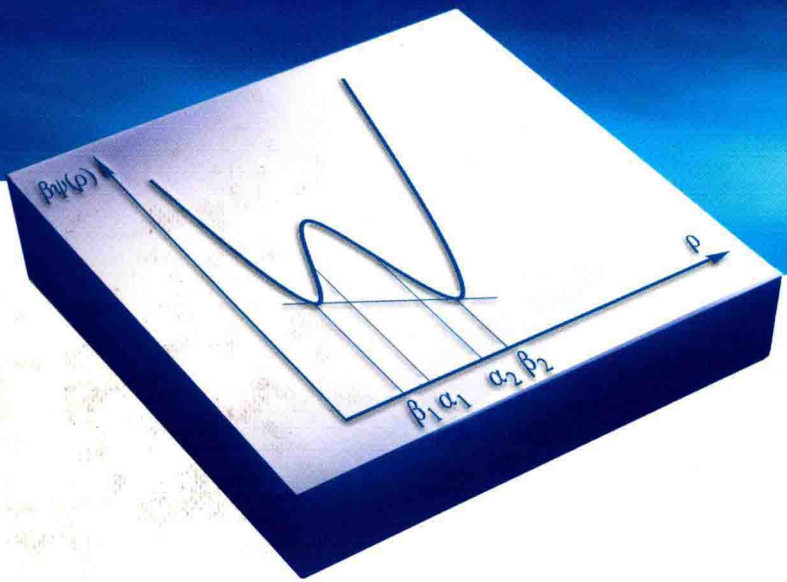
Series in Contemporary Applied Mathematics
CAM 18

# Hyperbolic Problems

## Theory, Numerics and Applications

Volume 2

Tatsien Li
Song Jiang
*editors*



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edited by
**Tatsien Li**
*Fudan University, China*
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# Preface

The 13th International Conference on Hyperbolic Problems: Theory, Numerics and Applications (HYP 2010) was held in Beijing, China, from June 15 to 19, 2010. Over 200 participants attended the conference and 162 among them came from abroad. There were 10 plenary lectures, 18 invited talks and around 140 contributed talks in parallel sessions.

The objective of the conference is to bring together experts, researchers and students with interest in theoretical analysis, numerical simulations, and applications of hyperbolic partial differential equations and related mathematical models appearing in applied sciences. The conference keeps the traditional balance of the HYP series, blending theory, numerics and applications, and an emphasis is put on nonlinear problems and applications in various fields such as fluid mechanics, elasticity, astrophysics, biomathematics, traffic flow, etc. As has been done in the past, a special effort has been made it possible for young scientists to attend this conference and to promote their interaction with the more senior researchers.

The biannual HYP series of international conferences on Hyperbolic Problems was initiated by C. Carasso, P.-A. Raviart and D. Serre with the first conference held in Saint-Étienne, France in 1986. Since then it has been organized in Aachen (Germany, 1988), Uppsala (Sweden, 1990), Taormina (Italy, 1992), Stony Brook (USA, 1994), Hong Kong (China, 1996), Zurich (Swizerland, 1998), Magdeburg (Germany, 2000), Pasadena (USA, 2002), Osaka (Japan, 2004), Lyon (France, 2006), and Maryland (USA, 2008). Throughout these years, it has become one of the highest quality and most successful conference series in applied mathematics.

Hyperbolic problems, which are probably originated from Euler's study on the acoustic wave in 1755, not only have a long history but also have extremely rich physical background. The development is highly stimulated by their applications to Physics, Biology, and Engineering Sciences, in particular by the design of effective numerical algorithms. Due to recent rapid development of computers, more and more scientists use hyperbolic partial differential equations and related evolutionary equations as basic tools when propose new mathematical models of various phenomena and related numerical algorithms. We believe that various fields in science and engineering will bring us further into future interests

in hyperbolic problems.

The scientific committee of this conference consists of Yann Brenier (France), Alberto Bressan (USA), Constantine Dafermos (USA), Xiaqi Ding (China), Ling Hsiao (China), Rolf Jeltsch (Switzerland), Song Jiang (China), Kenneth Karlsen (Norway), Shuichi Kawashima (Japan), Dietmar Kroener (Germany), Randall LeVeque (USA), Tatsien Li (China), Tai-Ping Liu (USA), Helena J. Nussenzveig Lopes (Brazil), Pierangelo Marcati (Italy), Denis Serre (France), Chi-Wang Shu (USA), Eitan Tadmor (USA) and Zhouping Xin (China). We thank all members of the scientific committee for recommending plenary and invited speakers and promoting this conference.

This conference is supported by the Academy of Mathematics and Systems Science, the Institute of Applied Physics and Computational Mathematics, Capital Normal University, Central China Normal University, Fudan University, Shanghai Jiaotong University, and Wuhan University. It is also sponsored by Shanghai Key Laboratory for Contemporary Applied Mathematics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, National Natural Science Foundation of China, and Tianyuan Foundation for Mathematics.

These two volumes contain 80 original research and review papers which are written by leading researchers and promising young scientists, and cover a diverse range of multi-disciplinary topics addressing theoretical, modeling and computational issues arising under the umbrella of "Hyperbolic Partial Differential Equations". All the articles are peer-reviewed. We would like to thank all the referees for their efforts done for reading and judging the submitted articles. Special thanks go to Dr. Tao Wang for his laborious job arranging the final layout of articles.

Finally, we are extremely thankful to our colleagues Professors Feimin Huang, Hailiang Li, Yaguang Wang, Huijiang Zhao and Changjiang Zhu, and Dr. Xiaoyun Zhai, Dr. Yi Wang, and many graduate students from the Academy of Mathematics and Systems Science, the Institute of Applied Physics and Computational Mathematics, Capital Normal University, Central China Normal University, Fudan University, Shanghai Jiaotong University and Wuhan University, for their assistance in making this conference successful and comfortable.

June 2011
Tatsien Li, Song Jiang

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**Plenary Talks**



**Invited Talks**

# Convergence Analysis of the Particle Method for the Camassa-Holm Equation*

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**Abstract**

The purpose of this paper is to establish a new method for proving the convergence of the particle method applied to the Camassa-Holm (CH) equation. The CH equation is a strongly nonlinear, bi-Hamiltonian, completely integrable model in the context of shallow water waves. The equation admits solutions that are nonlinear superpositions of traveling waves that have a discontinuity in the first derivative at their peaks and therefore are called peakons. This behavior admits several diverse scientific applications, but introduce difficult numerical challenges. To accurately capture these solutions, one may apply the particle method to the CH equation. Using the concept of space-time bounded variation, we show that the particle solution converges to a global weak solution of the CH equation for positive Radon measure initial data.

*The work of A. Chertock and T. Pendleton was supported in part by the NSF Grant DMS-0712898; the work of J.-G. Liu was supported in part by the NSF Grant DMS 10-11738.