HANDBOOK OF APPLICABLE MATHEMATICS

Chief Editor: Walter Ledermann

GUIDEBOOK 2

MATHEMATICAL METHODS IN MANAGEMENT

Geoffrey Gregory
Department of Management Studies
Loughborough University of Technology

This is one of the guidebooks to the core volumes of the Handbook of Applicable Mathematics. The multi-volume handbook covers in a single coordinated work the majority of applicable mathematics and is written for all users of mathematics, among whom managers in business and industry and students on management courses constitute an important group.

This guidebook is designed for the practising manager to give him an insight into the use that can be made of mathematical methods. It does this by treating applications on the various management disciplines such as production, accounting and personnel, and emphasizing the applications. Where mathematical results are required these are provided by cross references to the core volumes, so that the flow of the argument is not interrupted by excursions into mathematical analysis. Such use of the core volumes will also enable the reader to develop the application beyond the illustration described in the text.

This guidebook will also provide interesting information for students on management courses.

Contents

Editorial Note

Preface and Acknowledgements

Chapter 1 Production and Inventory Control—Basic Models

Chapter 2 Production and Inventory Control—Further Models

Chapter 3 Marketing

Chapter 4 Distribution Management

Chapter 5 Human Resource Management

Chapter 6 Accounting and Finance

Index

JOHN WILEY & SONS
Chichester · New York · Brisbane · Toronto · Singapore
A Wiley–Interscience Publication

ISBN 0 471 90131 8

HANDBOOK OF APPLICABLE MATHEMATICS
Chief Editor: Walter Ledermann

MATHEMATICAL METHODS
IN
MANAGEMENT

GEOFFREY GREGORY
Department of Management Studies
Loughborough University of Technology

A Wiley–Interscience Publication

JOHN WILEY & SONS
Chichester · New York · Brisbane · Toronto · Singapore

Library of Congress Cataloging in Publication Data:
Gregory, Geoffrey
Mathematical methods in management.
'A Wiley-Interscience publication'
Guidebook no. 2 to Handbook of applicable mathematics.
Includes index.
1. Management science—Mathematical models.
I. Title. II. Title: Handbook of applicable mathematics.
T56.G74 1983 658.4'033 82-23836

ISBN 0 471 90131 8

British Library Cataloguing in Publication Data:
Gregory, Geoffrey
Mathematical methods in management—(Handbook of applicable mathematics)
1. Management—Mathematics
I. Title II. Series
510'.24658 HD30.83

ISBN 0 471 90131 8

Filmset by Eta Services (Typesetters) Ltd, Beccles, Suffolk
Printed by The Pitman Press Ltd, Bath, Avon

HANDBOOK OF APPLICABLE MATHEMATICS

MATHEMATICAL METHODS
IN
MANAGEMENT

HANDBOOK OF APPLICABLE MATHEMATICS

CORE VOLUMES

Volume I: ALGEBRA
Edited by Walter Ledermann, *University of Sussex* and Steven Vajda, *University of Sussex*
Volume II: PROBABILITY
Emlyn Lloyd, *University of Lancaster*
Volume III: NUMERICAL METHODS
Edited by Robert F. Churchhouse, *University College Cardiff*
Volume IV: ANALYSIS
Edited by Walter Ledermann, *University of Sussex* and Steven Vajda, *University of Sussex*
Volume V: GEOMETRY AND COMBINATORICS
Edited by Walter Ledermann, *University of Sussex* and Steven Vajda, *University of Sussex*
Volume VI: STATISTICS
Edited by Emlyn Lloyd, *University of Lancaster*

GUIDEBOOKS

Guidebook 1: MATHEMATICAL METHODS IN SOCIAL SCIENCE
David J. Bartholomew, *London School of Economics and Political Science*
Guidebook 2: MATHEMATICAL METHODS IN MANAGEMENT
Geoffrey Gregory, *Loughborough University of Technology*
Guidebook 3: MATHEMATICAL METHODS IN MEDICINE
Part 1 Statistical and Analytical Technique
Edited by David Ingram, *St Bartholomew's Hospital*
Ralph Bloch, *McMaster University, Ontario*
Guidebook 4: MATHEMATICAL METHODS IN MEDICINE
Part II Applications in Clinical Specialities
Edited by Ingram & Bloch
Guidebook 5: MATHEMATICAL METHODS IN ENGINEERING
Edited by Glyn A. O. Davies, *Imperial College*
Guidebook 6: MATHEMATICAL METHODS IN ECONOMICS
Edited by Frederick van der Ploeg, *London School of Economics and Political Science*

To Brenda, Janet, Sarah and Kate

Editorial Note

The *Handbook of Applicable Mathematics* is chiefly addressed to persons who have had little or no academic training in mathematics but who, perhaps at a late stage of their careers, find it necessary to master mathematical concepts and skills that are relevant to their professional work.

We have therefore presented in this Handbook all those topics of mathematics which, we believe, are at least potentially useful in other disciplines. The mathematical material is arranged in six *core volumes* bearing the titles

I Algebra
II Probability
III Numerical Methods
IV Analysis
V Geometry and Combinatorics
VI Statistics

However, we feel that a mere collection of mathematical articles is not sufficient for our purpose. Very often the same branch of mathematics (matrix algebra, statistics, differential equations) is required in quite different contexts arising in disparate professions. In order to assist the members of those professions to identify the appropriate mathematical tools we are publishing a number of *guidebooks* of which the present volume is one. Each guidebook discusses why and how mathematics is needed in a particular profession or group of professions. It does not, as a rule, contain detailed expositions of mathematical techniques or results. These will be found in the core volumes, to which the guidebooks will make frequent reference. The two components of the Handbook, that is core volumes and guidebooks, are designed together as complementary aspects of a work which reflects the universal validity and importance of mathematics in our society.

To achieve our goal it is essential to have an efficient reference system at our disposal. This system is explained fully in the 'Introduction' to each core volume; we repeat here the following points. The core volumes are denoted by the Roman numerals mentioned above. Each mathematical item belongs to one of six categories, namely

(i) Definitions
(ii) Theorems, propositions, lemmas and corollaries
(iii) Equations and other displayed formulae
(iv) Examples

(v) Figures
(vi) Tables

A typical item is designated by a Roman numeral followed by three arabic numerals *a.b.c*, where *a* refers to the chapter, *b* to the section and *c* to the individual item enumerated consecutively in each category. For example, 'Theorem IV 6.2.3' is the third theorem (or proposition or lemma or corollary) of section 2 in Chapter 6 of the core volume IV. We might also have 'Equation IV 6.2.3', the same numbers being possible for items in different categories.

The style and scope of the guidebooks will inevitably differ in accordance with the level of mathematical expertise required in the profession it is intended to serve. But we trust that each guidebook will enhance the usefulness of the set of core volumes which provide the reader with the mathematical information he seeks.

Preface and Acknowledgements

Management is concerned with people. Its task is to lead and to motivate; to persuade people to act in the best interests of their organization. Buildings, equipment, profits, energy sources, and training all have to be used effectively, but it is people with their individual attitudes, their differing aptitudes, their social needs and their career aspirations who make a commercial organization really viable. It is up to management to recognize the heterogeneity in the people they are managing and to take this into account when they take decisions.

Mathematical modelling, on the other hand, attempts to represent a problem situation in terms of mathematical relationships, the purpose being either to 'solve the problem' or, more realistically, to focus attention on the relative importance of the various components of the problem. It must necessarily treat its representation objectively so that it can use the sharpness of mathematical logic in its analysis.

It would appear that management and mathematical modelling make strange bedfellows. Left to their own devices this might be true, but one of the purposes of this book is to demonstrate what mathematics has to offer to management through the process of representing managerial problems by mathematical models. Some of the uncertainty present in managerial deliberations can be overcome by the use of mathematical statistics and this is a common theme. Sensitivity analysis can also be used, where the affects on the decision outcomes of departures from the assumptions made in the model can be seen.

Managers have to take decisions, and any means whereby the implications of their decisions can be clarified must be welcomed. Managers will never be replaced by computers or by robots, but with advances in the technologies of communication, of data retrieval and storage as well as of data processing, they will be expected to make better informed decisions.

This guidebook is intended for the person who wishes to discover what mathematics has to offer in management. It does not claim to satisfy that ambitious intention, but it is hoped that it will whet the appetite. There is a certain pleasure in putting into genuine commercial practice a mathematical argument which originated perhaps in a totally abstract form in the mind of some distant academic. The author has experienced this on a number of occasions, the first being when as a recent graduate he witnessed an interpretation of a queueing theory result by a shop steward to his loom overlookers in the deafening clatter of a weaving shed. The problem with the guidebook has been in deciding what to include. Chapters have been devoted to the broad areas of production, marketing, distribution, human resources, accounting and finance. The choice of particular applications within these areas has been influenced by their importance (for example aggregate production planning) or by the interest in the mathematics used. As far as the latter is concerned care has to be taken not to 'let the tail wag the dog'. In recent years mathematicians and operational research workers have been, with some justification, accused of looking for applications for their sophisticated mathematical techniques. There is nothing wrong with this as long as the application necessitates the mathematics and also as long as the mathematician or worker can justify spending his time in this way.

The guidebook is not written exclusively for the mathematician, although familiarity with the basic ideas of algebra and calculus will enable the reader to follow the arguments without too much effort. Beyond this it is very much up to the reader. The flow of the arguments is uninterrupted by mathematical derivations since, for these, reference can be made to the core volumes. Undoubtedly, further insight will be obtained from careful reading of the indicated core volume material, and this is particularly recommended as few practical applications can be lifted without amendment.

Acknowledgements are due to many people, from Mr H. G. Hilton and Mr J. Bissell, whose strong views had a profound effect on me during my time as an industrial statistician, to my present colleagues who helped enormously with suggestions for the material in this book. Amongst the latter group, particular mention must be made of Dr K. J. Blois, Dr D. W. Cowell, Professor C. K. Elliott, Dr M. R. Hill, Dr R. H. Mole, Mr M. J. Robbie, and Professor J. Sizer. Finally, no book can be written without efficient secretarial support and here I have been particularly fortunate. The period of gestation of this book has spanned three secretaries which is, I am sure, a measure of my dilatoriness rather than the result of the imposition of this work. Linda Harland and Hilary Scoles typed the first four chapters. Yvonne Marshall completed the work and cheerfully undertook the task of preparing the final manuscript. I am deeply indebted to all of them.

Loughborough G. Gregory

Contents

Editorial Note ix
Preface and Acknowledgements. xi

1 Production and Inventory Control—Basic Models 1
1.1 Introduction 1
1.2 Aggregate Production Planning 2
1.3 The Basic Deterministic Model of Aggregate Planning . . . 2
1.4 The Basic Stochastic Model of Aggregate Planning 9
1.5 Aggregate Planning with Normally Distributed Production Levels and Demands 10
1.6 The Economic Order Quantity (EOQ) 14
1.7 The Fully Deterministic EOQ Model 16
1.8 The Fixed Re-order Quantity (Two-Bin) System 17
1.9 The Fixed Re-order Cycle System 25
1.10 Variants on the Basic EOQ Models 31
References 32

2 Production and Inventory Control—Further Models 33
2.1 Introduction 33
2.2 Quality Control and Acceptance Sampling 33
2.3 Assembly Line Balancing 44
2.4 Job Shop Scheduling, and Sequencing 48
2.5 In-Process Stocks 51
2.6 Multi-Product Inventory Systems 53
2.7 Other Production Planning Models 56
2.8 Maintenance and Replacement 58
2.9 Forecasting 63
2.10 Evolutionary Operation 64
References 67

3 Marketing 70
3.1 Introduction 70
3.2 System Models 71
3.3 Advertising Models 73
3.4 New Product Models 80

3.5 Models of Consumer Behaviour 89
3.6 Sales Force Models 93
3.7 Pricing Models 96
3.8 Distribution Models 97
Bibliography 97
References 98

4 Distribution Management 100
4.1 Introduction 100
4.2 Depot Location 101
4.3 Vehicle Routing 107
References 111

5 Human Resource Management 113
5.1 Introduction 113
5.2 Sample Surveys 114
5.3 Manpower Planning 115
5.4 Job Assignment 120
References 122

6 Accounting and Finance 124
6.1 Introduction 124
6.2 Portfolio Management 126
6.3 Project Appraisal 132
6.4 Cash Flow Management 138
6.5 Risk Analysis 139
6.6 Credit Scoring Models 142
6.7 Pricing Models and the Product Mix Problem 145
References. 149

Index 151

1

Production and Inventory Control—Basic Models

1.1 Introduction

Production is the means by which goods and services are converted into further goods and services. Thus, for example, to produce a roll of carpet we may need yarn, latex, jute and dyestuffs as well as electricity, gas and water. These are the raw materials of the process which are themselves the final product of some other production process. The carpet in its turn may be a raw material in the production of an office block, forming a link in a never-ending chain of processes. Such processes occur, for example, in factories, shops, banks, government offices and schools, in the sense that all have an input on which they carry out some operation before discharging it to further processes. Our concern in this section is with the efficient operation of such processes, where efficiency can be expressed in a variety of ways, not all of which need be in an explicit mathematical form.

The intention certainly is to look at mathematical models of production processes. Moreover, in this account there will almost inevitably be a proposal to optimize some mathematical expression in the operating characteristics of the process. The expression may represent cost, profit, output, time taken, or raw materials used, or a properly weighted combination. Incidentally, it will rarely be possible to optimize simultaneously more than one such criterion with the same operating conditions. We either have to optimize one, subject to others being above some satisfactory level, or we have to compromise by taking some appropriately weighted expression in the relevant criteria. For a fuller treatment of this problem, see Rivett (1977).

The basic production process is shown in Figure 1.1.

In this section we shall analyse a number of common problems in production management. Not all of the problems will be appropriate to any process, and it

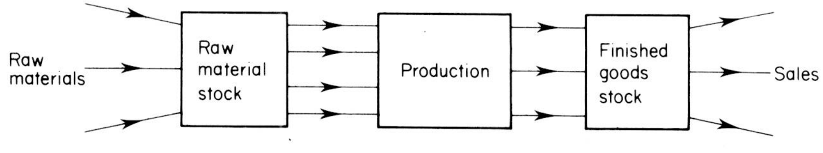

Figure 1.1.

is up to the reader to recognize when a model approximates sufficiently to his situation.

There are many important aspects of production and inventory control which will not be covered. Production design, plant layout, job design and work incentives are just a few of the topics with which the production manager will have to grapple. These and other less mathematical topics have been omitted. There are, however, a number of excellent books covering this material and, indeed, covering in much greater detail the methods of this chapter (Starr, 1972; Zimmerman and Sovereign, 1974; Buffa, 1973; Wagner, 1962).

1.2 Aggregate Production Planning

Consider first the problem of deciding the production level to be maintained for a single product process. It is a rare and fortunate organization that has a constant and known demand in the foreseeable future for its product, and can, therefore, simply match this demand with its own production level. It is even more rare if the organization can maintain its output consistently at the designed level. The more usual situation is to have a demand which is a combination of seasonal variations, short- and long-term trends and local random perturbations. This has to be met by a production process which, although set to a nominal level of output, is bedevilled by machine breakdowns, material variability and absenteeism. Some options are open to management beyond the basic one of deciding on the mean production level. Overtime can be introduced, work can be subcontracted or refused if there is an increased demand. Conversely, it may be possible to bring forward a maintenance shutdown, or to transfer operatives and machines to other work if the demand falls to a level where stocks are being built up to an uneconomic level.

In §§1.3–1.7 we shall examine models of production planning of the single product process. Strictly speaking, the results are only appropriate for such a process, but it is often useful to formulate plans for total output despite the fact that the actual product range may consist of a variety of package sizes, colours or qualities. Thus a carpet manufacturer may wish to plan for a known number of square metres each month, or a manufacturer of electric lamps for a quantity of lamps. The implicit assumption is that the mix between the different colours, qualities and widths of carpet and between the different lamp wattages will be approximately the same from month to month and that, therefore, the marginal demand on resources will be constant. In such cases aggregate planning, as it is known, is a useful tool, giving a broad plan from which detailed plans for production of individual product mixes can be ascertained. Models covering decisions for more detailed planning over a shorter horizon will be discussed in Chapter 2.

1.3 The Basic Deterministic Model of Aggregate Planning

Consider first a simple model where the following assumptions are made:

(a) The demand (units per month) is variable but known.
(b) A constant output (units per month) can be maintained.

Assumption (a) means in practice that we have accurate monthly forecasts, perhaps in the form of contracts with customers. The solution to the problem is mathematically trivial. Output is set at the mean demand level. The only problem that can arise is where the higher demands occur in the earlier part of the time over which production is planned, so that there could be shortages until production catches up. If customers are willing to accept late deliveries (a practice known as back-ordering) then the difficulty is removed. Some care will have to be taken with opening and closing stock levels. We would not wish to leave closing stocks at embarrassingly low or high levels, and we may therefore argue that in subsequent planning periods the demand patterns will be similar to the current one specified and that we should aim for a closing stock level optimal for such a demand pattern. If the assumption of back-ordering is taken literally, then obviously it would pay the organization to supply permanently in arrears, thereby never incurring any stockholding costs. In most cases, however, some compromise will be made, such as aiming to reduce stocks to zero at least once in the planning period.

On the other hand, suppose that orders not met in the month that they are placed are lost. If we are considering a twelve-month horizon then the following will be known:

I_0: the initial inventory
d_j: the demand for month j
x: the planned daily output for the year
m_j: the number of working days in month j
s_j: the inventory at the end of month j.

Our aim is to determine the daily output x for the forthcoming year. Clearly there are advantages in having a stable output level for as long as possible. If the forecast demands prove to be incorrect, then some emergency action on x will have to be taken, but otherwise it should be possible to hold x steady until the following year's level is planned.

Note that, because of public holidays, annual shutdowns and the different calendar month lengths, we have to correct the monthly output for the corresponding number of working days. Also we assume that orders for a particular month can be supplied at any time during the month and, if necessary, on the last day.

It follows that

$$s_1 = \max(0, I_0 + xm_1 - d_1) \tag{1.1}$$

and in general

$$s_j = \max(0, s_{j-1} + xm_j - d_j). \tag{1.2}$$

If our objective is to meet all the demands with a minimum stock carried of finished goods, then we would require

$$\min s_j = 0. \tag{1.3}$$

In other words, stock must fall to zero at least once during the year.

If we were to consider only the next twelve months, the optimum x would leave the final stock s_{12} at as low a level as possible. Clearly, this could be an embarrassment for the future, and what is needed is to leave the stock at an 'equilibrium' level where, assuming that future demands will repeat the current forecasts, a stable x value will continue thereafter. If we call

x^*: the equilibrium planned daily output
I_0^*: the equilibrium inventory at the beginning of each year
s_j^*: the equilibrium inventory at the end of month j, so that $s_j^* = s_{j+12}^*$

then x^* and I_0^* must satisfy equations (1.1), (1.2), and (1.3) and in addition

$$I_0^* = s_{12}^*. \tag{1.4}$$

Note that we are assuming that the number of working days in corresponding months will be the same in any year. Clearly this will not be true, but the effect will only be marginal.

We therefore have to solve equations (1.1)–(1.4) for I_0^* and x^*. Expressed as they are, this may appear difficult. In fact, under equilibrium the demand is exactly equal to the output, which determines x^* directly, and from this we select I_0^* so that at some time during the year the stock falls to zero (but never below).

Having found values for the equilibrium model, we then have to steer our existing situation towards this model. If we were to do this in one year, we would simply impose the condition that

$$s_{12} = I_0^*. \tag{1.5}$$

The intervening inventory levels s_j may or may not fall to zero in achieving this. If they do not then there will be a good case for a 'one-year transition' strategy, although there is a further consideration in the comparison of this year's daily output x with the subsequent equilibrium level of x^*. For example, if the organization had been carrying too much stock, the value of I_0 would be somewhat greater than the optimum I_0^*. The resultant output x for the current (transition) year would then be low, which would therefore run down the stock, after which a higher output x^* would maintain equilibrium conditions. Most organizations would very reasonably find abrupt oscillations in output levels unworkable, and would prefer a gentler approach to this equilibrium level, say over two or three years. In terms of the model this is simply a matter of putting $s_{24} = I_0^*$ or $s_{36} = I_0^*$ and setting $d_{j+12} = d_j$ and $m_{j+12} = m_j$. There is another point in favour of the longer horizon approach. Although our best forecast of the long-term future demand may, with perhaps some simple modification, be the one strictly appropriate only for next year, it would be foolish to attach too much credence to it. This sort of forecast is based on the assumption of stable economic conditions, an assumption which appears to become less warranted as the world digests more and more advances in technology. Nevertheless it is a goal towards which we should be aiming, and a policy which keeps its (changing) ideal equilibrium goal two or three years over the horizon should have little difficulty in adapting to all but the most abrupt changes.

Example of deterministic aggregate planning

An organization has the forecast demands for twelve months ahead shown in Table 1.1(a). What should be its future output level policy?

The total demand for the year is 520 to be supplied in 240 working days, and therefore x^*, the planned daily output, will be 520/240, i.e. 13/6 units each day. Thus, with an initial inventory of I_0^* the pattern of demands and stock shown in Table 1.1(b) would occur. From this table, which is equivalent to the solution of equations (1.1), (1.2), and (1.4), it can be seen that the minimum inventory will occur at the end of December. To make this equal to zero would require $I_0^* = 232/6$. Note that the peak inventory of 232/6 + 176/6, equalling 68 units, occurs at the end of August. A graph of the equilibrium stock levels is shown in Figure 1.2.

Suppose that the existing stock is 70 units. Clearly the organization has to reduce its stock level; if it is to reach equilibrium in one year, it will have a total production which will be less than the total demand by an amount

$$70 - \tfrac{116}{3} = \tfrac{94}{3} \text{ units.}$$

To achieve equilibrium in two years, the annual reduction would obviously be one-half of this, and in three years it would be one-third, etc. The corresponding daily output levels would be

$x = 2.04$ (1 year) $\quad x = 2.10$ (2 years)
$x = 2.12$ (3 years) $\quad x = 2.17$ (equilibrium).

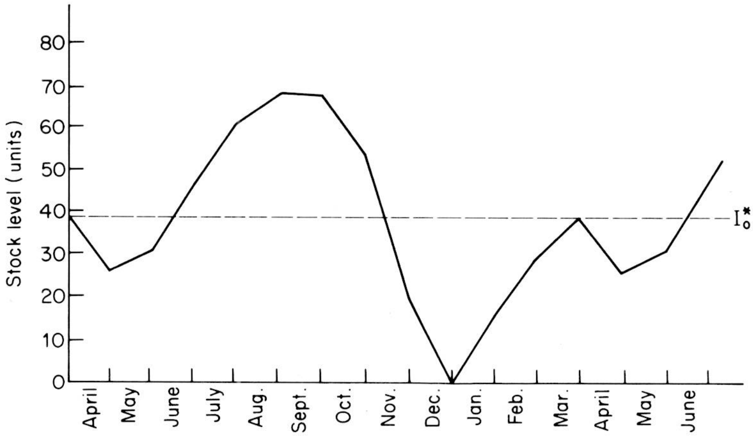

Figure 1.2.