyses	Chapter 7
21	One-Factor Analyses	Chapter 7
22	Control Charts	Chapter 7
23	Two-Factor Analyses	Chapter 8
24	2 ⁿ Analyses/ Q4	Chapter 8
25	Inference in Curve & Surface Fitting	Chapter 9
26	Inference in Curve & Surface Fitting	Chapter 9
27	Inference in Curve & Surface Fitting	Chapter 9
28	Inference in Curve & Surface Fitting/ Q5	Chapter 9
29	Fractional Factorials	Chapter 10
30	Fractional Factorials	Chapter 10

granted by ISU in the spring of 1989 was essential to getting me genuinely under way with the writing. Excellent teaching opportunities in the ISU Statistics and Industrial and Manufacturing Systems Engineering Departments have been both the principal motivation for putting this material down on paper and the laboratory in which many of the examples have been developed and the text debugged. I particularly appreciate the support of Professor Dean Isaacson, who has been the head of the ISU Statistics Department throughout the writing of this book. He has been a real advocate of efforts in engineering statistics and treated this project as important to the efforts of both the ISU department and the statistics profession to increase effective interaction with the engineering community. Dean David Kao of the ISU Engineering College has been another important supporter of the engineering/statistics interface; I appreciate his encouragement as well.

A number of instructors have used this book in class note form and have given me valuable suggestions for its improvement. These include especially Alan Zimmermann and Chuck Lerch, and also Peter Jones, Dean Isaacson, Jerry Hall, Ann Russey, Jean Pelkey, Peter Peterka, and Todd Sanger. Over the years, John Patterson has sent me a

number of useful examples from engineering journals. I also gratefully acknowledge the help of Thomas Fischer in finding several other telling examples.

Chuck Lerch not only taught from this book in note form, providing valuable suggestions and catching many typos, but he also meticulously prepared the answer section of the book and the Solutions Manual. In the process, he identified and helped eliminate many rough spots in the exercise sets. The value of this book as a university teaching tool will owe much to his kind and careful help.

The manuscript has benefited from the comments and suggestions of “volunteer” reviews made by interested colleagues Kim Erland, Marjorie Green, and Noel Artiles. In addition, I most gratefully acknowledge the help of formal reviewers Bill Fulkerson, Dan Wardrop, Bob Kasprzyk, Harry Wadsworth, Craig Van Nostrand, John Boyer, J. Peter Jones, John Ramberg, Phillip Beckwith, and especially Harvey Arnold. (Harvey caught a number of potentially embarrassing errors in the production manuscript and page proofs, some that I and generations of students had failed to identify.) Of course, not every suggestion made by these reviewers was adopted — but many were, and all have influenced the final product.

Minitab Inc. provided software that allowed me to conveniently generate the print-outs in the book. I am grateful for their kind help in this regard.

Sharon Shepard of the ISU Statistics Department spent many hours flawlessly converting an early electronic version of the manuscript into one that was easily edited into a production version. Without her patient help, my hair would be considerably grayer.

The people of PWS and the companies they have enlisted to work on the production of the book have been first-rate professionals, and I am glad to have had this project in their hands. Special thanks are due to Michael Payne, who signed the book for PWS, championed it, and gave me substantial freedom to make it what I think it needs to be. Tom Robbins stepped in where Michael left off, and I have greatly appreciated his advocacy of treating the book as an engineering title. I am most grateful as well to Mary Thomas, who helped organize the production effort, and Susan Caffey, who handled many of the production details for PWS, including the excellent interior design. David Hoyt of Hoyt Publishing Services was wonderfully meticulous in handling day-to-day production matters and did a first-rate job of copy editing and making my sometimes arcane prose readable. Integre Technical Publishing did a superb, especially clean job of technical typesetting. Thanks also go to Hayden Graphics for excellent art and putting up with my incessant fiddling with some of the figures. As I say, this was a most professional crew and it has been a real pleasure to be associated with them.

Finally, and most importantly, there is my family. The text represents what now seems to me a huge investment of time and effort over nearly a seven-year period. During that time, my wife Jo Ellen and sons Micah and Andrew have patiently put up with too many excuses: “I’m sorry, I’ve got to work on the book . . .” Jo Ellen cheerfully typed the original version of the manuscript, and I seriously doubt whether I would have ever finished even a first draft if she hadn’t taken on that task in addition to all the other demands on her time. Thanks, Vardemans!

I will be grateful for feedback from users of this book in both the academic and corporate worlds. I hope that it proves genuinely helpful in teaching and practice, and I am anxious to know how the various parts of the presentation “work” in different environments. Written comments can be directed to me at the Statistics Department, Iowa State University, Snedecor Hall, Ames, Iowa 50011, or at my e-mail address: vardeman@iastate.edu

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This chapter lays a foundation for all that follows. Because it contains a road map for the study of the subject of engineering statistics, Chapter 1 should be read carefully and referred to frequently as you make your way through this text. In it, the subject of engineering statistics is defined, and its importance to an engineer is described. Some basic terminology is introduced, and the important subject of measurement is discussed. Finally, the role of mathematical models in achieving the objectives of engineering statistics is investigated.

1-1 Engineering Statistics: What and Why

In general terms, what a working engineer does is to design, build, operate, and/or improve physical systems and products. Chemical engineers design and operate systems that produce products ranging from fertilizers to fuels; civil engineers build highways, waterworks, large buildings, etc.; aeronautical engineers design and improve a myriad of different aircraft; industrial engineers design and operate manufacturing facilities. On and on and on the list could go.

An engineer is often guided by basic mathematical and physical theories (typically learned in those trying days spent negotiating a demanding undergraduate engineering curriculum). As the engineer's experience grows, these quantitative and scientific principles form the basis for, and work alongside, sound engineering judgment. But as technology advances and new systems and products are encountered, the working engineer is inevitably faced with questions and situations for which available theory and experience provide little or no help. When this happens, what is to be done?

On occasion, consultants can be called in, but most often the engineer must independently find out "what makes things tick." It is necessary to collect and interpret data that will help in understanding how the new system or product works. Without proper training, the engineer's attempts at data collection and analysis can be haphazard and poorly conceived. When this is the case, valuable time and resources are wasted, and sometimes erroneous (or at least ambiguous) conclusions are reached. To avoid such circumstances, it is vital for a working engineer to have a tool kit that includes the best possible principles and methods for gathering and interpreting data.

DEFINITION 1-1

Engineering statistics is the study of how best to

1. collect engineering data,
2. summarize or describe engineering data, and
3. draw practical conclusions on the basis of engineering data,

all recognizing the reality of variation.

The subject of engineering statistics has as its goal to provide the concepts and tools needed by an engineer who faces a problem for which his or her background or theory do not serve as adequate guides to a solution. It supplies principles of how to efficiently acquire and process empirical information for use in understanding and manipulating engineering systems.

To better understand the definition of engineering statistics, it is helpful to consider a real situation and how the elements of the subject enter a real problem.

EXAMPLE 1-1

Heat Treating Gears. The article “Statistical Analysis: Mack Truck Gear Heat Treating Experiments” by P. Brezler, (*Heat Treating*, November, 1986) describes an illustrative situation. A process engineer was faced with the question, “How should untreated gears be loaded into a continuous carburizing furnace in order to minimize distortion during heat treating?” Various people had various semi-informed opinions about how it should be done — in particular, about whether the gears should be laid flat in stacks or hung on rods passing through the gear bores. But no one really knew the consequences of laying versus hanging.

In order to settle the question, the engineer decided to get the facts — to collect some data on “thrust face runout,” a measure of gear distortion, for gears laid and gears hung. Notice that exactly how this data collection should be performed required careful thought and planning. There were possible differences in gear raw material lots; machinists and machines that produced the gears; furnace conditions at different times and positions within the furnace; technicians and measurement devices that would produce the final runout measurements; etc. The engineer did not want these variations either to be mistaken for differences between the two loading techniques or to unnecessarily cloud the picture. Avoiding this required care.

In fact, the engineer conducted a well thought-out and executed study. Table 1-1 shows the runout values obtained for 38 gears laid and 39 gears hung after heat treating.

TABLE 1-1

Thrust Face Runouts (.0001 in.)

Gears Laid	Gears Hung
5, 8, 8, 9, 9	7, 8, 8, 10, 10
9, 9, 10, 10, 10	10, 10, 11, 11, 11
11, 11, 11, 11, 11	12, 13, 13, 13, 15
11, 11, 12, 12, 12	17, 17, 17, 17, 18
12, 13, 13, 13, 13	19, 19, 20, 21, 21
14, 14, 14, 15, 15	21, 22, 22, 22, 23
15, 15, 16, 17, 17	23, 23, 23, 24, 27
18, 19, 27	27, 28, 31, 36

In raw form, the runout values are hardly understandable. They lack organization; it is not possible to simply look at Table 1-1 and tell what is going on. The data needed to be summarized. One thing that was done was to compute some numerical summarizations of the data. For example, the process engineer found

$$\text{Mean laid runout} = 12.6$$

$$\text{Mean hung runout} = 17.9$$

Further, a simple graphical summarization was made of the data, as shown in Figure 1-1.

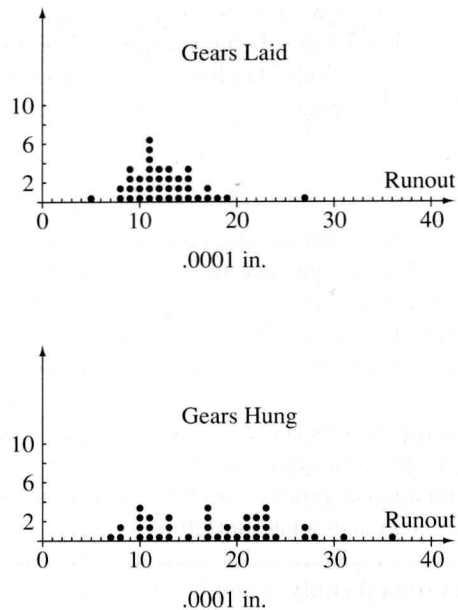
From these summaries of the runout data, several points are obvious. One is that there is variation in the runout values, even within a particular loading method. Variability is an omnipresent fact of life when one sets about to use data, and all statistical methodology and philosophy is conceived and applied in recognition of the reality of variation. In the case of the gears, it appears from Figure 1-1 that there is somewhat more variation in the hung values than in the laid values.

But in spite of the variability that complicates comparison between the loading methods, Figure 1-1 and the two group means calculated by the process engineer carry the message that the laid runouts are on the whole smaller than the hung runouts. By how much? One answer is

$$\text{Mean hung runout} - \text{Mean laid runout} = 5.3$$

FIGURE 1-1

Dot Diagrams of Runouts



But how “precise” is this figure? It is already clear that runout values are variable. Is there any assurance that the difference seen in the present means would hold up and reappear in further testing? Or is it possibly explainable as simply “stray background noise?” Laying gears is more expensive than hanging them. Can one know whether the extra expense is justified?

These questions point to the need for methods of formal statistical inference from data and subsequent translation of those inferences into practical conclusions. Methods studied later in this text can, for example, be used to support the following statements about hanging and laying gears.

1. One can be roughly 90% sure that the difference in mean runouts that would be produced under conditions like those of the engineer’s study is in the range

3.2 to 7.4

2. One can be roughly 95% sure that 95% of runouts for gears laid under conditions like those of the engineer’s study would fall in the range

3.0 to 22.2

3. One can be roughly 95% sure that 95 % of runouts for gears hung under conditions like those of the engineer’s study would fall in the range

.8 to 35.0

These are formal quantifications of what was learned from the study of laid and hung gears. To derive practical benefit from statements like these, the process engineer had to combine them with other information, such as the consequences of a given amount of runout and the comparative costs for hanging and laying gears, and apply sound engineering judgment to form a rational plan of action. Depending on the specifics of quality needs and costs, the engineer might or might not have been led to adopt the laying method for furnace loading. But as it turned out, the runout improvement was great enough to justify some extra expense, and the laying method was implemented.

The example shows how, in a fairly simple situation, the elements of the subject of statistics were helpful in solving an engineer's problem. Throughout this text, the intention is to convey the understanding that the topics discussed are not ends in themselves, but rather tools that engineers can use to help them do their jobs effectively.

1-2 Basic Terminology

A first step when entering any new subject area is to learn some basic terminology. Like most subjects, engineering statistics has both new words to be learned and new technical meanings for familiar words to be absorbed. This section introduces some common statistical jargon for types of statistical studies, types of data that can arise in those studies, and types of structures those data can have.

Types of Statistical Studies

When an engineer sets about to observe a system or process and gather data from it, it must be decided how active the observer's role will be. Will there be turning of knobs and manipulation of process variables, or will one simply let things happen and record the salient features of what transpires?

DEFINITION 1-2

An **observational study** is one in which the investigator's role is basically passive. A process or phenomenon is watched and data are recorded, but there is no intervention on the part of the person conducting the study.

DEFINITION 1-3

An **experimental study** (or more simply, an *experiment*) is one in which the investigator's role is active. Process variables are manipulated, and the study environment is regulated.

Most real statistical studies have both observational and experimental features, and the two definitions above should be thought of as representing opposite ends of a continuum. On this continuum, the experimental end is usually thought of as providing the most efficient and reliable ways to collect engineering data. This is true for several reasons. First, when one adopts a passive posture in data collection (hoping simply to observe one or more instances of favorable process behavior and then try to identify a cause from circumstances accompanying that good behavior), a long wait may be involved. It is typically much quicker to manipulate process variables and watch how a system responds to the changes.

In addition, it is far easier and safer to infer *causality* from an experiment than from an observational study. The problem with passive observation in this regard is the complexity of real systems. One may observe several instances of good process performance and note that they were all surrounded by circumstances X, without being safe in assuming that circumstances X cause good process performance. There may be important variables in the background — ones that are not recognized as part of circumstances X and perhaps not even observed — that are changing and are the true reason for instances of favorable system behavior. These so-called *lurking variables* may govern both process performance and circumstances X. Or it may simply be that many variables change haphazardly without appreciable impact on the system and that by chance, during one's limited period of observation, some of these happen to produce X at the same time that good performance occurs. In either case, an engineer's effort to create situation X, as a means of making things work well, will be wasted effort.

On the other hand, in an experiment, where the study environment is largely regulated except for a few variables the engineer changes in a purposeful way, an inference of causality is much stronger. If circumstances created by the investigator are consistently accompanied by favorable results, one can be reasonably sure that they cause the favorable results.

EXAMPLE 1-2

Pelletizing Hexamine Powder. Cyr, Ellson, and Rickard attacked the problem of reducing the number of nonconforming fuel pellets being produced in the compression of a raw hexamine powder in a pelletizing machine. This was a problem of long standing. (In fact, it has served as the basis for a number of student projects.) There were many possible factors influencing the percentage of nonconforming pellets: among others, Machine Speed, Die Fill Level, Percent Paraffin added to the hexamine, Room Temperature, Humidity at manufacture, Moisture Content, “new” versus “reground” Composition of the mixture being pelletized, and the Roughness of the chute entered by the freshly stamped pellets. Trying to passively keep track of these factors, to notice when a high percentage of the pellets produced happened to be acceptable and then to infer how best to run the pelletizing machine, was a hopeless proposition.

In fact, the students were able to make significant progress by conducting an experiment. They chose three of the factors above that seemed most likely to be important and purposely changed their values, while holding the other nonexperimental factors as close to constant as possible. The important changes they observed in the percentage of acceptable fuel pellets were appropriately attributed to the influence of the system variables they had manipulated.

Besides the distinction between observational and experimental statistical studies, it is helpful to distinguish between studies on the basis of the intended breadth of application of the results. Two relevant terms, popularized by W.E. Deming, are defined next.

DEFINITION 1-4

An **enumerative study** is one in which there is a concrete, well-defined group of objects under study. Data are collected on some or all of these objects, and conclusions are intended to apply only to these objects.

DEFINITION 1-5

An **analytical study** is one in which a process or phenomenon is investigated at one point in space and time, with the hope that the data collected will be representative of system behavior at other places and times where similar conditions hold. In this kind of study, there is rarely, if ever, a concrete group of objects to which conclusions are thought to be limited.

Most engineering studies tend to be of the second type, although some important engineering applications involve enumerative work. One such example is the reliability testing of one (or a few) of a kind of critical components — e.g., for use in a space shuttle. The interest is in the one (or the few) components actually in hand and how well they can be expected to perform, rather than on any broader problem like “the behavior of all components of this type.” *Acceptance sampling* (where incoming lots of raw materials or vendor-produced components are checked before taking formal receipt), can be thought

of as another important kind of enumerative study. But as indicated, most engineering studies are analytical in nature.

EXAMPLE 1-2

(continued)

The students working on the pelletizing machine were not interested in any particular batch of pellets, but rather in the question of how to make the machine work effectively. They hoped (or tacitly assumed) that what they learned in the spring of 1986 about making fuel pellets would remain valid at later times, at least under shop conditions like those they were facing. Their experimental study was analytical in nature.

Particularly when discussing enumerative studies, the next two definitions are helpful.

DEFINITION 1-6

A **population** is the entire group of objects about which one wishes to gather information in a statistical study.

DEFINITION 1-7

A **sample** is the group of objects on which one actually gathers data in a statistical study. In the case of an enumerative investigation, the sample is a subset of the population (and can in some cases include the entire population).

If a crate of 100 machine parts is delivered to a loading dock and five are examined in order to verify the acceptability of the lot, the 100 parts constitute the population of interest, and the five parts make up a (single) sample of size 5 from the population. (Notice the word usage here. There is one sample, not five samples.)

There are several ways in which the meanings of the words *population* and *sample* are often extended. For one, it is common to use the words in reference not only to objects under study but also to data values associated with those objects. For example, if one thinks of Rockwell hardness values associated with 100 crated machine parts, the 100 hardness values might be called a population (of numbers). Five hardness values corresponding to the parts examined in acceptance sampling could be termed a sample from that population.

EXAMPLE 1-2

(continued)

Cyr, Ellson, and Rickard identified eight different sets of experimental conditions under which to run the pelletizing machine. Several production runs of fuel pellets were made under each set of conditions, and each of these produced its own percentage of conforming pellets. These eight sets of percentages can be referred to as eight different samples (of numbers).

Also, although strictly speaking there is no concrete population being investigated in an analytical study, it is common to talk in terms of a *conceptual population* in such cases. Phrases like “the population consisting of all widgets that could be produced under these conditions” are sometimes used. This language is unfortunate, because it tends to encourage fuzzy thinking. But it is a common usage, and it is supported by the fact that one typically uses the same mathematics when drawing inferences from samples in enumerative and analytical contexts.