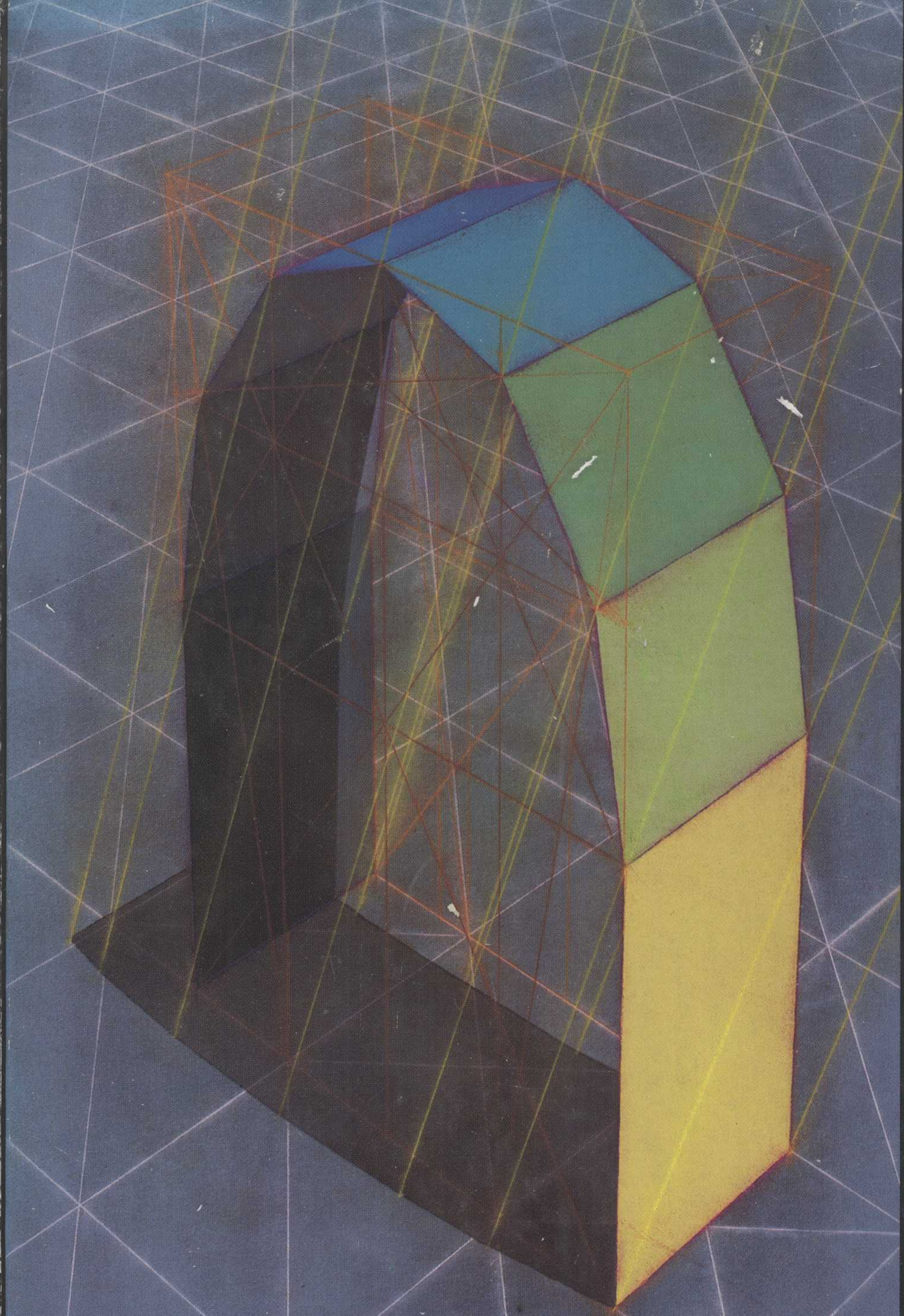

SECOND EDITION

Applied Calculus for Business and Economics, Life Sciences, and Social Sciences

RAYMOND A. BARNETT
MICHAEL R. ZIEGLER

APPLIED CALCULUS SECOND EDITION

FOR BUSINESS AND ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

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Preface

The material in this book is suitable for a two-quarter or two-semester course in calculus that emphasizes the topics most useful to students in business and economics, computer science, life sciences, and social sciences. The contents consist of a careful but well-motivated presentation of the calculus for functions of one variable (including the exponential and logarithmic functions) and selected topics in multivariable calculus, differential equations, infinite series, numerical techniques, and probability. The choice and organization of topics make the book readily adaptable to a variety of courses.

The book is designed for students who have had $1\frac{1}{2}-2$ years of high school algebra or its equivalent. However, because much of this material is forgotten through lack of use, Appendix A and Chapter 1 contain a review of basic topics from intermediate algebra that can be treated in a systematic way or referred to as needed. In addition, certain key topics are reviewed immediately prior to their use (see Section 5-1).

■ Major Changes from the First Edition

The second edition of *Applied Calculus for Business and Economics, Life Sciences, and Social Sciences* reflects the experiences and recommendations of a large number of users of the first edition. Additional examples and exercises have been included in almost every section to increase student support and to provide a better understanding of the material. In particular, a concentrated effort has been made to increase a student's ability to work with graphs and to visualize mathematical relationships through graphs. These additions have increased the overall length of the book, but it is still possible to cover most, if not all, of the material in two quarters or two semesters.

The most noticeable change from the first edition is the reorganization of the topics. The material on graphing has been expanded and rewritten and now occupies most of Chapter 4. The exponential and logarithmic functions are introduced at an earlier point so that Chapters 2–5 now deal exclusively with differential calculus and Chapters 6 and 7 deal with integral calculus. There are new sections on asymptotes, elasticity of demand, integral tables, power series, and interpolating polynomials; and expanded treatments of L'Hôpital's rule, error estimates for Taylor series, and error estimates for numerical integration.

■ General Comments

Chapters 2–5 present the differential calculus for functions of one variable, including the exponential and logarithmic functions. Limits and continuity are presented in an intuitive fashion, utilizing numerical approximations and one-sided limits. All the rules of differentiation are covered in Chapter 2. Various applications of differentiation are presented in Chapters 3 and 4, with a strong emphasis on graphing concepts. Finally, the exponential and logarithmic functions are covered in Chapter 5.

Chapters 6 and 7 deal with integral calculus. In Chapter 6, differential equations and exponential growth and decay are included as applications of antidifferentiation. The definite integral is intuitively introduced in terms of an area function and is later formally defined as the limit of a Riemann sum. Techniques of integration and improper integrals are covered in Chapter 7. Since the integral table used in Section 7-3 contains formulas for a variety of rational functions, the method of partial fractions is not included among the techniques of integration.

The remaining five chapters cover a variety of more advanced topics. In order to provide maximum flexibility in the selection of topics, these chapters are relatively independent of each other (see the diagram of chapter dependencies following this Preface).

Chapter 8 provides an introduction to multivariable calculus, including both partial derivatives and double integrals. No previous experience with three-dimensional coordinate systems is assumed. Three-dimensional figures are included to illustrate the concepts, but graphing techniques in three dimensions are not discussed.

Differential equations are first introduced in Section 6-2, but Chapter 9 contains a much more thorough presentation of this important topic, including second-order linear differential equations and systems of first-order linear equations. All the growth laws discussed in Section 6-2 are reviewed, and many other applications are considered.

Chapter 10 begins by discussing the approximation of functions by Taylor polynomials. Infinite series follow naturally from this presentation, eliminating the need for a lengthy treatment of series of constants and all the associated convergence tests. The operations that can be performed on Taylor series are carefully discussed, since this is the most practical procedure for finding the Taylor series for many functions.

A variety of numerical techniques are covered in Chapter 11. This material can be presented effectively using only a hand-held calculator, but sample computer output is included in each section as an illustration of the way these techniques are put to use in actual practice. If students have access to computing facilities, it would be a simple matter to incorporate the use of a computer in the presentation of this material. To facilitate this, a computer supplement containing the programs used in this chapter, along with many other programs, additional examples, and exercises, is

available from the publisher (see the lists of student and instructor aids later in this Preface).

Chapter 12 covers the application of calculus to the theory of probability. The presentation, though comprehensive, assumes no previous experience with probability.

■ Important Features

Emphasis

Emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

Examples and Matched Problems

The book contains over 320 completely worked examples. Each example is followed by a similar problem for the student to work while reading the material. The answers to these matched problems are included at the end of each section for easy reference.

Exercise Sets

The exercise sets are designed so that an average or below-average student will experience success, and a very capable student will be challenged. They are divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theoretical) levels.

Applications

This book includes sufficient applications to convince even the most skeptical student that mathematics is really useful. The majority of the applications are included at the ends of exercise sets and are generally divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own fields of interest, or if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems taken from professional journals and professional books associated with the given subjects. No specialized experience is required to solve any of the applications included in this book.

■ Student and Instructor Aids

Student Aids

Dotted **"think boxes"** are used to enclose steps that are usually performed mentally (see Section 1-1).

Examples and developments are often **annotated** to help students through critical stages (see Section 1-2).

A **second color** is used functionally to indicate key steps (see Section 1-1).

Boldface type is used to introduce new terms and important comments.

Answers to odd-numbered problems are included in the back of the book.

Chapter review sections include a review of all important terms and symbols, a comprehensive review exercise, and a practice test. Answers to all review exercises and practice test problems are included in the back of the book.

A **solutions manual** is available at a nominal cost through a book store. The manual includes detailed solutions to all odd-numbered problems, all chapter review exercises, and all practice test problems.

A **computer applications supplement** by Carolyn L. Meitler and Michael R. Ziegler is available at a nominal cost through a book store. The supplement contains examples, computer program listings, and exercises that demonstrate the use of a computer to solve a variety of problems. No previous computing experience is necessary to use this supplement.

Instructor Aids

A **test battery** designed by Carolyn L. Meitler can be obtained from the publisher without charge. The test battery contains six different tests with varying degrees of difficulty for each chapter. All tests have easy-to-grade solution keys. The format is $8\frac{1}{2}$ by 11 inches for ease of reproduction.

An **instructor's manual** can be obtained from the publisher without charge. The instructor's manual contains some remarks on selection of topics and answers to even-numbered problems, which are not included in the text.

A **solutions manual** (see Student Aids) is available to instructors without charge from the publisher.

A **computer applications supplement** by Carolyn L. Meitler and Michael R. Ziegler (see Student Aids) is available to instructors without charge from the publisher. The programs in this supplement are also available on diskettes for APPLE II® and IBM® PC computers.* The publisher will supply one of these diskettes without charge to institutions using this book.

■ Acknowledgments

In addition to the authors, many others are involved in the successful publication of a book. We wish to thank personally all those who reviewed the manuscript: Gary Brown, Washington State University; Robert Clark,

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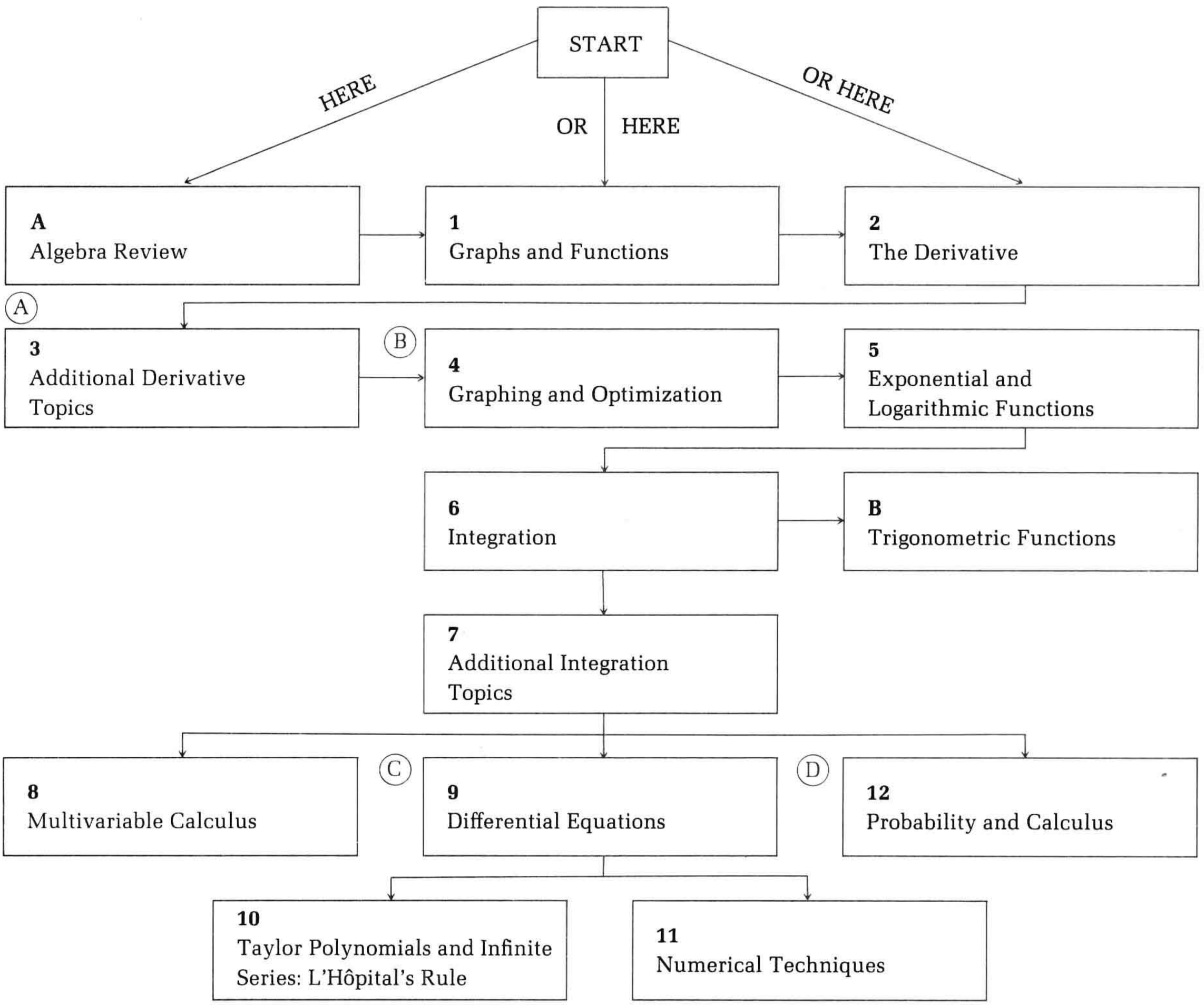
The production of this new edition with the help of all of these extremely competent people has been a most satisfying experience.

R. A. Barnett
M. R. Ziegler

Chapter Dependencies



(A) Section 3-2 can be omitted without loss of continuity.

(B) Section 4-4 can be omitted without loss of continuity.

(C) Chapter 9 should be covered first, since Chapters 10 and 11 contain techniques for approximating the solution to a differential equation.

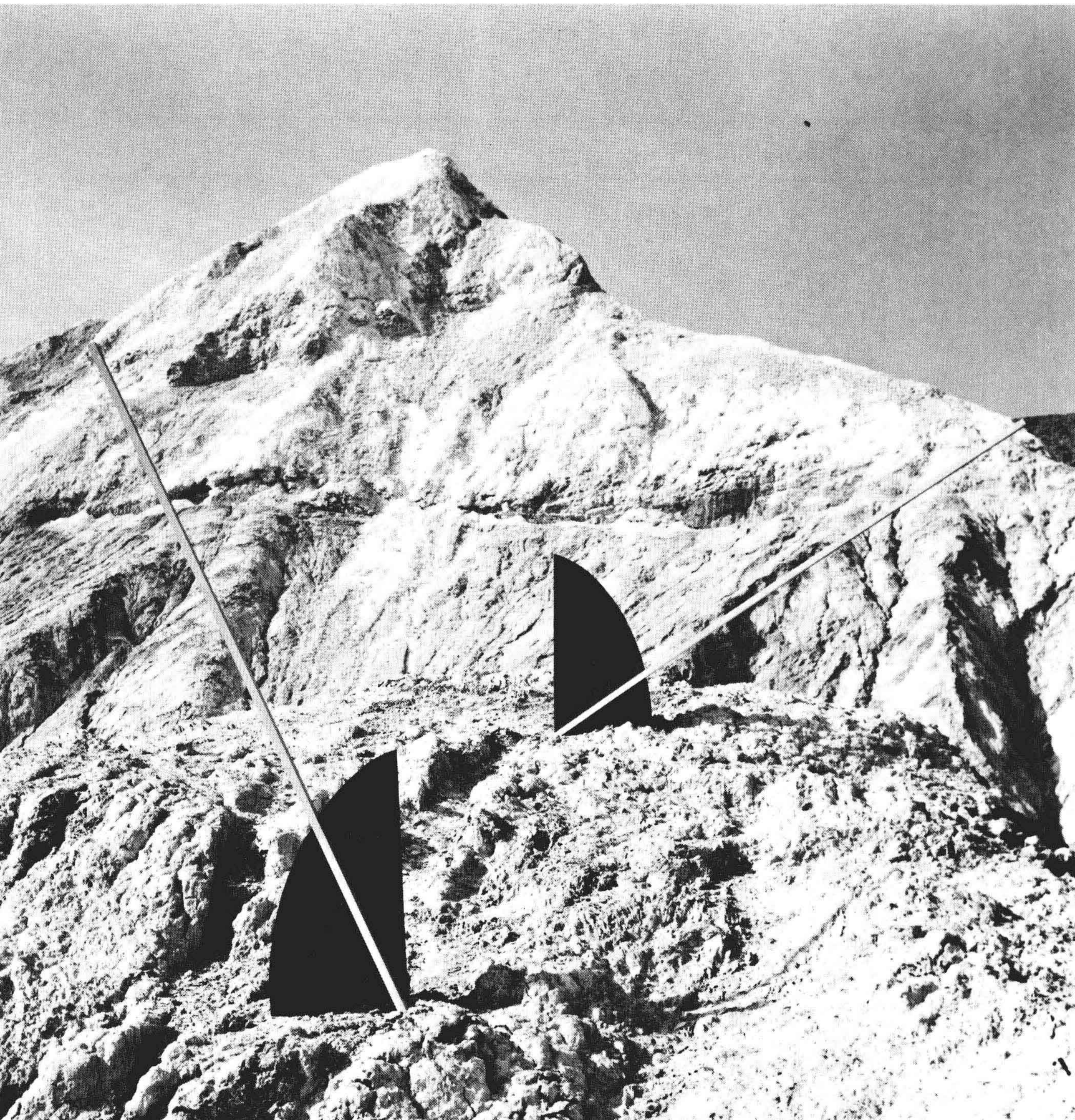
(D) Section 12-5 requires Sections 8-7 and 8-8.

Note: The instructor's manual has a detailed discussion of chapter and section interdependencies to aid instructors and departments in designing a course for their own particular needs.

Contents

Graphs and Functions

1



CHAPTER 1 Contents

1-1 Linear Inequalities

- The Real Number Line
- Inequality Relations
- Solving Linear Inequalities
- Application

■ The Real Number Line

Recall from algebra that a one-to-one correspondence exists between the set of real numbers and the set of points on a line; that is, each real number corresponds to exactly one point, and each point corresponds to exactly one real number. A line with a real number associated with each point and vice versa, as shown in Figure 1, is called a **real number line,** or simply, a **real line.** Each number associated with a point is called the **coordinate** of the point. The point with coordinate 0 is called the **origin.** The arrow indicates a positive direction; the coordinates of all points to the right of the origin are called **positive real numbers** and those to the left of the origin are called **negative real numbers.**

■ Inequality Relations

We now define *less than* and *greater than* for the set of real numbers.

Definition of $a < b$ and $b > a$

For real numbers a and b, we say that ***a* is less than *b*** or ***b* is greater than *a*** and write

$$a < b \quad \text{or} \quad b > a$$

if there exists a positive real number p such that

$$a + p = b \qquad \text{(or, equivalently, } b - a = p\text{)}$$

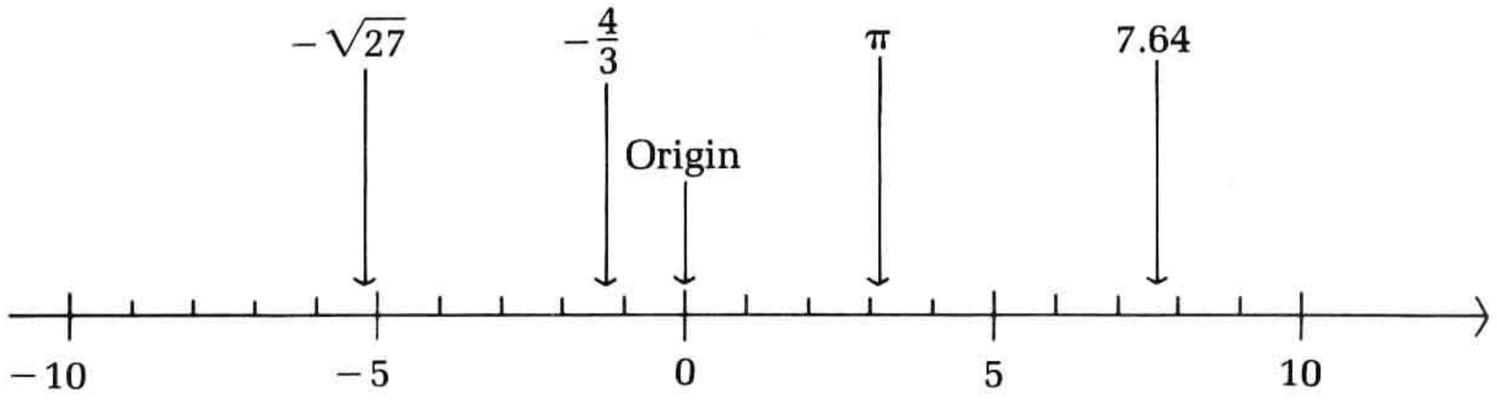


Figure 1 A real number line

We would certainly expect that if a positive number were added to *any* real number, the sum would be larger than the original number. That is essentially what the definition states. When we write $a \leqslant b$ we mean ***a* is less than or equal to *b*,** and when we write $a \geqslant b$ we mean that ***a* is greater than or equal to *b*.**

The inequality symbols $<$ and $>$ have a very clear geometric interpretation on the real number line. If $a < b$, then a is to the left of b; if $c > d$, then c is to the right of d (see Figure 2).

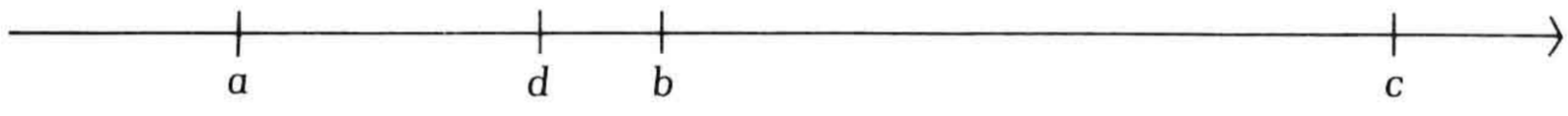


Figure 2 $a < b$, $c > d$

It is an interesting and useful fact that if a and b are any two real numbers, then either $a < b$, $a > b$, or $a = b$. This property is called the **trichotomy property.**

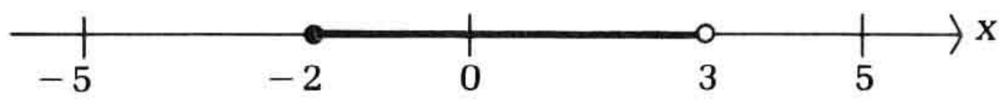
The double inequality $a < x \leqslant b$ means that $a < x$ and $x \leqslant b$; that is, x is between a and b, including b but not including a. Other variations on the theme, as well as a useful **interval notation,** are indicated in Table 1 (see page 4).

Example 1 Write each of the following in inequality notation and graph on a real number line.

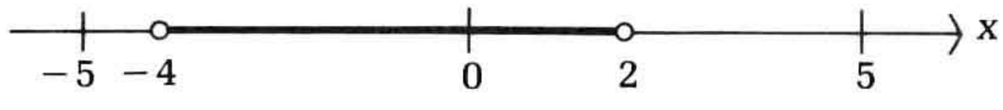
(A) $[-2, 3)$ (B) $(-4, 2)$
(C) $[-2, \infty)$ (D) $(-\infty, 3)$

Solution

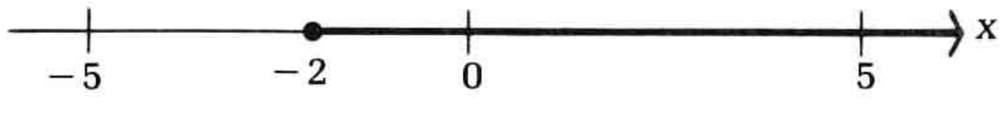
(A) $-2 \leqslant x < 3$



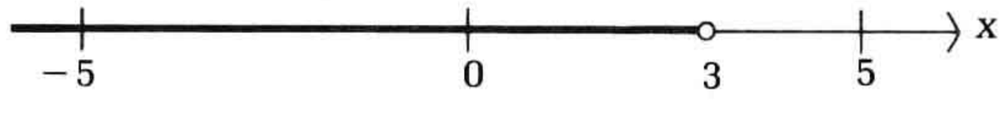
(B) $-4 < x < 2$



(C) $x \geqslant -2$



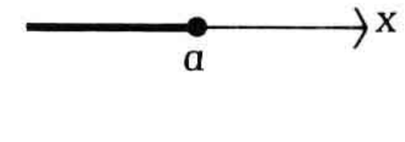
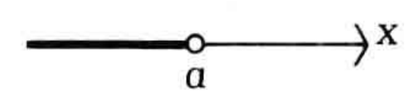
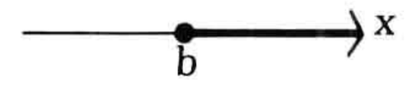
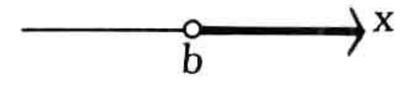
(D) $x < 3$



Problem 1 Write each of the following in interval notation and graph on a real number line.

(A) $-3 < x \leqslant 3$
(B) $-1 \leqslant x \leqslant 2$
(C) $x > 1$
(D) $x \leqslant 2$

Table 1

Interval Notation	Inequality Notation	Line Graph
$[a, b]$	$a \leqslant x \leqslant b$	
$[a, b)$	$a \leqslant x < b$	
$(a, b]$	$a < x \leqslant b$	
(a, b)	$a < x < b$	
$(-\infty, a]^*$	$x \leqslant a$	
$(-\infty, a)$	$x < a$	
$[b, \infty)$	$x \geqslant b$	
(b, ∞)	$x > b$	

* The symbol ∞ (read "infinity") is not a number. When we write $[b, \infty)$, we are simply referring to the interval starting at b and continuing indefinitely to the right. We would never write $[b, \infty]$.