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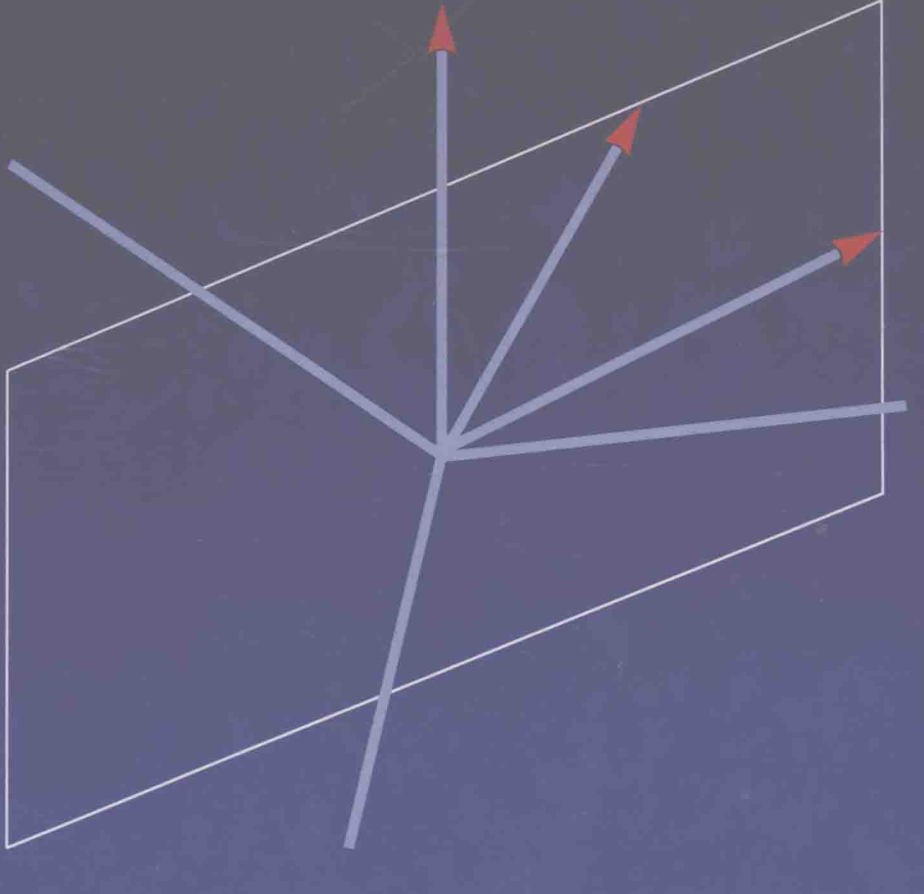
Edition

LINEAR ALGEBRA
With
APPLICATIONS



Steven J. Leon

FOURTH EDITION

LINEAR ALGEBRA WITH APPLICATIONS

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To Judith Russ Leon

Preface

This book is suitable for either a sophomore-level course or for a junior/senior-level course. The student should have some familiarity with the basics of differential and integral calculus. This prerequisite can be met by either one semester or two quarters of elementary calculus.

If the book is used for a sophomore-level course, then one should probably spend more time on the early chapters and omit many of the sections in the later chapters. For more advanced courses one could quickly review many of the topics in the first two chapters and then do a more complete coverge of the later chapters. The explanations in the text are given in sufficient detail so that beginning students should have little trouble reading and understanding the material. To further aid the student, a large number of examples have been worked out completely. Additionally there are computer exercises at the end of each chapter that give students the opportunity to perform numerical experiments and try to generalize the results. Applications have been scattered throughout the book. These applications can be used to motivate new material or to illustrate the relevance of material that has already been presented.

The text contains more material than can be covered in a one-quarter or one-semester course. It is the author's feeling that it is easier for an instructor to leave out or skip material than it is to supplement a book with outside material. Even if many topics are omitted, the book still should provide students with a feeling for the overall scope of the subject matter. Furthermore, many of the students may use the book later as a reference and consequently may end up learning many of the omitted topics on their own.

Ideally, one could cover the entire book in a two-quarter or two-semester sequence. Although two semesters of linear algebra have been recommended by a special NSF-sponsored Linear Algebra Curriculum Study Group (LACSG), it is still not practical at many universities and colleges. In a later section of this preface a number of outlines are provided for one-semester courses at either the sophomore-level or the junior/senior-level and with either a matrix-oriented emphasis or a slightly more theoretical emphasis. To further aid the instructor in the choice of topics, three sections have been designated as optional and are marked with an asterisk in the table of contents. These sections are not prerequisites for any of the following sections in the book. They may be skipped without any loss of continuity.

Earlier editions of this book have been used at a large number of colleges and universities for a wide variety of linear algebra courses. Thanks to the support and enthusiasm of its many users, the book is now in its fourth edition. With each new edition the book continues to evolve. Although the success of the earlier editions indicates that there is no need for fundamental changes, there are always sections and topics that can be enhanced and clarified. The author teaches two or three linear algebra classes per year and is constantly seeking better ways to present the material. Reviewers and users have also contributed many helpful suggestions. Consequently this new edition, while retaining the essence of the previous editions, incorporates a wide array of substantive improvements.

WHAT'S NEW IN THE FOURTH EDITION?

1. Computing Exercise Sets for Each Chapter

   The MATLAB computing exercises have been greatly expanded. The new edition now includes a section of MATLAB exercises at the end of each chapter. These exercise sections run from 2 to 7 pages depending on the length of the chapter. The exercises are all carefully designed to fulfill a number of teaching objectives. The exercises involve much more than just mechanical computations. They require students to perform computations and to answer questions about the results of the computations. The questions serve to bring out the mathematical significance of the computations. Students should not only gain experience doing matrix computations but should also gain new insights into the subject matter.

2. More Geometrical Motivation

Each successive edition of this book has had a greater emphasis on geometry. The new edition includes still further additional geometrical motivation for some topics and nine new geometrical figures.

3. New Application Involving Graph Theory and Networks

An application involving graphs and networks has been added to Section 3 of Chapter 1 and to the MATLAB exercises. New problems have also been added to the exercise sets in Chapter 1, and a number of the worked examples in the chapter have been revised and improved.

4. Additional Motivation for the Definition of the Determinant

New material has been added to the first section of Chapter 2 in order to provide better motivation for the definition of the determinant of a matrix. As a result, most of the section was rewritten. The determinant is introduced as a number associated with a matrix whose value indicates whether or not the matrix is nonsingular. Before considering the general definition of the determinant of an $n \times n$ matrix A, the special cases $n = 1, 2, 3$ are examined. In each case a condition is derived for determining whether or not A is row equivalent to the identity matrix, based on whether or not an expression involving the entries is nonzero. The general definition is presented as a generalization of these expressions.

New exercises have also been added to all three sections of Chapter 2.

5. Change of Basis Section Moved to Chapter 3

The section "Change of Basis" has been moved from Chapter 4 to Chapter 3 for this edition. Much of this section has been rewritten. Students should find the revised version much more user friendly. In addition, four of the six exercise sets in Chapter 3 have been expanded.

6. Major Revisions in the Section on Inner Product Spaces

The section "Inner Product Spaces" was revised extensively. It includes a different proof of the Cauchy–Schwarz inequality. The new proof should be more meaningful to students than the proof given in the previous editions. Also new to this section is the introduction of an inner product for the vector space $R^{m \times n}$. The Frobenius norm is then introduced as the norm derived from the inner product.

7. Section on Matrix Norms Moved to Chapter 7

   The Frobenius matrix norm is now introduced in Section 3 of Chapter 5; the rest of the material on matrix norms in Section 4 has been revised and moved to Chapter 7. Matrix norms are now contained in Section 4 of Chapter 7, which has been retitled "Matrix Norms and Condition Numbers."

8. New Application: Approximation of Functions by Trigonometric Polynomials

   Some major revisions were also made in the section "Orthonormal Sets" in Chapter 5. A new subsection was added showing how to find the best least squares approximation to a function in $C[a, b]$ by a trigonometric polynomial of degree less than or equal to n. Some of the examples in this section were also revised and new material about projection matrices has been added to the section.

9. Chapter 6 Revisions

   New exercises have been added to most of the sections in Chapter 6. There are also a number of new examples.

10. The section "Iterative Methods" has been dropped from Chapter 7 in this edition. This was an optional section in previous editions and I suspect was rarely covered in actual linear algebra courses. With all of the improvements added to this edition it was necessary to cut out some material in order to keep the page count (and cost to the student) down.

11. In preparing the fourth edition, the author has carefully reviewed every section of the book. In addition to the major changes that have been listed, numerous minor improvements have been made throughout the text.

COMPUTER EXERCISES

This edition contains a section of computing exercises at the end of each chapter. These exercises are based on the software package MATLAB. The MATLAB Appendix in the book explains the basics of using the software. MATLAB has the advantage that it is a powerful tool for matrix computations and yet it is easy to learn. After reading the Appendix, students should be able to do the computing exercises without having to refer to any other software books or manuals. To help students get started, we recommend one 50-minute classroom demonstration of the software. The assignments can be done either as ordinary homework assignments or as part of formally scheduled computer labs.

Although the course can be taught without any reference to the computer, we believe that computer exercises can greatly enhance student learning and provide a new dimension to linear algebra education. This view seems to be gaining wide support in the greater mathematics community. The Linear Algebra Curriculum Study Group has recommended that technology be used in a first course in linear algebra. At meetings of all three major mathematics societies there are now sessions devoted primarily to using computers in teaching linear algebra. The National Science Foundation and the International Linear Algebra Society are sponsoring a project called ATLAST (Augmenting the Teaching of Linear Algebra through the use of Software Tools). The purpose of the project is to encourage and facilitate the use of software in the teaching of linear algebra. ATLAST has conducted ten faculty workshops using the MATLAB software package. Participants from these workshops are designing computer exercises for linear algebra courses and contributing these exercises to a project database. Exercises from the ATLAST database will be collected in a book that has been tentatively titled *ATLAST Computer Exercises for Linear Algebra*. The editors of this book are Steven J. Leon, Richard Faulkenberry, and Eugene Herman.

SUGGESTED COURSE OUTLINES

I. Two-Semester Sequence

In a two semester sequence it is possible to cover all 39 sections of the book. Additional flexibility is possible by omitting any of the three optional sections in Chapters 2, 5, and 6. One could also include an extra lecture demonstrating how to use the MATLAB software.

II. One-Semester Sophomore-Level Course

A. Basic Sophomore-Level Course

Chapter 1	Sections 1–5		7 lectures
Chapter 2	Sections 1–2		2 lectures
Chapter 3	Sections 1–6		9 lectures
Chapter 4	Sections 1–3		4 lectures
Chapter 5	Sections 1–6		9 lectures
Chapter 6	Sections 1–3		4 lectures
		Total	35 lectures

B. LACSG Matrix-Oriented Course

The core course recommended by the Linear Algebra Curriculum Study involves only the Euclidean vector spaces. Consequently for this course one should omit Section 1 of Chapter 3 (on general vector spaces) and all references and exercises involving function spaces in Chapters 3–6. All of the topics in the LACSG core syllabus are included in the text. It is not necessary to introduce any supplementary materials. The LACSG recommended 28 lectures to cover the core material, but the author feels that the following schedule of 35 lectures is perhaps more reasonable.

Chapter 1	Sections 1–5	7 lectures
Chapter 7	Section 2 (LU factorization)	1 lecture
Chapter 2	Sections 1–3	3 lectures
Chapter 3	Sections 2–6	6 lectures
Chapter 4	Section 1	1 lecture
Chapter 5	Sections 1–6	9 lectures
Chapter 6	Sections 1, 3–5	8 lectures
	Total	35 lectures

III. One-Semester Junior/Senior-Level Courses

The coverage in an upper-division course is dependent on the background of the students. Below are two possible courses with 35 lectures each.

A. Course 1

Chapter 1	Sections 1–5	6 lectures
Chapter 2	Sections 1–2	2 lectures
Chapter 3	Sections 1–6	7 lectures
Chapter 5	Sections 1–6	9 lectures
Chapter 6	Sections 1–6 Section 7 if time allows	9 lectures
Chapter 7	Section 6 Part of Section 8 if time allows	2 lectures

B. Course 2

Review of Topics in Chapters 1–3		5 lectures
Chapter 4	Sections 1–3	3 lectures
Chapter 5	Sections 1–6	9 lectures
Chapter 6	Sections 1–6 Section 7 if time allows	9 lectures
Chapter 7	Sections 4–8 If time allows, Sections 1–3	9 lectures

SUPPLEMENTARY MATERIALS

A solutions manual is available to all instructors teaching from this book. The manual contains complete solutions to all of the nonroutine exercises in the book. The manual also contains answers to any elementary exercises that were not already listed in the answer key section of the book.

ACKNOWLEDGMENTS

The author would like to express his gratitude to the long list of reviewers that have contributed so much to all four editions of this book. Thanks also to the many users who have sent in comments and suggestions. Special thanks to Wayne Barrett and Germund Dahlquist for their suggestions for the second and third editions.

Many of the revisions and new exercises in this latest edition are a direct result of the comments and suggestions of the reviewers: Timothy Hardy, The University of Northern Iowa; Inessa Levi, University of Louisville; Dennis McLaughlin, Princeton University; Hiram Paley, University of Illinois at Urbana; Sandra Shields, College of William and Mary; Ilya Spitkovsky, College of William and Mary; Mo Tavakoli, Chaffey Community College; and Santiago Tavares, University of Florida at Gainesville.

The author would also like to thank a number of individuals who have helped to shape this edition. Thanks to Cleve Moler for suggesting two of the MATLAB exercises. Thanks also to Roger Horn and Kermit Sigmon for their suggestions and a special thanks to Philip Bacon for providing detailed commentary on many of the sections of the third edition. The mathematics community suffered a great loss when Philip passed away in November 1991. He will be greatly missed by the students and faculty of his home institution, the University of Florida, and by his many friends.

Thanks to Judith Russ Leon and Ann Cox for independently proofreading the manuscript for the fourth edition. Thanks are also due to Ann Cox for working the exercises and checking the answers in the back of the book.

The final revisions on the manuscript were done while the author was on sabbatical visiting the Swiss Federal Institute of Technology (ETH) and Stanford University. The author would like to thank his hosts Walter Gander and Gene Golub for making those visits possible. In particular the author greatly appreciated the opportunity to use Sun Workstations to prepare the manuscript for this edition.

Thanks to Mathematics Editor Bob Pirtle and to the rest of the editorial, production, and sales staff at Macmillan College Publishing Company for the work they have done on all four editions.

Finally, the author would like to acknowledge the contributions of Gene Golub and Jim Wilkinson. Most of the first edition of the book was written in 1977–78 while the author was a Visiting Scholar at Stanford University. During that period the author attended courses and lectures on numerical linear algebra given by Gene Golub and J. H. Wilkinson. Those lectures have greatly influenced this book.

S. L.

Linear Algebra with Applications

Contents

*The asterisk indicates optional sections. See the first section of the Preface for an explanation.

MATRICES AND SYSTEMS OF EQUATIONS

Probably the most important problem in mathematics is that of solving a system of linear equations. Well over 75 percent of all mathematical problems encountered in scientific or industrial applications involve solving a linear system at some stage. By using the methods of modern mathematics, it is often possible to take a sophisticated problem and reduce it to a single system of linear equations. Linear systems arise in applications to such areas as business, economics, sociology, ecology, demography, genetics, electronics, engineering, and physics. Therefore, it seems appropriate to begin this book with a section on linear systems.

1 SYSTEMS OF LINEAR EQUATIONS

A *linear equation in n unknowns* is an equation of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, a_2, \ldots, a_n$ and b are real numbers and $x_1, x_2, \ldots, x_n$ are variables. A *linear system* of m equations in n unknowns is then a system of the form

$$
\begin{array}{rcl}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &=& b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &=& b_2 \\
\vdots && \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &=& b_m
\end{array} \tag{1}
$$

where the a_{ij}'s and the b_i's are all real numbers. We will refer to systems of the form (1) as $m \times n$ linear systems. The following are examples of linear systems:

$$
\text{(a)}\quad \begin{aligned} x_1 + 2x_2 &= 5 \\ 2x_1 + 3x_2 &= 8 \end{aligned} \qquad
\text{(b)}\quad \begin{aligned} x_1 - x_2 + x_3 &= 2 \\ 2x_1 + x_2 - x_3 &= 4 \end{aligned} \qquad
\text{(c)}\quad \begin{aligned} x_1 + x_2 &= 2 \\ x_1 - x_2 &= 1 \\ x_1 \quad &= 4 \end{aligned}
$$

System (a) is a 2×2 system, (b) is a 2×3 system, and (c) is a 3×2 system.

By a solution to an $m \times n$ system, we mean an ordered n-tuple of numbers $(x_1, x_2, \ldots, x_n)$ that satisfies all the equations of the system. For example, the ordered pair $(1, 2)$ is a solution to system (a), since

$$
\begin{aligned} 1 \cdot (1) + 2 \cdot (2) &= 5 \\ 2 \cdot (1) + 3 \cdot (2) &= 8 \end{aligned}
$$

The ordered triple $(2, 0, 0)$ is a solution to system (b), since

$$
\begin{aligned} 1 \cdot (2) - 1 \cdot (0) + 1 \cdot (0) &= 2 \\ 2 \cdot (2) + 1 \cdot (0) - 1 \cdot (0) &= 4 \end{aligned}
$$

Actually, system (b) has many solutions. If α is any real number, it is easily seen that the ordered triple $(2, \alpha, \alpha)$ is a solution. However, system (c) has no solution. It follows from the third equation that the first coordinate of any solution would have to be 4. Using $x_1 = 4$ in the first two equations, we see that the second coordinate must satisfy

$$
\begin{aligned} 4 + x_2 &= 2 \\ 4 - x_2 &= 1 \end{aligned}
$$

Since there is no real number that satisfies both of these equations, the system has no solution. If a linear system has no solution, we say that the system is *inconsistent*. Thus system (c) is inconsistent, while systems (a) and (b) are both consistent.

The set of all solutions to a linear system is called the *solution set* of the system. If a system is inconsistent, its solution set is empty. A consistent system will have a nonempty solution set. To solve a consistent system, one must find its solution set.