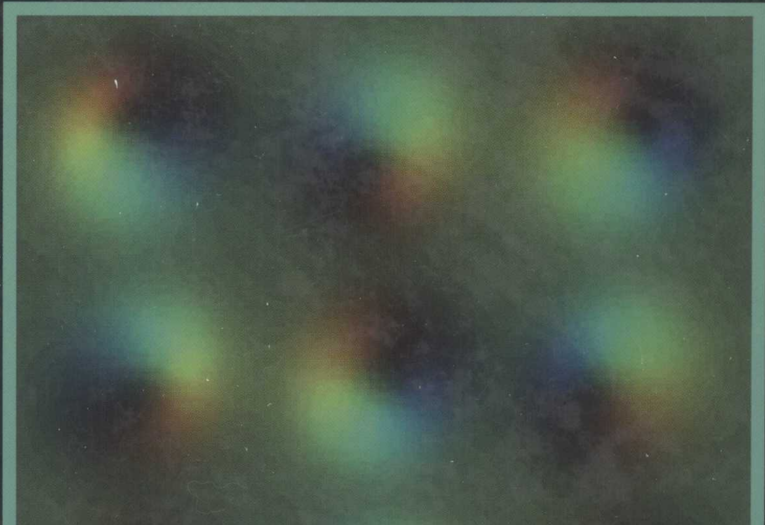
HAL R. VARIAN
EDITOR

COMPUTATIONAL ECONOMICS AND FINANCE

MODELING AND ANALYSIS WITH *MATHEMATICA*®



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Editor

Computational Economics and Finance

Modeling and Analysis with *Mathematica*®

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Preface

In 1992 I edited *Economic and Financial Modeling with Mathematica*. In the Preface to that work I said "I have been pleased and impressed with the materials developed by the contributors to this book. But there is a lot more that can be done. If the demand for this book, and the supply of new applications are large enough we may well publish a second volume." As it turned out both conditions were satisfied: many readers bought the book and several authors came forth with new and exciting applications of *Mathematica*. The result is the book you hold in your hands.

Overview of the Book

As before, I have divided the book into three main sections: economics, finance, and statistics.

The first few chapters in economics have to do with various forms of optimization. Michael Carter leads off with a study of *Mathematica*'s capabilities for linear programming. He provides both a tutorial in using *Mathematica*'s built-in functions and goes on to describe extensions to these functions that make them much more useful for economic applications. Jean-Christophe Culioli follows with a very nice overview of nonlinear programming techniques that can be implemented in *Mathematica*. Whereas Culioli is primarily concerned with convex optimization problems, Paul Rubin shows how *Mathematica* can be used to get a complete solution to a certain class of non-convex problems.

Moving on to economic applications, Eduardo Ley examines "data envelopment analysis," which is a way to measure the efficiency of production units using linear programming techniques. I demonstrate another way of doing efficiency analysis that applies to both production and consumption decisions. William Sharkey shows how to use *Mathematica* to allocate fixed costs using some of the tools of cooperative game theory. Finally, Luke Froeb and Gregory Werden demonstrate the use of *Mathematica* for simulation of mergers in an oligopolistic industry.

Turning to financial applications, John Dickhaut, Steve Gjerstad, and Arijit Mukherji have written a nice set of tools for studying auctions, both theoretically and experimentally. Given the recent interest in auctions of the electromagnetic spectrum, I expect to see much further research in practical problems of auction design, and this chapter provides a very nice demonstration of how *Mathematica* can be used for this purpose.

Ward Hanson looks at yield management of perishable assets—how to price goods such as airline seats, hotel rooms, and other dated services. In addition to the mathematical analysis, this chapter also presents a neat example of how to integrate *Mathematica* with a spreadsheet using MathTalk.

Simon Benninga, Raz Steinmetz, and John Stroughair show how *Mathematica* can be used to compute option values in a variety of cases. Their treatment of binomial option pricing is especially interesting. Mark Fisher and David Zervos use *Mathematica* to fit yield curves, and Luke Froeb examines the use of *Mathematica* in spectral analysis of financial time series.

These two chapters provide a nice lead-in to the statistical applications that follow. Robert Stine shows how to use *Mathematica* to do "exploratory data analysis." This involves making use of both the computational and the graphical capabilities of the program. David Belsely provides a very nice illustration of the use of *Mathematica* for Monte Carlo problems. Although *Mathematica* is significantly slower than dedicated statistical packages, the ease and flexibility of its programming language more than makes up for this lack of speed. Finally, Colin Rose and Murray Smith provide a very useful set of tools for manipulating probability distributions.

Acknowledgments

The first volume of *Economic and Financial Modeling with Mathematica* owed its existence to Allan Wylde, Publisher of TELOS/Springer-Verlag, and the same is true of the second volume. His words of encouragement (and his persistence) were necessary ingredients to produce the final product.

I would also like to thank Leszek Sczaniecki at Wolfram Research who was kind enough to ask his staff to read preliminary versions of the chapters; they provided a number of useful comments and suggestions. Finally, Paul Wellin has done a wonderful job of converting the authors' highly eclectic manuscripts to a coherent set of chapters.

The most enjoyable part of the project has been working with the authors; their creativity and energy never ceases to amaze me. I hope that you, the reader, will be as pleased with the results as I am.

Hal R. Varian
Berkeley
May 1996

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Contents

Part I

Economics

1 Linear Programming with *Mathematica*: The Simplex Algorithm

Michael Carter

Linear programming and the simplex algorithm are fundamental to the theory and practice of optimization. In this chapter, we exploit the symbolic manipulation capability of *Mathematica* to elucidate the simplex algorithm clearly and intuitively. This provides the foundation for the following chapter, in which we develop functions that enhance *Mathematica*'s in-built linear programming facility.

> *The subject of linear programming is surrounded by notational and terminological thickets. Both of these thorny defenses are lovingly cultivated by a coterie of stern acolytes who have devoted themselves to the field. Actually, the basic ideas of linear programming are quite simple.*
>
> Press, Teukolsky, Vetterling, and Flannery (1992, p. 431)

1.1 Introduction

In his book *Methods of Mathematical Economics*, Joel Franklin (1980) relates the story of a visit to the headquarters of the Mobil Oil Corporation in New York in 1958. The purpose of his visit was to study Mobil's use of computers. In those days, computers were rare and expensive and Mobil's installation had cost millions of dollars. Franklin recognized the person in charge; they had been post-doctoral fellows together. Franklin asked his former colleague how long he thought it would take to pay off this investment. "We paid it off in about two weeks," was the surprise response. Elaborating, he explained that Mobil was able to make massive cost savings by optimizing production decisions using linear programming, decisions that had previously been made heuristically.

It would be hard to exaggerate the importance of linear programming in practical optimization, in applications such as production scheduling, transportation and distribution, inventory control, job assignment, capital budgeting, and portfolio management. Franklin's anecdote highlights the enormous benefits that can accrue from optimizing recurrent decisions.

There are two main reasons for the practical success of linear programming. Many production processes and economic systems are linear or nearly so. Linear programming provides an appropriate mathematical model for such processes. Also, there exists a very efficient algorithm (the simplex algorithm) for solving most linear programming problems. The postwar conjunction of the availability of digital computers and the discovery of the simplex algorithm by George Dantzig paved the way for successful industrial application as exemplified by Mobil's experience.

Linear programming is also important to economists and game theorists. Although most economic models are nonlinear, linear models of exchange, production, and capital accumulation serve an important didactic role (Dorfman et al. 1958). Furthermore, an understanding of the simplex algorithm and the duality theorem enhances comprehension of nonlinear optimization, mastery of which is central to economic analysis. In game theory, the solution of zero-sum games is a linear programming problem and the minimax theorem is formally equivalent to the fundamental duality theorem of linear programming. The efficient solution of nonzero-sum games uses a modification of the simplex algorithm called the complementary pivot algorithm (Lemke and Howson 1964; Wilson 1992). Much of cooperative game theory reduces to the application of linear programming theory and techniques (Carter 1993).

Despite recent discoveries of alternative "interior point" algorithms, the simplex method and its variants remain by far the most common practical method for solving linear programming problems. The simplex algorithm is based upon a very simple intuitive idea of successive improvement. However, because most textbook treatments aim at describing a mathematical formulation suitable for implementation by conventional programming languages, their discussion tends to hide the intuitive simplicity of the algorithm. The symbolic manipulation capability of *Mathematica* enables the simplex algorithm to be elucidated more clearly and intuitively, and the significance of all the results understood.

Because our implementation of the simplex algorithm is designed for explanation rather than execution, our code emphasizes clarity rather than efficiency.

1.2 The Problem

To make the exposition easier to follow, we start with a simple specific example. A furniture maker can produce three products: bookcases, chairs, and desks. Each product requires machining, finishing, and some labor. The supply of these resources is limited. Unit profits and resource requirements are listed in the following table.