PROCEEDINGS OF THE JOHNS HOPKINS WORKSHOP
ON
CURRENT PROBLEMS IN PARTICLE THEORY 16

GÖTEBORG, 1992
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PATHWAYS TO FUNDAMENTAL THEORIES

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PREVIOUS JOHNS HOPKINS WORKSHOP PROCEEDINGS

1	BALTIMORE	1974	....
2	BALTIMORE	1978	....
3	FLORENCE	1979	....
4	BONN	1980	Lattice Gauge Theories. Integrable Systems.
5	BALTIMORE	1981	Unified Field Theories and Beyond.
6	FLORENCE	1982	Lattice Gauge Theories. Supersymmetry and Grand Unification.
7	BONN	1983	Lattice Gauge Theories. Supersymmetry and Grand Unification.
8	BALTIMORE	1984	Particles and Gravity.
9	FLORENCE	1985	New Trends in Particle Theory.
10	BONN	1986	Infinite Lie Algebras and Conformal Invariance in Condensed Matter and Particle Physics.
11	LANZHOU	1987	Frontiers in Particle Theory.
12	BALTIMORE	1988	TeV Physics.
13	FLORENCE	1989	Knots, Topology and Quantum Field Theory.
14	HUNGARY	1990	Nonperturbative Methods in Low Dimensional Quantum Field Theories.
15	BALTIMORE	1991	Particle Physics from Underground to Heaven.

THE JOHNS HOPKINS WORKSHOPS ON CURRENT PROBLEMS IN PARTICLE THEORY ARE ORGANIZED BY THE FOLLOWING INSTITUTIONS:

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FOREWORD

The sixteenth Johns Hopkins Workshop on Current Problems in Particle Theory was held in Göteborg in Sweden. This was the first time the group in Göteborg organized this workshop. However, as in the past the purpose was to provide a forum for theoretical physicists from around the world to discuss problems at the forefront of particle physics. Also as in previous meetings the discussions were concentrated around invited talks with ample time to continue the discussions during lunches and evenings.

The title of the workshop was "Pathways to Fundamental Theories". One trend in particle physics during the last twenty years has been the strive to find a unified theory for all interactions. It led us to investigate extensions of the standard model of strong, weak and electromagnetic interactions into Grand Unified Models and extensions of gravity into supergravity theories. These two lines were merged into superstring theory which in the mid 80's was shown to avoid many of the problems of the other two approaches. A very intense effort from the theory community has been put into the development of string theory since then. It has confirmed that superstring theory is a viable candidate for a unified theory, but it has also shown that it will be quite hard to prove that it is the unique and correct approach. String effects are expected to play an important and decisive role at Planck energies, but at energies accessible today, the string models look like typical GUT's. We have also learned that string theory contains a wealth of information and that we are quite far from a detailed understanding of the quantum physics of it. On the other hand since the string sweeps out a two-dimensional surface, the world-sheet, when it propagates, string theory can be viewed as two-dimensional physics. This understanding has led to a lot of insight into low-dimensional physics. It has focused on the possibility that a limited number of models can describe this type of physics. These low-dimensional models that can underly both string theory and low-dimensional physics were the main theme of the workshop.

Specifically the workshop discussed different conformal theories, string theories and 2-d gravity. One theme was black holes which recently have been shown to appear naturally in certain conformal field theories. Another was new results in string theory such as strings on curved non-compact manifolds, mirror symmetry, point-like structure and strings based on W-symmetries. There was also one talk on testable predictions from unified models.

The workshop took place in a hotel in downtown Göteborg. Since all participants stayed in the hotel and had their meals there it gave a lot of room for discussions. The weather was so perfect it can only be in a northern country at midsummer time. One evening the whole group made an excursion to a cottage by a lake close to the city. We feel this was a perfect break in the rather hectic programme. One British participant spent most of the evening swimming around the lake. Many participants used the sauna and we think that it gave memories for life for some participants from southern countries.

The Workshop was financed mainly by the Nobel Committee for Physics. They paid the expenses for all lecturers. The expenses for the Nordic participants were covered by Nordita. The local expenses were covered by grants from Wilhelm och Martina Lundgrens Vetenskapsfond and The Royal Society of Arts and Sciences in Gothenburg. The Johns Hopkins university has continued their generous support and has paid for the advertizing and all expenses in connection with these proceedings. We are very grateful to all these contributors who made the workshop possible.

Last but not least we must thank Annika Hofling who single-handedly took care of almost all administration. She was the secretary, negotiator, travel agent and inspirator. She made life really easy for the organizers. We would also like to thank the members of the group in Göteborg for their assistance in organizing the excursion and in our editing of this volume.

Finally we wish to thank Prof. K.K. Phua and Ms. H. M. Ho of the World Scientific Publishing Company for their skilful and enthusiastic handling of this volume.

The Organizers

Feza Gürsey, J.W. Gibbs Professor Emeritus at Yale University died in April, 1992. The physics community lost one of the most influential theoretical physicists of our time. Many participants at this workshop also lost a teacher and a close friend. We will painfully miss him. We have been most fortunate to have had Feza contribute frequently to these volumes. His ideas influenced many a talk presented here and at past workshops.

These Proceedings are dedicated to Feza Gürsey's memory.

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In addition to the above a talk was presented orally at the Conference: N. Seiberg "2d – gravity"

AFFINE TODA SOLITONS

Abstract of talk given by

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For more than ten years there has been interest in the affine Toda field theories. Associated with each affine untwisted Kac Moody algebra $\hat{\mathbf{g}}$, there exists a set of $r =$ rank $\mathbf{g}$ coupled, relativistic equations in r scalar fields which are integrable in two space-time dimensions in the sense of possessing an infinite number of local conservation laws. Their form can be deduced from zero-curvature conditions satisfied by Lie algebra-valued gauge potentials dependent on the fields. More recent developments have provided further motivations for their study. The first is that they furnish a beautiful and instructive illustration of the idea of Zamolodchikov that integrable theories can be realised as deformations of conformally invariant theories. The action for affine Toda theory differs by the inclusion of a single term from the action of conformal Toda theory, which possesses conformal symmetry augmented by W-symmetry. Just as affine Toda theory is thereby a deformation of conformal Toda theory, so its conservation laws constitute non-chiral relics of the chiral conservation laws expressing conformal and W-symmetry. For $\mathbf{g}$ equals $\mathbf{SU(2)}$ the conformal and affine Toda equations reduce respectively to the Liouville and Sinh-Gordon theories.

Since the affine Toda theories possess a single mass scale parameter, that multiplying the additional term, the r masses associated with the r fields should possess ratios independent of this parameter and could display interesting patterns. This was confirmed by Corrigan et al and others who found that the masses were given by the components of the right Perron-Frobenius eigenvector of the Cartan matrix of $\mathbf{g}$. This is the eigenvector to the lowest eigenvalue and hence the only eigenvector whose components can all be taken to be positive. Even more surprising is that this result is respected by quantum corrections, at least when $\mathbf{g}$ is simply laced. The results are intriguing as this eigenvector has occurred in other contexts recently, for example, integrable lattice models and the theory of type II_1 subfactors of Von Neumann algebras. As a consequence of the result, there is a danger that a heavy affine Toda particle could be unstable with respect to a decay into two light ones. But the equations themselves govern the couplings for this decay and have the remarkable property that all such decays are strictly forbidden. This selection rule is encapsulated in terms the "fusing rule" found by Dorey and formulated in terms of properties of the Lie algebra $\mathbf{g}$. This structure plays a role in the construction of S-matrices for the quantum mechanical scattering of the particles. These obey all known consistency criteria, such as unitarity, analyticity, crossing and the bootstrap properties.

Putting the coupling constant of Sinh-Gordon theory pure imaginary produces Sine-Gordon theory which is still a real theory, but now with degenerate vacua. Solutions interpolating adjacent vacua are topologically stable and are known as "solitons". These behave like classical models of particles with localised energy density and enjoy a host of interesting properties, much studied over the years. When the coupling constant of any other affine Toda theory is made imaginary, again emerge vacua degenerate with the vanishing solution, but unfortunately the equations are now irredeemably complex. Undeterred, Hollowood constructed soliton solutions interpolating adjacent vacua of affine $\mathbf{su}(\mathbf{N})$ Toda theory and calculated their energy and momentum to be real, despite their densities being complex.

This remarkable result triggered much activity aimed at extending the class of solutions and pinpointing the explanation of reality. I shall describe two related pieces of progress that my collaborators and I have made.

The result that the complexity of the densities is irrelevant suggests that they may actually be total space derivatives. This is known to be so for Sine-Gordon and many other, more complicated solitons, including ones in three space dimensions. The solutions are then governed by first order "Bogomolny" equations rather than the second order equations. The presence of a Bäcklund transformation, provides a candidate, first order equation and has long been known for the affine $\mathbf{su}(\mathbf{N})$ Toda theory at least. Accordingly, Liao, Olive and Turok solved these equations and showed that the solutions possessed a real mass. Explicit construction of the solutions, by elementary methods, yielded Hollowood's results, obtained by Hirota's method, and were extended to find multi-soliton solutions. These results left open the question of other choices of $\mathbf{g}$.

For this, Olive, Turok and Underwood reconsidered earlier work by Olive and Turok, based in turn, on even earlier work by Leznov and Saveliev. Within the most general solution to the affine Toda equations, specialisations corresponding to soliton solutions were presented. These involved ratios of matrix elements of a Kac-Moody "group element" parametrised in an unorthodox way, as a product of p exponential factors, one for each soliton. Each factor contained two parameters, one for the coordinate and one for the momentum of the soliton. Thus one could think of the group factors as creating individual solitons. This picture was very attractive and reproduced previously known solutions in a more unified way once it was understood that each exponential series terminated in a well-defined way. The attractiveness was enhanced by strong formal similarities to the structure underlying Dorey's fusing rule, mentioned above and the particle mass formulae. Despite the generality of this solution it was possible to insert it into the energy momentum tensor and show that that reduced to a total space divergence (or more precisely, took the form of an "improvement"). Thus the integrated energy and momentum was a surface term which could be evaluated precisely. It took the form of a sum of contributions of the individual solitons. Each energy was positive and definite mass formulae emerged. There were again r species of soliton, just as for the particles. The mass was again proportional to the components of the Perron Frobenius vector, but this time the left eigenvector.

Key elements in this work concern the concept of conformal affine Toda theory and the consideration of $\hat{\mathbf{g}}$ in a new basis consisting of an infinite dimensional Heisenberg subalgebra, known as the principal Heisenberg subalgebra, together with r "fields" that diagonalise their ad-action and play the role of creating each of the r species of soliton.

Besides opening intriguing new perspectives on soliton theory and relating it more to concepts of particle physics, these results raise two interesting issues. The structure of the soliton mass spectrum together with the fact that they too satisfy Dorey's fusing rule in a classical analogue, suggest that the soliton field theory may also be an affine Toda theory, this time based on the dual Lie algebra to $\mathbf{g}$, namely that obtained by interchanging roots and coroots, and with the coupling constant replaced by its inverse. Secondly, the fact that the soliton spectrum is real suggests that we have a mechanism whereby a unitary theory is consistently embedded in a non-unitary theory. In the quantum theory we might expect a no-ghost theorem analogous to that of string theory with an associated BRST complex. Maybe this relates to ideas already proposed in this direction.

The work I have described here is now available in either published or preprint form:

HC Liao, DI Olive and N Turok: "Topological Solitons in $\mathbf{A_r}$ Affine Toda Theory", Physics Letters **B298** (1993), 95-102

DI Olive, N Turok and JWR Underwood: "Solitons and the Energy-Momentum Tensor for Affine Toda Theory", preprint Imperial/TP/91-92/35 and Swansea preprint SWAT/3.

Detailed references to all the other work I have cited in the above abstract can be found in the above two papers. I would like to mention that subsequent to the production of these two papers we learnt of an earlier preprint by Niedermaier with related material: MR Niedermaier: "The Spectrum of the Conserved Charges in Affine Toda Theories", University of Hamburg preprint.

Seventy Relatives of the Monster Module

A.N. Schellekens

CERN

The simplest conformal field theories, from the point of view of the modular group or the fusion rules, are those with just one primary field with respect to some integer spin chiral algebra. It is elementary to show that unitary conformal field theories of this kind must have a central charge that is a multiple of 8. They transform according to a one-dimensional representation of the modular group with $S = 1$ and T a cubic root of unity. Furthermore, if the central charge is a multiple of 24 the single character is modular invariant by itself, and can be written as a polynomial in the absolute modular invariant j,

$$j = \frac{1}{q} + 744 + 196884q + 21493760q^2 + \dots ,$$

with a leading term q^{-n} if the central charge $c = 24n$; here $q = e^{2\pi i\tau}$. Since the character $\mathcal{X}$ is modular invariant by itself one may consider, instead of the usual "diagonal" CFT with partition function $\mathcal{X}\mathcal{X}^*$, a purely chiral conformal field theory with partition function $\mathcal{X}$. Such a theory will be called a *meromorphic* conformal field theory, and denoted MCFT.

The classification of these theories is an essential part of the programme of classification of rational conformal field theories, initiated a few years ago. Indeed, one can argue that the entire RCFT classification problem can be embedded in that of classification of MCFT's, provided that one can show that any RCFT has a *complement*. This is a RCFT with the same number of primary fields and complex conjugate S and T matrices. (A complement can easily be constructed for all WZW-models and for all coset theories without field identification fixed points). Then any diagonal RCFT with a modular invariant $\sum_i \mathcal{X}_i(\mathcal{X}_i)^*$ can be mapped to a meromorphic one with partition function $\sum_i \mathcal{X}_i\mathcal{X}_i^C$, where '$C$' denotes the complement.

In any case it is clear that the RCFT classification problem is not solved as long as we cannot even classify the theories with just one primary field. This is bad news, since for $c \geq 32$ the number of such theories grows so fast with the central charge that listing them is simply impossible. Indeed, for $c = 32$ the number of such theories is known to be larger that 8×10^7. The problem looks substantially easier for $c \leq 24$, and with some (unfounded) optimism one may hope that the information contained in the $c \geq 32$ theories will never really be needed in practice.

The fact that enumeration is impossible for $c \geq 32$ may dampen one's enthusiasm for attempting a enumeration for $c \leq 24$. Nevertheless, there are indications that the $c \leq 24$ theories (and in particular those with $c = 24$) are of some intrinsic interest. In physics, $c = 24$ is special because of the bosonic string, whose transverse dimension is 24; in mathematics the number 24 plays a special role in many contexts, such as the theory of sphere packings or the Monster group (the largest of the sporadic simple finite groups), for which a meromorphic $c = 24$ theory provides a "natural" q-graded representation, the "monster module" [1]. These rather vague motivations will probably turn out to be the most important ones for attempting to classify the meromorphic $c = 24$ CFT's. A somewhat more practical motivation is that a listing of such theories will enable us to complete another classification problem, that of ten-dimensional heterotic strings. Yet another unsolved problem about which we will learn a few interesting new facts (without solving it, though) is that of the classification of Kac-Moody modular invariants.

A large class of MCFT's can be constructed by taking $8n$ free bosons with momenta quantized on an even self-dual lattice. This gives $1, 2$ and 24 [2,3] distinct theories for $c = 8, 16$ and 24 respectively (and more than 8×10^7 for $c = 32$). This class can be enlarged by a $\mathbf{Z}_2$ orbifold twist, using the symmetry that sends every boson X to $-X$ [4,5]. This gives back the same $E_{8,1}$ theory for $c = 8$, and maps the two $c = 16$ MCFT's $(E_{8,1})^2$ and $D_{16,1}([0] + [s])$ to one another (the argument denotes the conjugacy classes that appear). The result is more interesting when this twist is applied to the Leech lattice and the 23 Niemeier lattices: The former gives a new MCFT, the monster module, while from the latter one gets other Niemeier lattices in 9 cases, and new MCFT's in the 14 remaining cases [5]. Altogether this gives us thus $1, 2$ and 39 MCFT's for $c = 8, 16$ and 24.

Clearly there are other orbifold twists one might consider, but it becomes rather difficult to prove the consistency of the resulting theories. More importantly, even an exhaustive classification of all orbifolds of known theories is not sufficient to show that the result is complete. The same is true for other kinds of constructions. For example, one could study all tensor products of Kac-Moody algebras with total central charge $8n$, and determine their meromorphic modular invariants. Even though this is a finite problem, there is no guarantee that the answer will be complete, since in general only part of the central charge will be saturated by (non-abelian) Kac-Moody algebras. As soon as one allows rational $U(1)$ factor the problem is not finite anymore, and it gets still worse if one adds factors without spin-1 currents (e.g. coset theories). In any case, it was already known for some time that the number of MCFT's with $c = 24$ is larger than the 39 mentioned so far: two additional candidates were presented in [6], one of which can certainly be constructed.

While explicit constructions approach the set of solutions from below it is possible in some cases to limit the set of solutions from above, i.e. to derive necessary rather than sufficient conditions for the existence of solutions. An example is the set of $c = 8$ and $c = 16$ solutions. Any such theory can be used to build a supersymmetric heterotic string theory in 10 dimensions. It can be shown in general ([7], see also [8]) that modular invariance of such a theory implies that all gauge and gravitational anomalies of the resulting field theory must factorize à la Green-Schwarz [9]. But all possibilities for such anomaly cancellations are known [9] [10], and this immediately reduces the $c = 8$ and $c = 16$ theories to $(E_{8,1})^2$ and $D_{16,1}$. There cannot exist more such theories, and since both can be constructed using self-dual lattices, there are no fewer either.

It turns out that a similar argument can be applied, with a considerably larger effort, to the $c = 24$ theories [11]. Beyond $c = 24$ the nature of the problem changes drastically, and these methods become useless, not just in practice but even in principle. The basic idea is to write down a character valued partition function for a given $c = 24$ theory analogous to similar functions introduced in [7] for the chiral sector of heterotic strings. This function generalizes the ordinary one-loop partition function

$$P(q) = \sum_{n=1}^{\infty} d_n q^n \ ,$$

by replacing the multiplicities d_n by Chern-characters of the representation at each level. Thus we get

$$P(q, F) = \sum_{n=1}^{\infty} \mathrm{Tr}\, e^F q^n \ .$$

Here F is some representation matrix of a semi-simple Lie-algebra, in the representation