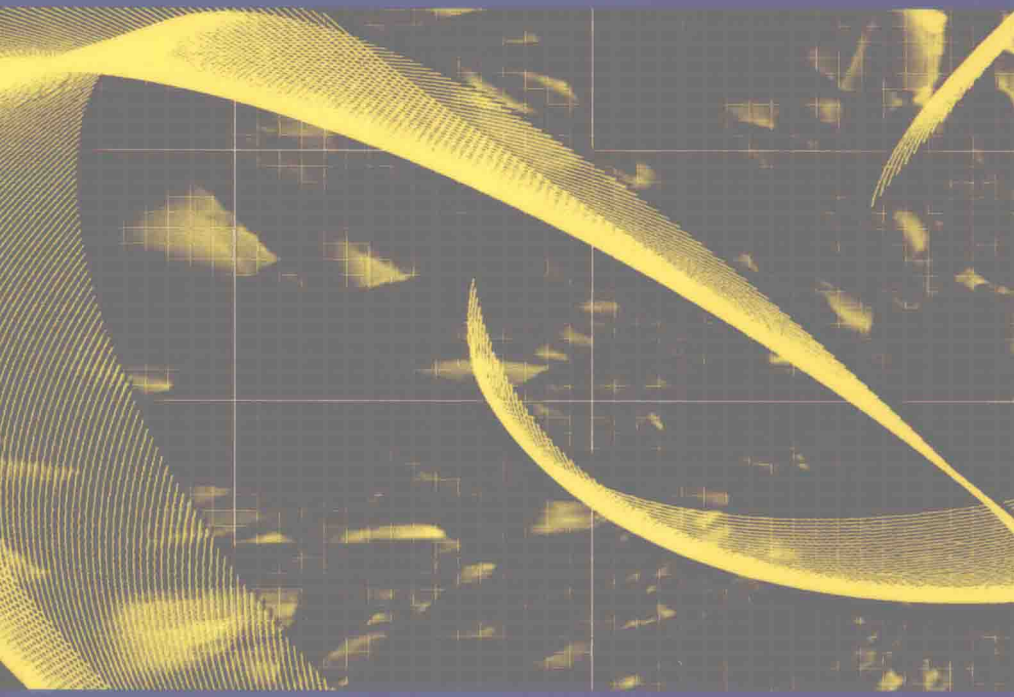
Extended Finite Element Method for Crack Propagation



Sylvie Pommier, Anthony Gravouil
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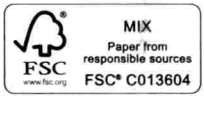

Extended Finite Element Method for Crack Propagation

Foreword

This book, offered to us by Sylvie Pommier, Anthony Gravouil, Nicolas Moës, and Alain Combescure, is undoubtedly the most awaited book today in the solid mechanics community. It certainly fills a gap in the very important subject of the new "eXtended" Finite Element Method, known only very recently and available only through specialized articles. The fact, that one of the authors of this new "X-FEM" method is also the joint author of this book, makes it all the more valuable.

The *raison d'être* of this book is to provide a method, certainly the only truly efficient one, to model real crack propagation, from the initiation stage to final fracture.

We know that the conventional Finite Element Methods using fixed meshes can only deal with this type of problem, either if the crack path travels through mesh nodes, or if we remove mesh elements. This is an extremely important limitation in industrial applications.

The new "X-FEM" method no longer requires the mesh to be constantly stuck to discontinuous surfaces or propagating cracks. It introduces discontinuous finite elements. It uses, as in other known methods, the partition of unity and introduces level sets to model the crack surface and the crack front, while maintaining the same mesh.

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In addition to the discussion of the "X-FEM" method, in Chapters 2 and 3, there is also a discussion on the non-linear behavior of the cracked body and the related crack growth laws (Chapter 4); brittle, ductile, static, and dynamic fracture (Chapter 5), as well as an investigation of the voluminal forces and thermal loading.

I would also like to add that the numerous illustrations have conferred a particularly enticing feel to this book, which, I do not doubt, will have great success in the solid mechanics community.

H. D. Bui, Institute Member

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For this reason, we would particularly like to thank ANR, Cetim, DGA, SAFRAN, AREVA, EDF. We ask forgiveness from those who we may have forgotten.

A big thank you also goes to all our colleagues who actively encouraged the drafting of this collective work, and who allowed us to improve its quality. We also direct our gratitude to R. Desmorat, J. Lemaître, A. Benallal and H.D. Bui who all read this document, in part, or as a whole.

Finally, to finish, we would like to send warm thanks to the PhD students and post-doctorals who contributed to this work and to whom we are highly indebted.

List of Symbols

$\boldsymbol{x}$	Vector position of a point.
$\boldsymbol{u}(\boldsymbol{x})$, $\boldsymbol{v}(\boldsymbol{x})$	Displacement vector, and velocity of a point.
$\boldsymbol{\sigma}(\boldsymbol{x})$, $\boldsymbol{\epsilon}(\boldsymbol{x})$	Stress and strain tensors.
$\boldsymbol{C}$	Hooke tensor.
K_I, K_{II}, K_{III}	Stress intensity factors of Modes I, II, and III.
ψ and ϕ	Level set functions associated with the surface and front of the crack.
$H(\boldsymbol{x})$	Generalized Heaviside function.
$\boldsymbol{\varphi}(\boldsymbol{x})$	Displacement modes of the enriched Finite Element approximation.
$\dot{\tilde{K}}_I$, $\dot{\tilde{K}}_{II}$	Rate of elastic displacement field intensity factors.
$\dot{\rho}_I$, $\dot{\rho}_{II}$	Rate of complementary field intensity factors.
$\boldsymbol{G}$, (G_I, G_{II})	Energy release rate vector (and the components).
G	Energy release rate in crack plane.

Introduction

The predicted lifespan of parts and components is a key question regarding the safety of certain machines, such as airplanes, cars, production factories, or with regard to the reliability of micro-electronic components or implants in the human body.

Mechanical parts are designed and controlled in order to guarantee that they do not contain a macroscopic crack, i.e. detectable by standardized test methods such as metal sweating or ultrasonic control. These increasingly sophisticated devices, together with strict and standardized procedures, are used in order to guarantee that at the end of the production cycle the assembled parts are free from detectable cracks. In addition, defects can be implemented during the assembly stage (e.g., during welding). Lastly, a mechanical system, even when completely healthy at the end of manufacturing and assembly stages, may still be damaged when in use due to the encountered stresses, naturally mechanical ones as well as thermal or environmental (chemical or biological attack, scratches or wearing, minor impacts, etc.). Sometimes, synergies exist between these damage mechanisms which can then lead to anticipated failures. Stress-corrosion or fatigue-corrosion are both well-known examples.

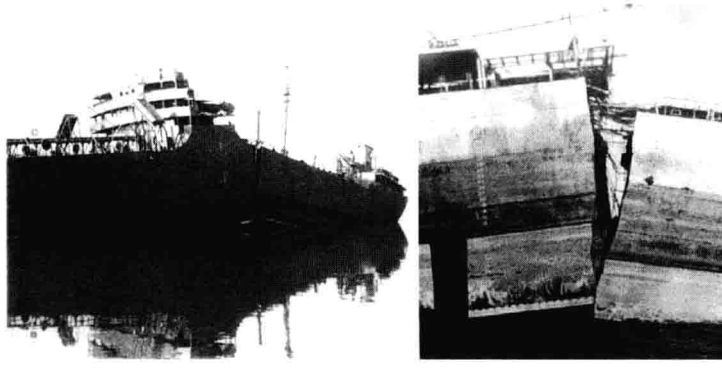

Figure 1. *Photographs of broken, moored liberty-ships during the winter of 1941*

Some examples of failures have made us more aware of the importance of the potential presence of these defects (from manufacturing, assembly, or usage) and of the risk of fracture they induce. These examples can be regarded as "founders" of fracture mechanics. The most spectacular example is undoubtedly the brittle fracture of liberty-ships during the Second World War. These ships were built as a series in order to transport men and materials across the Atlantic Ocean. Some of these boats were found split through by enormous cracks, as seen in Figure 1. The breaks occurred during the night, in the winter of 1941, when the boats were moored and waiting to depart. The role of the subjected temperature was quickly identified: with low-temperatures and close-welding, the material became brittle and the residual stresses introduced during welding were sufficient for the manufacturing defects to propagate and lead to the unstable fracture of the ship.

The development of fracture mechanics began at this time. This discipline, originating at the beginning of the 20th century, tries to predict or rather avoid, the breaking of parts and components of structures.

The initially vague and invisible-to-the-eye damage gradually leads to the appearance of macroscopic cracks. These cracks can then propagate and lead to fracture.

Figure 2. *High-speed "ICE" train in Eschede, Germany in 1998. This accident was caused by a crack in the wheel caused by fatigue*

The sudden fracture of a component can sometimes have very serious consequences for people's safety. For example, in Eschede (Germany) in 1998, a dramatic train accident took place (Figure 2) due to crack initiation, and then fatigue crack propagation in a wheel. This was followed by a brutal breaking of the wheel at full speed. A fracture can also have important financial consequences for the operators, with direct costs (replacement of broken or damaged parts) and indirect costs coming from the unforeseen unavailability of the systems (airplanes, trains, electricity power stations, etc.).

Inspections are also carried out on safety components by devices which also carry risks. The vital areas of the parts are periodically tested to detect the possible appearance of cracks and to estimate their size (by non-destructive testing). Then, procedures are applied to decide whether the part must be replaced or not, as a preventive measure. This is a classic procedure in aviation, railways, or the nuclear industry. As for such preventative maintenance, the decisions are made by relying primarily on statistical results from the systematic and organized monitoring of aircraft fleets in service.

Thus, all the progress of crack prediction or non-destructive test methods bears great economic interest. Certainly, to

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reduce the costs of preventive maintenance (replacements or periodic part testing) without reducing the safety, it is necessary to reduce uncertainty over the potential lifespan of parts, which requires an improvement in the methods used to predict lifespan.

Normally, the lifespan of mechanical components is divided into two stages: a stage known as "initiation" during which the defects are developed, becoming detectable macro-cracks. Then, there is a propagation stage during which these cracks begin to propagate. Each stage has a specific time duration: incubation time T_i, for the first stage, and propagation time T_p for the second stage. The lifespan is the sum of these two durations. When we can guarantee that the incubation time is much higher than the propagation time ($T_p \ll T_i$), there is little point in simulating crack propagation. In the opposite case, we will adhere to calculating T_p, in order to predict the evolution of the crack, from the detection threshold until the critical dimension where the break occurs.

Thus, during each periodic inspection of a safety component, the following questions are raised:

– Considering the loading applied in service, and the safety coefficients of this loading, are the (detected or potential) cracks critical?

– If yes: the part must be replaced or the device be stopped.

– If no: how much time remains before it does become critical?

– If this time is lower than an inter-inspection interval, considering the safety margins over the calculated lifespan, the part must be replaced or the device stopped.

– Otherwise, the device is put back in to service until the next periodic test.

This process is determined by experiment, the possibilities of simulation, and common-sense and it is commonly accepted in literature on this subject. It allows the risk of fracture to

be minimized in operating conditions, by swooping down to pessimistic (or conservative) assumptions. However, this conservative procedure is expensive and can sometimes involve ineffectual or even harmful interventions. In the aviation industry, for example, the probability of a defect creation during repair or replacement of a part is taken into account for certain components.

It is therefore useful to limit the amount of interventions and to spread out the time between inspections, i.e. to increase the calculated lifespan for the part. For a material, a load, and a given geometry, considering the process, the lifespan can be increased mainly by three ways:

– by lowering the crack detection threshold, which requires improving the non-destructive test methods;

– by lowering the safety coefficients on the load, which requires an improved loading knowledge, which can be obtained, *inter alia*, by the instrumentation of parts in operating conditions (or by health *monitoring*);

– by lowering the safety margins on the calculated lifespans, which requires the improvement of lifespan calculation methods, which is the objective of this book.

Many books have already been devoted to fracture mechanics. The reader may wonder what a new book might bring to this already well-known subject. What prompted the writing of this work was that the recent scientific developments make it possible to raise two strong hypotheses, which are usually put forward in considering residual lifespan and which are sometimes debatable:

– any crack that is propagated will propagate until breakage;

– the stress state stays the same during propagation.

These hypotheses result from the difficulty of taking into account the effect of spatial and temporal variations of the crack propagation path. For example, a crack that started for

a certain a stress concentrator may stop, bifurcate, or propagate in an even more critical plane when its tip moves away. The crack then becomes a curved surface of complex form.

This book aims to present the important recent advances in research that makes it possible to raise the restrictive hypotheses usually used for the remaining lifespan prediction of the cracked parts. This new progress stems from two conceptual jumps in modeling which arrive almost simultaneously:

–*The appearance of new numerical methods* that allow modeling of complex shaped cracks in three-dimensional media (independently of the mesh), and therefore, to consider spatial variations of crack loading.

The crack front is modeled by a continuous function of the three-dimensional medium (*level* set), which gives the signed distance to the crack plane at each spatial point. A second level set makes it possible to define the crack front.

This modeling is associated with a calculation method by enriched finite elements (X-FEM) based on the partition of unity, which makes it possible for the elements to be completely or partially intersected by the crack. Jump-type discontinuous functions are added to normal displacement interpolation functions for the completely intersected elements, while the asymptotic displacements fields resulting from the linear fracture mechanics are added for the elements where the crack front exists.

Thus we can easily simulate crack propagation in a three-dimensional medium by finite elements: it is useless to remesh when the crack propagates. It is sufficient to update the *level sets* and to modify the base displacement field of X-FEM elements related to the new crack position.

–*Appearance of incremental crack growth laws* which integrate effects of the confined plastic strain at the crack tip under mixed mode and variable amplitude loading conditions.

The approach is based on a projection of the velocity field around the crack tip on a base of reference fields (space functions only). The intensity factor of each of these fields constitutes a *condensed measure* of elastic and plastic strain rates for each mode in the crack tip region.

Determining the evolution laws of these intensity factors makes it possible to equip eXtended finite elements (X-FEM) with cyclic and multiaxial elastic-plastic extended behavior laws. They are used to predict the growth rate of the cracked surface area during loading paths including non-proportional mixed mode and variable amplitude loadings schemes.

These two advances make it possible to demonstrate effective numerical simulations in three-dimensions, with reasonable meshes and calculation times.

Here, we could question the relevance of these "global" methods, which are based on neighboring fields at the crack tip with respect to "local" approaches, which are based on local stress values of strains or strain velocities.

Two brief answers can be put forward: first of all, the efficiency of the numerical solutions is much higher with the "global" methods, a very fine mesh around the crack front not being required as the field shape was given *a priori*. Secondly, if the "local" approaches are well adapted to the fracture initiation, they are less so for the propagation simulation. Certainly, in practice, the local methods, which use the continuous finite elements also represent, *a priori,* the displacement fields selected for the elements (linear or quadratic) and can somewhat inadequately apprehend the presence of inherent discontinuities due to the presence of a crack.

The first chapter of this book is devoted to recalling the elementary concepts of the fracture process.

In Chapter 2, the numerical modeling of fixed or moving discontinuities is explained. Here, the discontinuities are

cracks, but the same methods apply to represent other interfaces, such as a fluid-solid front in a foundry simulation.

The third chapter is devoted to the presentation of eXtended finite elements X-FEM.

The fourth chapter concerns non-linear constitutive laws for a cracked body and the strategies used to identify them for a given material.

Chapter 5 shows fracture applications through fatigue, brittle, ductile, static, and dynamic means. Some three-dimensional cases are also compared with experimental results.

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