

Jaime Marquez

# Estimating Trade Elasticities

Advanced Studies in  
Theoretical and Applied Econometrics



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by

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Jaime Marquez

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# Preface

I have made a living out of estimating how international trade responds to changes in income and prices. And by pure happenstance, I have seen how estimates of these responses percolate into policy discussions. For example, one key question during the 1982 debt crisis of developing countries was whether their exports would grow fast enough to provide the earnings needed for servicing their debts. A key element of the answer was how responsive their exports were to changes in foreign income. In another case, the unprecedented appreciation of the U.S. dollar during the 1980s raised the question of how much of the deterioration of the U.S. current account was due to that appreciation. Again, an important element of the answer to this question was how responsive U.S. exports and imports were to price changes. Finally, the question raised by the 1997 Asian crisis was whether the decline in Asian economic activity would translate into a decline in U.S. economic activity; the answer to this question depended on the responsiveness of U.S. exports to foreign income and prices.

I am not alone in being interested in quantifying those trade responses, and like most other researchers, my focus has been on measuring those responses as elasticities: the percent response of trade induced by a 1 percent change in income or prices. Moreover, like previous researchers, I have sought to estimate these elasticities by relying on statistical methods, and in the process, I joined a rather active field of research.

Greatly simplified, some economists estimate these elasticities as a necessary step in translating assumptions about future expenditures and relative prices into projections for external imbalances. With such projections, one can anticipate the implications for economic performance and design possible policy responses. Therefore, these practitioners favor a framework in which predictive accuracy is of the essence, and they might be willing to put aside conflicts with theory for the sake of predictive accuracy. Historically, this

approach has involved assuming that income and price elasticities are constant because it greatly facilitates the estimation of elasticities and allows researchers to evaluate the role of choice of techniques in accounting for differences in the estimates. Thus, with few exceptions, assuming constancy of elasticities is the trademark of this area of research.

For a second group of practitioners, however, estimating income and price effects is central to the process of discriminating among competing theories explaining movements in international trade. With that understanding, one can frame meaningful policy discussions—meaningful in the sense that they do not contradict economic theory. Therefore, these practitioners are willing to sacrifice predictive accuracy for formal consistency with economic theory.

The tension between these groups arises because models consistent with economic theory do not fit the data as well as the models that conflict with theory. Indeed, the record reveals that predictive accuracy calls for constant-elasticity models whereas theoretical consistency calls for varying-elasticity models. This record raises two questions. First, how can one justify using variable-elasticity models to craft policy prescriptions for an economy if they cannot explain the functioning of that economy? Second, how can one justify using constant-elasticity models to craft policy prescriptions if these models contradict economic theory?

These questions have no clear-cut answers, but what is clear is that, in the absence of a generally accepted method for deciding between predictive accuracy and theoretical consistency, all prediction errors become equally important, all theoretical implications become equally important, and the professional divisiveness lives on. For me, this tension has translated into one question: How can one judge the usefulness of a collection of elasticities for studying global interdependencies? The essays in this book show how, as a practical economist, I have dealt with this question.

The extent to which I may offer something useful here comes, in no small measure, from criticisms and remarks that I have received, over many years, from F. Gerard Adams, Neil Ericsson, Jon Faust, Joe Gagnon, David Gordon, Dale Henderson, David Hendry, William Helkie, Peter Hooper, Hendrik Houthakker, David Howard, Karen Johnson, Lawrence Klein, Andrew Levin, Steve Magee, Cathy Mann, Kathryn Morisse, Charles Pearson, Ralph Tryon, and Ted Truman. Bill Helkie and Hendrik Houthakker read the entire manuscript and provided detailed criticisms.

I have also benefited from remarks during various presentations: Midwest International Economics Meetings (Spring 1998, Michigan State Uni-

versity; Fall 1998, University of Michigan; Spring 1999, University of Illinois, Champaign-Urbana); Federal Reserve Meetings (FRB International workshop and the Spring 1999 meetings of the System's Committee on International Economic Analysis); the U.S. International Trade Commission, Johns Hopkins' SAIS, the Summer 1999 meetings of the Econometric Society (University of Wisconsin, Madison), and the 2000 World Congress of the Econometric Society (University of Washington, Seattle). Remarks from Laura Adams, John Ammer, Anjit Bajwa, Bill Donnelly, Michael Ferrantino, Jeff Frankel, Kishore Gawande, Linda Goldberg, Morris Goldstein, David Gould, Keith Head, Jane Ihrig, Wolfgang Keller, Peter Kennedy, Kala Krishna, Prakash Loungani, Priya Ranjan, J. David Richardson, Raymond Robertson, Wendy Takacs, Kei-Mu Yi, and Joachim Zietz are gratefully acknowledged. I am also grateful to Molly Wetzell and Lisa Workman for their research assistance, to Cathy Tunis for implementing literature searches, and to Ellen Dykes for superb editorial suggestions. I alone own the remaining errors in this work. The views in this work are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

Finally, this book is dedicated to my daughters, Cecilia and Bianca, who have taught me how to wear many hats.

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# Chapter 1

## Introduction

### 1.1 Policies, Parameters, and Practices

One cannot exaggerate the usefulness of knowing how changes in income and price affect trade. Questions involving the stability of the foreign exchange market, the willingness to participate in a monetary union, the strength of international interdependencies, and the sustainability of the external deficits are hard to answer without estimates of the effects of income and price on trade.<sup>1</sup>

Krugman offers a useful expression that illustrates how important information of these effects can be in practice. His formulation (Krugman, 1989) gives the rate of depreciation of a country's real exchange rate consistent with external balance as a function of economic activity and trade elasticities:

$$\hat{r} = \frac{-(\eta_x \cdot \hat{y}^* - \eta_m \cdot \hat{y})}{(\epsilon_x + \epsilon_m - 1)}, \quad (1.1)$$

where  $\hat{r}$  is the rate of change of the real exchange rate ( $\hat{r} > 0$  indicates a real depreciation);  $\eta_x$  is the income elasticity of export demand;  $\hat{y}^*$  is the growth rate of foreign income;  $\eta_m$  is the income elasticity of import demand;  $\hat{y}$  is the growth rate of domestic income;  $\epsilon_x$  is the price elasticity of export demand; and  $\epsilon_m$  is the price elasticity of import demand. All of these elasticities

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<sup>1</sup>See Magee (1975), Stern et al. (1976), Goldstein and Khan (1985), Kohli (1991), Hooper and Marquez (1995), Sawyer and Sprinkle (1996), Mann (1999). The classical paper is Orcutt (1950).

are defined to be non-negative. The numerator of equation (1.1) gives the response of the balance of trade to changes in foreign and domestic income whereas the denominator gives the response of the balance of trade to changes in the country's real exchange rate.

The usefulness of this equation hinges on three assumptions. First,  $\epsilon_x + \epsilon_m \neq 1$ ; otherwise,  $\hat{r}$  is not defined. Second, changes in either exports or imports do not affect world prices. Third, elasticities are constant. To illustrate the usefulness of this last property, I assume unitary export elasticities and unitary (annual) growth rates to compute

$$\hat{r} = \frac{-(1 \cdot 1 - \eta_m \cdot 1)}{(1 + \epsilon_m - 1)} = \frac{-(1 - \eta_m)}{\epsilon_m}$$

for alternative values of  $\eta_m$  and  $\epsilon_{dm}$ . Figure 1.1 shows the calculations.

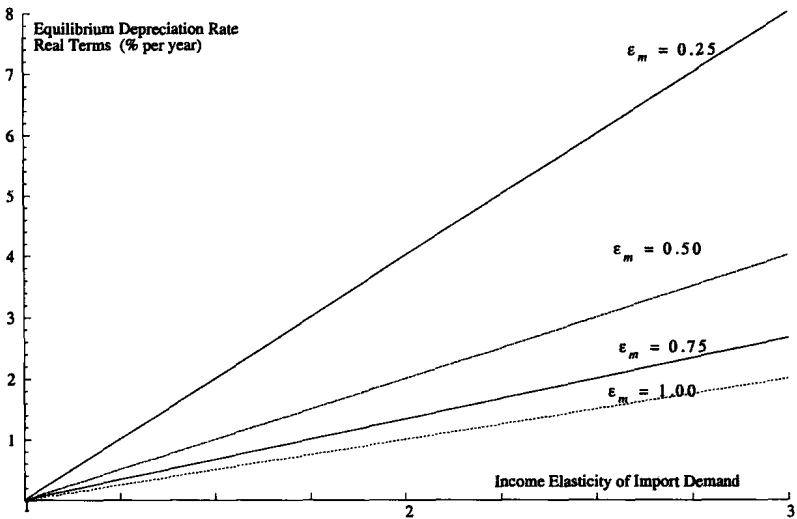


Figure 1.1: Import Demand Elasticities and Equilibrium Real Depreciation

The calculations show that the equilibrium real depreciation decreases in direct relation to the price elasticity of imports. Intuitively, an increase in the price elasticity of imports reflects an increase in the availability of domestic substitutes for these imports. Thus, a decline in the change in the relative price of imports is needed to switch expenditures and maintain external balance. Second, the equilibrium real depreciation increases in direct

relation to the income elasticity of imports. Intuitively, an increase in the income elasticity for imports reflects an increase in the spillover effect from the demand for domestic products onto imports. Thus, one needs a greater increase in the relative price of imports to offset the additional demand for imports and maintain external balance. Finally, if one knows the values of income and price elasticities, then one can estimate the path of the equilibrium rate of depreciation, draw inferences about future adjustments in the actual real exchange rate, and develop potential policy responses.

The ease with which equation (1.1) translates estimated elasticities into policy applications hinges on the assumption that these elasticities are constant. But assuming constancy of elasticities comes at a price: Constancy of elasticities translates into helpful policy discussions as long as everyone agrees on the magnitude of the elasticities. If there is not such agreement, conclusions implied by the estimates from one study could be easily contradicted by choosing the estimated values from another study.

To illustrate this point, table 1.1 reports estimates of the equilibrium real depreciation rates for G-7 countries from four studies that assume constant elasticities; section 2.4.2 reports the details of the calculations. That existing elasticity estimates do not offer a precise characterization of the equilibrium rate of depreciation for the real exchange rate is evident. For Japan, for example, the estimates imply that the real yen should be either appreciating at a rate of 19 percent per year or depreciating by 8 percent; for the other countries, the lack of precision is less pronounced, but the message is the same.

One might be tempted to dismiss this imprecision by arguing that the elasticity estimates of the earlier studies of Cline and of Houthakker and Magee are no longer relevant—that they exclude years of important economic developments that could affect their magnitudes in a way relevant for policymaking. But if the usefulness of estimated elasticities hinges on whether they account for recent developments, would it not be better to rely on models that allow these elasticities to change in response to economic developments?

The appeal of relying on formulations that allow elasticities to change is strengthened when one recognizes that, by definition, an elasticity is the ratio between a marginal function and an average function. Specifically, the income elasticity  $\eta_m$  is  $\frac{\partial q_m}{\partial y} / \frac{q_m}{y}$ , where  $q_m$  is the quantity demanded of imports,  $\frac{\partial q_m}{\partial y}$  is the marginal propensity to import, and  $\frac{q_m}{y}$  is the average propensity to

import. The general tendency for the GDP share of imports ( $\frac{q_m}{y}$ ) to increase in the post-war period implies that income elasticities will change unless  $\frac{\partial q_m}{\partial Y}$  changes so as to offset the change in the GDP share of imports, a pattern for which optimization theory offers no justification. Reliance on empirical models with varying elasticities, however, undermines the usefulness of equation (1.1) because predicting the real depreciation rate would involve predicting first how elasticities respond to the changes in income and prices.

Table 1.1: Equilibrium Real Depreciation (Percent, annual rates, local/foreign):

Study	Alternative Studies						
	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Houthakker and Magee (1969)	-0.6	0.0	0.1	2.8	-18.7	-1.2	0.8
Cline (1989)	0.4	nd	nd	-1.3	-6.4	0.1	0.6
Carporale and Chui (1999)	-4.9	1.1	-2.9	-5.5	8.2	-11.2	0.1
Hooper et al. (2000)	0.8	0.6	-1.2	-2.6	-3.7	1.2	2.8

*Note:* nd means not defined because  $\epsilon_x + \epsilon_m = 1$ . *Source:* see section 2.4.3.

One could bypass all the ambiguities and complications of estimation and instead impose suitable assumptions about firm or consumer behavior to obtain values for the parameters that statistical methods cannot pin down. And, indeed, if elasticities are assumed to be both constant and consistent with economic theory, then they are found to be equal to one. But if constant elasticities are known to be equal to one, why is there so much work devoted to estimating them?

The answer is that empirical models with constant (and non-unitary) elasticities explain the data much better than theoretically consistent models do. Unfortunately, we lack generally accepted criteria for trading off key features of a model: ease of implementation, predictive accuracy, and consistency with economic theory. These essays offer one approach – a middle way, so to speak – of undertaking that kind of tradeoff.

## 1.2 Outline of the Essays

The following chapter outlines the methods that I use in these essays. I start by reviewing how the logic of optimizing behavior yields the values of the elasticities if they are assumed constant. I also review optimization models that do not assume constancy of elasticities and document the tension between their theoretical consistency and their predictive power. I then focus on the choice of method for parameter estimation. For this, I rely on the method of Johansen because it addresses two features central to the empirical modeling of international trade: dynamic adjustments and interdependencies among income, prices, and trade (see Banerjee et al. 1993 for details). Finally, I outline an additional criterion for judging the usefulness of a collection of elasticities: estimated elasticities should not imply a violation of the identity between the value of world exports and the value of world imports. This criterion is not new, but it has received little attention in empirical work as reflected in available elasticity estimates that contradict this identity. Fortunately, finding that a given collection of elasticity estimates violates this identity helps in identifying avenues to explore in response. Here I explore two possibilities.

The first possibility, examined in chapter 3, arises from addressing the implications of the puzzling estimate of the income elasticity for U.S. imports; the bulk of this book is devoted to this possibility. Indeed, existing estimates of this elasticity are greater than one implying that, in the absence of relative price increases, the United States will change from a largely self-sufficient economy to one that cannot pay for its imports. This puzzling result, first noted by Houthakker and Magee (1969), has received much attention, and chapter 3 reviews various proposals to resolve this puzzle.<sup>2</sup> I will show that models avoiding the puzzle face a deteriorating explanatory power relative to models that embody the puzzle, and I suggest that a more fruitful approach involves recognizing that imports depend on factors other than income and relative prices. I then show what these models imply for real exchange rates and how they assist in restoring the consistency between elasticity estimates and the world trade identity.

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<sup>2</sup>Houthakker and Magee (1969) was cited nearly 300 times between 1972 and 1998, the largest number of citations from all the articles in the spring issue of five journals: the *American Economic Review* (June), *Econometrica* (April), the *Journal of Political Economy* (May/June), the *Quarterly Journal of Economics* (May), and the *Review of Economics and Statistics* (May).

The second possibility, examined in chapter 4, arises from addressing the neglect of the role of Asian countries in modeling world trade.<sup>3</sup> Indeed, the literature has focused on explaining the trade of G-7 countries and has neglected quantifying the role of income and prices in determining the expansion of Asian trade. This neglect raises the question of whether our models, even if accurate and theoretically consistent, account for enough of international trade to be useful for issues involving global interactions. Thus in chapter 4 I estimate income and price elasticities for exports and imports of Hong Kong, Indonesia, Korea, Malaysia, the Philippines, Singapore, Taiwan, and Thailand with quarterly data through 1997. I then show how these estimates assist in restoring the consistency between elasticity estimates and the world trade identity. Finally, chapter 5 outlines the limitations of this work and proposes lines of research that might be of interest.

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<sup>3</sup>I performed electronic searches on four databases: the Social Science Research Network, EconLit as supported by WebSpirs, JSTOR, and Rubinni's Website. I found Ghose and Karas (1993), Reinhardt (1995), Riedel and Athukorala (1995), Muscatelli, Stevenson, and Montagna (1995), Mody and Yilmaz (1997), and Mah (1999).

# Chapter 2

## Modeling Considerations

When I took his course, Lawrence Klein introduced his lectures in econometrics by indicating that econometricians wear two hats. In formulating behavioral relations, we wear a theorist's hat since we assume the parameters of the behavioral relations to be known. In estimating the parameters, we wear a statistician's hat since we take the behavioral relations as given.

This observation conveys to me the sense that sole reliance on theory or statistics is not enough for drawing inferences about the world we live in. We need to have a way, eclectic as it might be, of combining the two sources of knowledge. In this chapter I present the tools that I have used to create my rendition of that combination.

### 2.1 Theorist's Hat

I now review how the logic of optimization yields the values of trade elasticities if they are assumed constant. I focus on the case of imports and consider them as either intermediate products to production decisions or as final products to consumption decisions. For both cases, I assume that imports and domestic products are imperfect substitutes for each other and that the supplies of both these goods are perfectly elastic. The material for intermediate products comes from Kohli (1991), and the material for consumer goods comes from Deaton and Muellbauer (1980b). For a seminal paper see Armington (1969) and for an alternative approach, see Bergstrand (1985).

## 2.1.1 Optimization and Constant Elasticities

### Intermediate Products

If one assumes that producers find the mix of foreign and domestic inputs that minimizes the cost of attaining a given level of output, then the optimization problem is

$$C(p_m, p_d, q_y) \equiv \min_{q_m, q_d} \{q_m \cdot p_m + q_d \cdot p_d \ni f(q_m, q_d) \geq q_y\},$$

where

$p_m$  is the price of imports,

$p_d$  is the price of the domestic bundle of capital and labor,

$q_y$  is the level of production – total sales or gross output,

$q_m$  is the quantity of imports,

$q_d$  is the quantity of the domestic bundle of capital and labor, and

$f(\cdot)$  is the production function (Kohli, 1991, equation 5.2, p. 64).

The demand for imports that minimizes cost is

$$q_m = \frac{\partial C(p_m, p_d, q_y)}{\partial p_m}.$$

If one assumes that  $f(\cdot)$  exhibits constant returns to scale, then

$$C(p_m, p_d, q_y) = C(p_m, p_d) \cdot q_y,$$

and the demand for imports is homogenous of degree 1 in  $q_y$ :

$$q_m = \frac{\partial [C(p_m, p_d) \cdot q_y]}{\partial p_m} = C_{p_m}(p_m, p_d) \cdot q_y,$$

where  $C_{p_m}(p_m, p_d) = \frac{\partial C(p_m, p_d)}{\partial p_m}$ . Thus if one interprets  $q_y$  as income, then the income elasticity equals 1.

One question is whether this result extends to the case of profit maximization. Thus, following Kohli, I assume as an alternative that the producer chooses the levels of production and imports that maximize profits subject to a given level of aggregate factor endowments ( $q_d$ ):

$$\pi(p_m, p_y, q_d) \equiv \max_{q_y, q_m} \{q_y p_y - q_m p_m \ni f(q_m | q_d) \geq q_y\},$$



where  $\pi(\cdot)$  is the profit function,  $p_y$  is the output price, and the term  $q_y p_y - q_m p_m = p_d q_d$  is GNP in nominal terms. The demand for imports is

$$q_m = \frac{-\partial\pi(p_m, p_y, q_d)}{\partial p_m},$$

and, if the production function exhibits constant returns to scale, then

$$q_m = \left( \frac{-\partial\pi(p_m, p_y)}{\partial p_m} \right) \cdot q_d.$$

Thus the derived import demand function is homogenous of degree one in  $q_d$ . If one interprets  $q_d$  as income, then the income elasticity equals 1. Note that because  $q_d$  and  $q_y$  are different measures of income, the result of a unitary income elasticity could be sensitive to the objective function.

Unlike income elasticities, price elasticities are not known in advance because they depend on the choice of a domestic substitute for imports, a choice dictated by the specification of the optimization problem. Indeed, with  $w_m$  as the GNP share of imports ( $\frac{q_m p_m}{q_y p_y}$ ) and  $\sigma$  as the elasticity of substitution between imports and the composite domestic factor, table 2.1 shows how price elasticities vary in response to the firm's objectives and constraints.

Table 2.1: Measurement and Price Elasticities

Objective	Scale	Specification	Price Elasticity
1. Cost Minimization	$q_y$	$q_m(\frac{p_m}{p_d}, q_y)$	$-\sigma(1 - w_m)$
2. Profit Maximization	$q_d$	$q_m(\frac{p_m}{p_y}, q_d)$	$-\frac{\sigma}{(1-w_m)}$
3. Cost Minimization	$q_d$	$q_m(\frac{p_m}{p_d}, q_d)$	$-\sigma$
4. Profit Maximization	$q_y$	$q_m(\frac{p_m}{p_y}, q_y)$	$-\sigma$

Source: Kohli, 1991, table 5.1.

Thus, without information on both  $\sigma$  and  $w_m$ , economic theory does not provide the values of the price elasticities. However, if one changes the specification of the constraints for a given objective (formulation 3 instead of 1, formulation 4 instead of 2), then the price elasticity is  $\sigma$ . Though  $\sigma$  is unknown, the literature focusing on characterizing the technological possibilities is ample, and a commonly used production function is the Cobb-Douglas specification, for which  $\sigma = 1$ . If one relies on this finding, then both the price and the income elasticities are known.