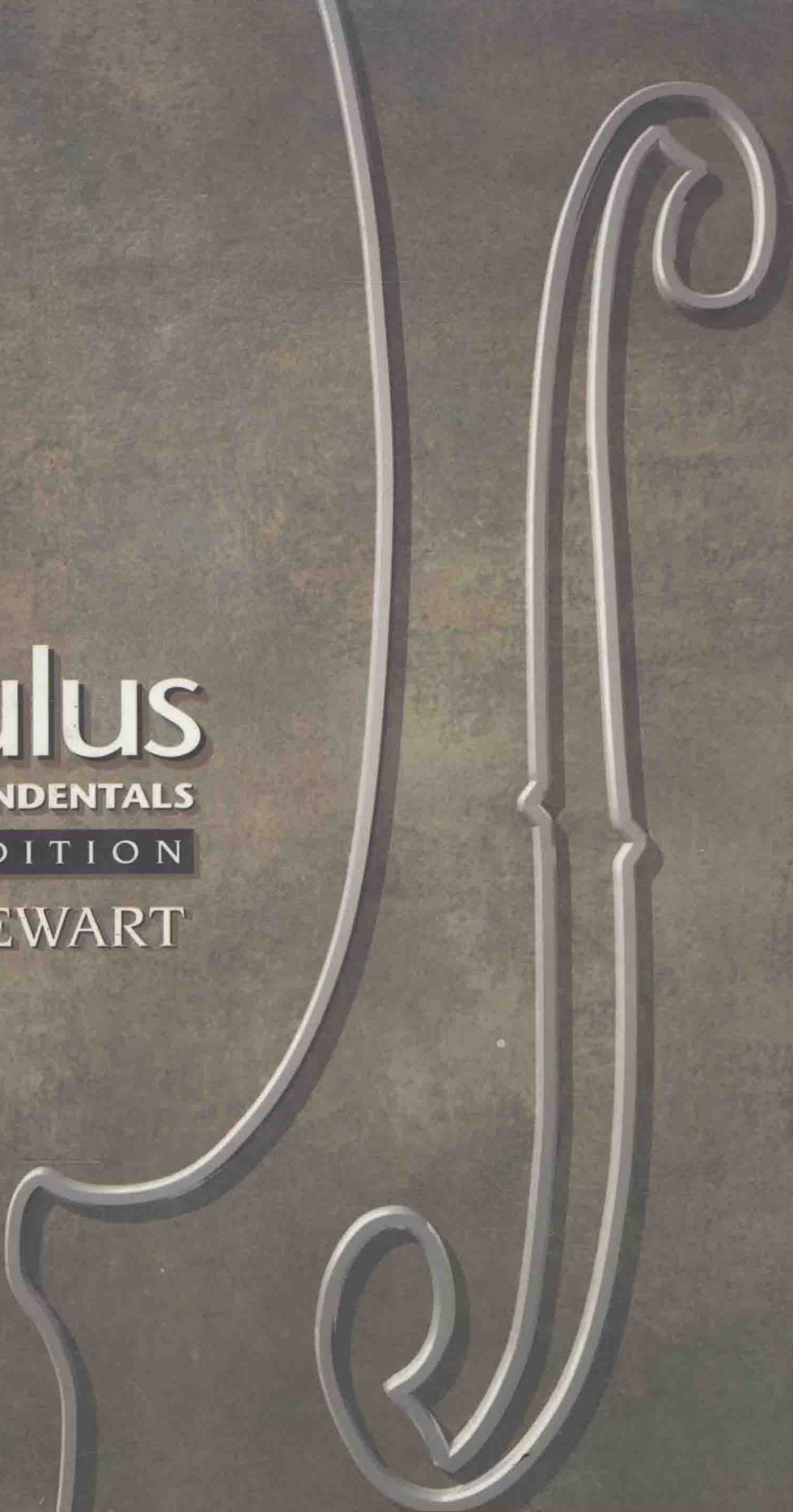
Calculus

EARLY TRANSCENDENTALS

FOURTH EDITION

JAMES STEWART



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Early Transcendentals

FOURTH EDITION

JAMES STEWART
McMaster University

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Preface

> A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.
>
> *George Polya*

The art of teaching, Mark Van Doren said, is the art of assisting discovery. I have tried to write a book that assists students in discovering calculus—both for its practical power and its surprising beauty. In this edition, as in the first three editions, I aim to convey to the student a sense of the utility of calculus and develop technical competence, but I also strive to give some appreciation for the intrinsic beauty of the subject. Newton undoubtedly experienced a sense of triumph when he made his great discoveries. I want students to share some of that excitement.

The emphasis is on understanding concepts. I think that nearly everybody agrees that this should be the primary goal of calculus instruction. In fact, the impetus for the current calculus reform movement came from the Tulane Conference in 1986, which formulated as their first recommendation:

Focus on conceptual understanding.

I have tried to implement this goal through the *Rule of Three*: "Topics should be presented geometrically, numerically, and algebraically." Visualization, numerical and graphical experimentation, and other approaches have changed how we teach conceptual reasoning in fundamental ways. More recently, the Rule of Three has been expanded to become the *Rule of Four* by emphasizing the verbal, or descriptive, point of view as well.

In preparing the fourth edition my premise has been that it is possible to achieve conceptual understanding and still retain the best traditions of traditional calculus. The book contains elements of reform, but within the context of a traditional curriculum. (Instructors who prefer a more streamlined curriculum should look at my book *Calculus: Concepts and Contexts.*)

Features

Conceptual Exercises

Pages 99, 131, 144, 711, 894, 905

Page 161

The most important way to foster conceptual understanding is through the problems that we assign. To that end I have devised various types of new problems. Some exercise sets begin with requests to explain the meanings of the basic concepts of the section. (See, for instance, the first few exercises in Sections 2.2, 2.5, 2.6, 11.2, 14.2, and 14.3.) Similarly, all the review sections begin with a Concept Check and a True-False Quiz. Other exercises test conceptual understanding through graphs or tables (see Exercises 1–3 in Section 2.8,

Pages 172–173, 237, 848
Pages 856, 883–885, 936, 938, 947, 974–975
Pages 1046, 1057–1058, 1066–1067
Pages 131, 173, 304, 532–533
Pages 145, 206, 620

Exercises 33–36 in Section 2.9, Exercises 1–4 in Section 3.7, Exercises 1–2 in Section 13.2, Exercise 27 in Section 13.3, Exercises 1, 2, 5 and 31–34 in Section 14.1, Exercises 1, 2, and 36 in Section 14.6, Exercises 3–4 in Section 14.7, Exercises 5–10 in Section 15.1, Exercises 11–18 in Section 16.1, Exercises 17, 18, and 43 in Section 16.2, and Exercises 1, 2, 11, and 23 in Section 16.3). Another type of exercise uses verbal description to test conceptual understanding (see Exercise 8 in Section 2.5, Exercise 44 in Section 2.9, Exercises 57 and 58 in Section 4.3, and Exercise 67 in Section 7.8). I particularly value problems that combine and compare graphical, numerical, and algebraic approaches (see Exercises 33 and 34 in Section 2.6, Exercise 21 in Section 3.3, and Exercise 2 in Section 9.5).

Graded Exercise Sets

More than 30% of the exercises are new. Each exercise set is carefully graded, progressing from basic conceptual exercises and skill-development problems to more challenging problems involving applications and proofs.

Real-World Data

Pages 11, 15, 172, 377, 406
Page 874
Page 895
Pages 912, 926
Pages 972, 1041

My assistants and I spent a great deal of time looking in libraries, contacting companies and government agencies, and searching the Internet for interesting real-world data to introduce, motivate, and illustrate the concepts of calculus. As a result, many of the new examples and exercises deal with functions defined by such numerical data or graphs. See, for instance, Figures 1, 11, and 12 in Section 1.1 (seismograms from the Northridge earthquake), Exercise 32 in Section 2.9 (smoking rates among high school seniors), Exercise 12 in Section 5.1 (velocity of the space shuttle *Endeavour*), and Figure 4 in Section 5.4 (San Francisco power consumption). Functions of two variables are illustrated by a table of values of the wind-chill index as a function of air temperature and wind speed (Example 2 in Section 14.1). Partial derivatives are introduced in Section 14.3 by examining a column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity. This example is pursued further in connection with linear approximations (Example 3 in Section 14.4). Directional derivatives are introduced in Section 14.6 by using a temperature contour map to estimate the rate of change of temperature at Reno in the direction of Las Vegas. Double integrals are used to estimate the average snowfall in Colorado on December 24, 1982 (Example 4 in Section 15.1). Vector fields are introduced in Section 16.1 by depictions of actual velocity vector fields showing San Francisco Bay wind patterns.

Projects

Page 602
Page 958
Page 655
Page 511
Pages 811, 1018
Page 1025

One way of involving students and making them active learners is to have them work (perhaps in groups) on extended projects that give a feeling of substantial accomplishment when completed. I have included four kinds of projects: *Applied Projects* involve applications that are designed to appeal to the imagination of students. The project after Section 9.3 asks whether a ball thrown upward takes longer to reach its maximum height or to fall back to its original height. (The answer might surprise you.) The project after Section 14.8 uses Lagrange multipliers to determine the masses of the three stages of a rocket so as to minimize the total mass while enabling the rocket to reach a desired velocity. *Laboratory Projects* involve technology; the one following Section 10.2 shows how to use Bézier curves to design shapes that represent letters for a laser printer. *Writing Projects* ask students to compare present-day methods with those of the founders of calculus—Fermat's method for finding tangents, for instance. Suggested references are supplied. *Discovery Projects* anticipate results to be discussed later or encourage discovery through pattern recognition (see the one following Section 7.6). Others explore aspects of geometry: tetrahedra (after Section 12.4), hyperspheres (after Section 15.7), and intersections of three cylinders (after Section 15.8). Additional projects can be found in the *Instructor's Guide* (see, for instance, Group Exercise 5.1: Position from Samples) and also in the *CalcLabs* supplements.

Technology The availability of technology makes it not less important but more important to clearly understand the concepts that underlie the images on the screen. But, when properly used, graphing calculators and computers are powerful tools for discovering and understanding those concepts. This textbook can be used either with or without technology and I use two special symbols to indicate clearly when a particular type of machine is required. The icon ▦ indicates an example or exercise that definitely requires the use of such technology, but that is not to say that it can't be used on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, Mathematica, or the TI-92) are required. But technology doesn't make pencil and paper obsolete. Hand calculation and sketches are often preferable to technology for illustrating and reinforcing some concepts. Both instructors and students need to develop the ability to decide where the hand or the machine is appropriate.

Problem Solving Students usually have difficulties with problems for which there is no single well-defined procedure for obtaining the answer. I think nobody has improved very much on George Polya's four-stage problem-solving strategy and, accordingly, I have included a version of his problem-solving principles following Chapter 1. They are applied, both explicitly and implicitly, throughout the book. After the other chapters I have placed sections called *Problems Plus,* which feature examples of how to tackle challenging calculus problems. In selecting the varied problems for these sections I kept in mind the following advice from David Hilbert: "A mathematical problem should be difficult in order to entice us, yet not inaccessible lest it mock our efforts." When I put these challenging problems on assignments and tests I grade them in a different way. Here I reward a student significantly for ideas toward a solution and for recognizing which problem-solving principles are relevant.

Content

The following chapter descriptions highlight some of the changes in this edition.

A Preview of Calculus The book begins with an overview of the subject and includes a list of questions to motivate the study of calculus.

Chapter 1 Functions and Models From the beginning, multiple representations of functions are stressed: verbal, numerical, visual, and algebraic. A discussion of mathematical models leads to a review of the standard functions, including exponential and logarithmic functions, from these four points of view.

Chapter 2 Limits and Derivatives The material on limits is motivated by a prior discussion of the tangent and velocity problems. Limits are treated from descriptive, graphical, numerical, and algebraic points of view. Section 2.4, on the precise ε-δ definition of a limit, is an optional section. Sections 2.8 and 2.9 deal with derivatives (especially with functions defined graphically and numerically) before the differentiation rules are covered in Chapter 3. Here the examples and exercises explore the meanings of derivatives in various contexts.

Chapter 3 Differentiation Rules All the basic functions, including exponential, logarithmic, and inverse trigonometric functions, are differentiated here. When derivatives are computed in applied situations, students are asked to explain their meanings.

Chapter 4
Applications of Differentiation

The sections on monotonic functions and concavity have been combined into a single section that explains how derivatives affect the shape of a graph. Graphing with technology emphasizes the interaction between calculus and calculators and the analysis of families of curves. Some substantial optimization problems are provided, including an explanation of why you need to raise your head 42° to see the top of a rainbow.

Chapter 5
Integrals

The area problem and the distance problem serve to motivate the definite integral, with sigma notation introduced as needed. (Full coverage of sigma notation is provided in Appendix E.) I decided to make the definition of an integral easier to understand by mainly using subintervals of equal width. Emphasis is placed on explaining the meanings of integrals in various contexts and on estimating their values from graphs and tables. To this end I have divided the section on the Fundamental Theorem of Calculus into two sections.

Chapter 6
Applications of Integration

Here I present the applications of integration—area, volume, work, average value—that can reasonably be done without specialized techniques of integration. General methods are emphasized. The goal is for students to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral.

Chapter 7
Techniques of Integration

A briefer account of the section on rationalizing substitutions has been incorporated into the partial fractions section, but the other techniques are fully covered. The use of computer algebra systems is discussed in Section 7.6.

Chapter 8
Further Applications of Integration

Here are the applications of integration—arc length and surface area—for which it is useful to have available all the techniques of integration, as well as applications to biology, economics, and physics (hydrostatic force and centers of mass). I have also included a new section on probability. There are more applications here than can realistically be covered in a given course. Instructors should select applications suitable for their students and for which they themselves have enthusiasm.

Chapter 9
Differential Equations

Modeling is the theme that unifies this introductory treatment of differential equations. Direction fields and Euler's method are studied before separable and linear equations are solved explicitly, so that qualitative, numerical, and analytic approaches are given equal consideration. These methods are applied to the exponential, logistic, and other models for population growth. The first five or six sections of this chapter serve as a good introduction to first-order differential equations. An optional final section uses predator-prey models to illustrate systems of differential equations.

Chapter 10
Parametric Equations and Polar Coordinates

Parametric curves are well suited to laboratory projects; the two presented here involve families of curves and Bézier curves. A brief treatment of conic sections in polar coordinates prepares the way for Kepler's Laws in Chapter 13.

Chapter 11
Infinite Sequences and Series

The convergence tests now have intuitive justifications (see page 714) as well as formal proofs. Numerical estimates of sums of series are based on which test was used to prove convergence. The emphasis is on Taylor series and polynomials and their applications to physics. Error estimates include those from graphing devices.

Chapter 12
Vectors and the Geometry of Space

The chapter on three-dimensional analytic geometry and vectors has been has been split into two chapters. Chapter 12 deals with vectors, the dot and cross products, lines, planes, surfaces, and cylindrical and spherical coordinates.

Chapter 13
Vector Functions

This chapter covers vector-valued functions, their derivatives and integrals, the length and curvature of space curves, and velocity and acceleration along space curves, culminating in Kepler's laws.

Chapter 14
Partial Derivatives

Functions of two or more variables are studied from verbal, numerical, visual, and algebraic points of view. In particular, I introduce partial derivatives by looking at a specific column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity. Directional derivatives are estimated from contour maps of temperature, pressure, and snowfall.

Chapter 15
Multiple Integrals

Contour maps and the Midpoint Rule are used to estimate the average snowfall and average temperature in given regions. Double and triple integrals are used to compute probabilities, surface areas, and (in projects) volumes of hyperspheres, and the volume of intersection of three cylinders.

Chapter 16
Vector Calculus

Vector fields are introduced through pictures of velocity fields showing San Francisco Bay wind patterns. The similarities among the Fundamental Theorem for line integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem are emphasized.

Chapter 17
Second-Order Differential Equations

Since first-order differential equations are now covered in Chapter 9, this final chapter deals with second-order linear differential equations, their application to vibrating springs and electric circuits, and series solutions.

Ancillaries

Calculus, Early Transcendentals, Fourth Edition is supported by a complete set of ancillaries developed under my direction. Each piece has been designed to enhance student understanding and to facilitate creative instruction.

The following resources are available, free of charge, to adopters of the text.

Instructor's Guide

This essential guide includes suggestions on how to implement innovative teaching ideas into the calculus course, serving as a practical roadmap to topics and projects in the text. Each section of the main text is discussed from several viewpoints and contains suggested time to allot, points to stress, text discussion topics, core materials for lecture, workshop/discussion suggestions, group work exercises in a form suitable for handout, and suggested homework problems.

Single Variable Calculus by Harvey Keynes, James Stewart, Douglas Shaw, and John Hall
Multivariable Calculus by Harvey Keynes and James Stewart

Complete Solutions Manual

Provides detailed solutions to all exercises in the text.

Single Variable Calculus by Daniel Anderson, Daniel Drucker, and Jeffery A. Cole
Multivariable Calculus by Dan Clegg and Barbara Frank

Printed Test Items

Organized according to the main text, this complete set of printed Test Items contains both multiple-choice and open-ended questions, offering a range of model problems, including short-answer questions that focus narrowly on one basic concept; items that integrate two or more concepts and require more detailed analysis and written response; and application problems, including situations that use real data generated in laboratory settings.

Single Variable Calculus by William Tomhave and Xueqi Zeng

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Study Guide

by Richard St. Andre

Offering additional explanations and worked-out examples, and formatted to provide guided practice, each section in this guide corresponds to a section in the text. Every section contains a short list of key concepts; a short list of skills to master, with worked examples for each; a brief introduction to the ideas of the section; an elaboration of the concepts and skills, including extra worked-out examples.

Single Variable Calculus ISBN 0-534-36820-4

Multivariable Calculus ISBN 0-534-36445-4

Student Solutions Manual

Contains strategies for solving and provides completely worked-out solutions to all odd-numbered exercises within the text, the review sections, the True-False Quizzes, the Problem Solving sections, and to all the exercises in the Concept Checks, giving students a way to check their answers and ensure that they took the correct steps to arrive at the answer.

Single Variable Calculus by Daniel Anderson, Daniel Drucker, and Jeffery A. Cole, ISBN 0-534-36301-6

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Lab Manuals

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JAMES STEWART

To the Student

Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper and calculator at hand to sketch a diagram or make a calculation.

Some students start by trying their homework problems and read the text only if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should look at the definitions to see the exact meanings of the terms.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected, step-by-step fashion with explanatory sentences—not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix H. Some exercises ask for a verbal explanation or interpretation or description. In such cases there is no single correct way of expressing the answer, so don't worry that you haven't found the definitive answer. In addition, there are often several different forms in which to express a numerical or algebraic answer, so if your answer differs from mine, don't immediately assume you're wrong. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you're right and rationalizing the denominator will show that the answers are equivalent.

The icon indicates an exercise that definitely requires the use of either a graphing calculator or a computer with graphing software. (Section 1.4 discusses the use of these graphing devices and some of the pitfalls that you may encounter.) But that doesn't mean that graphing devices can't be used to check your work on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, Mathematica, or TI-92) are required. You will also encounter the symbol ⊘, which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

The icon JTC indicates a reference to the CD-ROM *Journey Through*™ *Calculus*. The symbols in the margin refer you to the location in *Journey* where a concept is introduced through an interactive exploration or animation. The symbols in the exercise sets refer you to test questions with automatic grading.

Calculus is an exciting subject, justly considered to be one of the greatest achievements of the human intellect. I hope you will discover that it is not only useful but also intrinsically beautiful.

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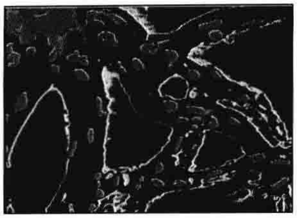

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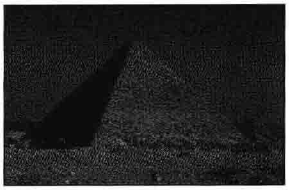

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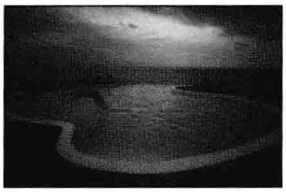

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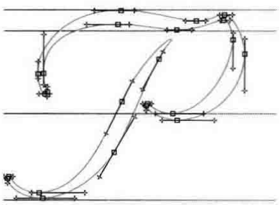

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