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RUDOLF PEIERLS AND THEORETICAL PHYSICS

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PROCEEDINGS OF THE SYMPOSIUM HELD IN OXFORD, ON JULY 11TH & 12TH 1974, TO MARK THE OCCASION OF THE RETIREMENT OF PROFESSOR SIR RUDOLF E. PEIERLS, F.R.S., C.B.E.

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PREFACE

The Peierls Symposium, in honour of Sir Rudolf Ernst Peierls on the occasion of his retirement from the Wykeham Chair of Theoretical Physics in the University of Oxford, was held at Oxford on July 11th & 12th, 1974. The two hundred participants all had some professional connection with Peierls, and the invited speakers were each asked to talk on some development in physics with which Peierls had been particularly associated. It is these invited talks, together with some of the discussion that followed them, which are reproduced in this volume. They range in style from the historical reminiscence to the up-to-date review survey. The topics covered span a wide range of theoretical physics, including nuclear theory, statistical mechanics, solid state physics and elementary particle physics, mirroring the great breadth of Peierls' own interests.

The invited talks were recorded, and from the tapes we produced a draft version, which was put into final form by the speakers themselves. We are greatly indebted to them for the care and speed with which they corrected the draft manuscripts.

We deliberately asked the speakers not to eliminate all colloquialisms. The occasion had the spirit, which we wished to preserve, of a gathering of an "extended family", the creation of which may be regarded as a significant part of Peierls' scientific achievements. We hope that the personal flavour of the contributions will still come across, even in this printed form.

We know we speak for very many physicists, and their families, all over the world, when we offer to Professor Sir Rudolf and Lady Peierls our sincere thanks and very best wishes.

Oxford, May 1976

I. J. R. Aitchison
J. E. Paton

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The Scattering of Pions by Nuclei

H. A. BETHE

I want to talk about the scattering of pions by nuclear matter and by finite nuclei. It is assumed in this that you know the scattering by a nucleon, and the question is to derive the scattering by a complex system from the scattering by one nucleon.

We begin with an equation which is familiar from the theory of the refractive index n in electrodynamics and optics, namely that the wave number in the medium k is related to the wave number in vacuum k_o by the equation

$$k^2 = k_o^2 + 4\pi\rho f(k, 0) \tag{1.1}$$

which contains as the main part the forward scattering amplitude f(k,o). The k_o, of course, in the case of pions, is related to the energy ω by the equation

$$k_o^2 = \omega^2 - \mu^2 \tag{1.2}$$

If I consider pions of energy, let's say, up to 300-400 MeV, which is what I am interested in here, then the scattering is mostly by S- and P-waves and therefore the forward scattering amplitude is expected to be of the form

$$f(k,o) = a_o + a_1 k^2 \tag{1.3}$$

where the constant a_1 represents the P-wave scattering.

Now if you put these things together, and this has been done many years ago - although not in exactly this form - then you find k^2 in terms of the energy

$$k^2 = \frac{\omega^2 - \mu^2 + 4\pi\rho a_o(\omega)}{1 - 4\pi\rho a_1(\omega)} \tag{1.4}$$

and the important part here is the denominator which is $1 - 4\pi\rho a_1(\omega)$ where ρ is the density of scatterers (density of nucleons) and $a_1(\omega)$ is the P-wave scattering amplitude (apart from the factor k^2). This is a minor catastrophe; namely, a_1 might very well be just big enough so that the denominator becomes zero. You may say it can't become zero because a_1 is complex - the scattering amplitude is complex - but nevertheless we are still in trouble. For one thing, at very low energy the scattering amplitude is real; but for another thing, the imaginary part of the scattering amplitude always tells you the probability of having processes which remove the particles - the pions - from the incident beam. Now in the case of pion scattering by an assembly of nucleons, that is the probability of scattering in a collision with a single nucleon which is usually called a quasi-elastic collision if it happens in a nucleus. Now the imaginary part of $a_1(\omega)$ therefore surely is different when the nucleons are inside the nucleus from what it is for free nucleons, because for free nucleons I have no impediment to scattering and inside the nucleus I have. So it is surely wrong to consider a_1 to be the same quantity which you derive from the scattering of a pion by a free nucleon. Now if I take zero energy then the denominator is indeed zero, at a density of $\frac{1}{3}\mu^3$, which is just about two-thirds of the density of normal nuclear matter. The density of normal nuclear matter is .16 per (Fermi)^3 and that's just about one-half μ^3. (μ is the reciprocal Compton wavelength of the pion and is about .7 inverse Fermis).

So this has been the status of the subject, more or less, for some twelve years, until a paper by Barshay, Rostokin and Vagradov (1) in 1973. Another paper putting the same idea in a much simpler and more transparent fashion followed, by Barshay, Brown and Rho (2). They took the point of view that really what happens in pion-nucleon scattering is essentially that the pion makes the nucleon into a resonant particle, into a Δ particle, and they then proved that this is the only part of the pion scattering which will survive inside nuclear matter. Then you automatically find that your $a_1(\omega)$ contains the wavenumber of the pion inside nuclear matter and everything is all right. What I have to say today is rather in the spirit of this but it goes more explicitly into the pion dynamics.

The fundamental theory of the pion scattering in nuclear matter was developed by Dover and Lemmer, published about a year ago in Physical

Review (3). They made the very simple statement that what you have to calculate is the self-energy of the pion (which is given by the diagram in Fig. 1.1(a)) inside the nuclear matter, and you then have in this diagram a loop of a nucleon, which you can cut at one or two points. If you cut it at one point you get the diagram Fig. 1.1(b), which is the familiar scattering diagram for pions which is responsible for (3,3) scattering, and if you cut it at the other point, you get the diagram of Fig. 1.1(c), in which you get first absorption, and then re-remission of the pion.

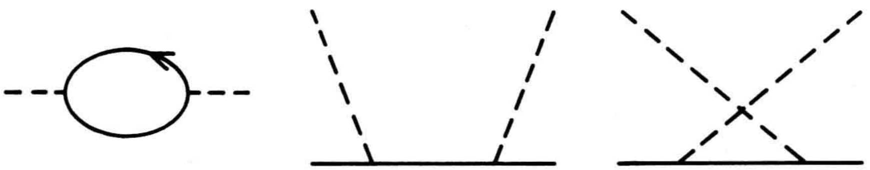

Fig. 1.1. The pion self-energy diagram, and its appearance when it is cut at one or another point on the nucleon line.

Now to put this in terms of a formula, what you want to calculate is the self-energy which I call Π. (By the way, the self-energy is defined by Dover and Lemmer in such a way that k, the wavenumber in nuclear matter is positive if Π is positive, in other words positive Π means an attractive potential).

Remembering the diagram of Fig. 1.1, where I cut the nucleon line, you will see that the scattering is essentially the derivative of the pion self-energy with respect to the number of nucleons in that state which I am considering. I have here one nucleon line and have, in fact, many nucleons that I can choose from, so the Π depends on the occupation number of the states in nuclear matter N(p), p being the momentum of the nucleon. (By the way, I find it useful to avoid confusion to always use p for nucleon and always k for pions, then you can recognise what you see in every formula). So the derivative of the self-energy with respect to the occupation number is just the forward scattering amplitude which we had before.

Now we have to look at this occupation number. What happens when I build up the density in nuclear matter? I start out by putting nucleons in the lowest momentum state, and then higher momentum states as the density goes up, so when I change the density from ρ to $\rho + d\rho$, what I add is simply occupation in those nucleon states which are just at the surface of the Fermi sea, and therefore have a nucleon momentum equal to the Fermi momentum, p_F. And so $d\Pi/d\rho$ will be an average over the direction of the nucleon relative to the pion of the forward scattering amplitude, and this average I call f_{av}. What you have then because of this derivative is something very similar to the original formula, (Eq. 1.1) from the optical theorem, only in that expression I had, as you will remember, ρ times the forward scattering amplitude, while now I have an integral of the forward scattering amplitude over $d\rho$ - not much difference.

What I have talked about so far is the scattering of a pion by a single nucleon. Of course, it is perfectly possible to have also scattering in which more than one nucleon is involved. I can have the pion come in, put one nucleon up into an excited state and then have an interaction of this with another nucleon. I make another particle-hole pair, and then the pion goes back out again. This is illustrated by Fig. 1.2.

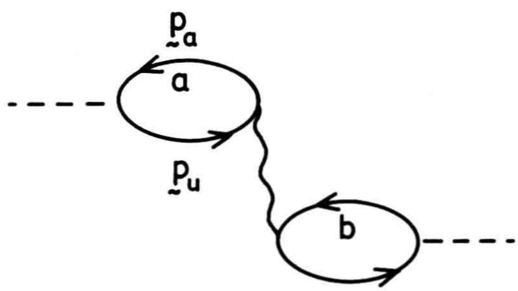

Fig. 1.2. A two-nucleon contribution to the pion self-energy; a and b are the two nucleons.

This two-nucleon scattering can, of course, be fairly complicated, but it has been shown, especially by Gerry Brown and a few of his collaborators, that the essential point in this diagram is the fact that two nucleons never can come very close to each other. There is a correlation function between nucleons which is zero at zero distance because of the strong repulsion at short distance, and once this is the case, once you have a strong anti-correlation of this type, this two-nucleon process reduces simply to the Lorentz-Lorentz factor, that is, instead of having $4\pi\rho a_1$ (this holds only for the P-wave scattering) you should divide this by $1 + \frac{4\pi}{3}\rho a_1$. So you have reduced it to a problem already solved in electrodynamics. The rest of the two-nucleon correlation apparently makes very little difference.

With this amendment then, we have essentially the same as I had in Eq. 1.4. The critical density now goes up, as you can easily convince yourself, to three-halves of what it was before and therefore just about to the density of nuclear matter. So you are still in trouble. The relation between wavenumber inside and wavenumber outside is

$$k^2 = \left[\omega^2 - \mu^2 + 4\pi\rho a_o(\omega) \right] \frac{1 + (4\pi/3)\rho a_1(\omega)}{1 - (8\pi/3)\rho a_1(\omega)} \qquad (1.5)$$

I repeat that Π of course is complex because the scattering amplitude is complex, the state k of the pion always decaying by quasi-elastic scattering.

The next step is to calculate the forward scattering amplitude. Fortunately, this can be done in essentially the same way as it is done for free pions by the Chew-Low theory. In the Chew-Low theory, as you may remember, the pions are treated relativistically, but the nucleons are treated statically or non-relativistically. In the Chew-Low theory the main thing is that you have an incident nucleon, and an incident pion, and you first emit a second pion, the nucleon thereby goes into state A, then you absorb the incident pion in state B, etc., etc., until the final pion comes out. The point is now, that in the Chew-Low theory, nearly all the interaction comes from intermediate pions of very high energy, because the dispersion integral which I will show you in a minute is linearly divergent with the momentum of the pion. Therefore this pion momentum is likely to be very high and therefore the corresponding nucleon momenta are also high until your final pion is emitted. This has two consequences:

1. The nucleon intermediate states have high momenta and therefore presumably are not affected by the Pauli principle.
2. Because of the formula

$$k^2 = k_o^{\ 2} + \Pi \qquad (1.6)$$

you can show that the self-energy of the pion remains finite or even goes down at high energy. Therefore as you go to high energy the momentum of the pion inside the nucleus is about the same as it is outside, and therefore the pion propagators which you have in the Chew-Low expansion will be the same as if the pion were free. Therefore, not only can you use the formalism of Chew and Low, but even the number should remain the same because the main part of the dispersion integral - namely the high energy part - remains unchanged. That's the main point of the theory.

In the Chew-Low theory you separate the scattered amplitude

$$f_{\underset{\sim}{k}\underset{\sim}{k}'}(\omega) = - \frac{2\pi}{(\omega_{\underset{\sim}{k}}\omega_{\underset{\sim}{k}'})^{1/2}} \sum_{\alpha} h_{\alpha}(\omega) P_{\alpha}(\underset{\sim}{k},\underset{\sim}{k}') \tag{1.7}$$

into an angular factor and a factor h depending on the energy of the pion. The angular factor corresponds to the various isospin and spin states. The interesting thing is the energy dependent factor, for which you get a dispersion integral

$$h_{\alpha}(\omega) = \frac{\lambda_{\alpha}}{\omega} + \frac{1}{\pi}\int d\omega_q \, q^3 v^2(q) \times \left\{ \frac{|h_{\alpha}(\omega_q)|^2}{\omega_q - \omega - i\varepsilon} + \Sigma A_{\alpha\beta} \frac{|h_{\beta}(\omega_q)|^2}{\omega_q + \omega} \right\} \tag{1.8}$$

The first term in the dispersion integral denotes the fact that you can emit a pion of a different momentum and then re-absorb it, and in doing so, you get each time a factor h_{α}, and so you get $|h_{\alpha}|^2$, with the requisite energy denominator.

Now to solve this equation, Chew and Low have shown that it is most convenient to use not h_{α} but to define a new quantity

$$g = \lambda/zh_{\alpha}(z) \tag{1.9}$$

where z is simply a generalisation, a complex generalisation, of the energy. It's just the energy taken in the complex plane. Now why is that convenient? It is convenient because the term $|h_{\alpha}|^2$ is determined from unitarity, and unitarity is expressed in the fact that h, the scattering amplitude, is essentially $\sin\delta\, e^{i\delta}$. As you know if you take the reciprocal of this, you get $e^{-i\delta}/\sin\delta$, which has the very agreeable property that its imaginary part is simply minus i and does not depend in any way on the process you are actually considering, and it is this which then permits the solution of the problem; and so the g_{α} goes to $\cot\delta_{\alpha} - i$.

I said already that you have to consider unitarity but in order to consider unitarity, it is essential that you use real values of the momentum k. As soon as you use complex values you get into all sorts of troubles and I know that very well because for about five months I struggled with just this problem: how can I put in unitarity when k is a complex number? And a complex number it ought to be from Eq. 1.6 because the self-energy Π is complex, and k_o is real, because it is $\omega^2 - \mu^2$. Well, the help for that was provided by a suggestion in a seminar at the University of Washington just about a year ago at which Professor Peierls was present. The suggestion was to make full use of the flexibility of complex variables which were introduced into the problem by Chew and Low, namely: what do you have in the complex energy plane, in the complex omega plane? Well, you have an origin, and you have a branch point at μ, the mass of the pion; then usually you take a branch cut along the real omega axis, but nobody tells you where to put the branch cut as long as it goes from the branch point to infinity. So why not make it go differently, why not make it go along the line on which k is real and omega therefore complex?

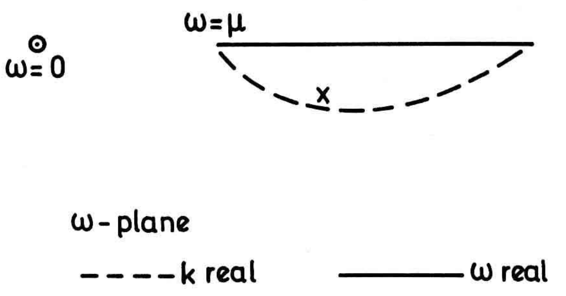

Fig. 1.3 The complex ω plane. ⊙ = origin, ------- line of real k, x = a possible value of z; solid line is branch cut along line of real ω.

I say that for real k, omega will be complex. Why do I need real values of k? This is most easily seen from the old Rayleigh scattering formula; you know that you have a plane wave e^{ikz} plus spherical waves, as you generally expand in incoming and outgoing spherical waves which have the form exp(±ikr). Clearly you get into trouble if you make k complex, either with the ingoing or with the outgoing spherical waves. So you have to choose k real to be able to define the phase shift. So I go along the solid line in Fig. 1.3 taking k real, and then I can use the fact that I know, from the phase shift expression for h, the imaginary part of the quantity zg(z), and I know this just on the positive side of the branch cut. That represents the physically permitted values which lead to outgoing spherical waves. These still have a negative imaginary part of omega but they are on the positive side of the branch cut. So you have this imaginary part because of the form of h, which is directly given to you, and is some simple expression in terms of k. Unfortunately, since the branch cut now has complex omega, you no longer get a very simple expression for what you want to know, namely zg(z) is given by the expression

$$zg(z) = z - \frac{z^2}{\pi}\int_{\mu}^{\infty} dx' \left[\frac{F_\alpha(x')}{x' - z} + \frac{G_\alpha(x')}{x' + z} \right] \tag{1.10}$$

What you really would like to know is the quantity F_α in this expression and that is what normally is determined from the imaginary part of g. But unfortunately, z itself is now complex so you get really three contributions to Im g, of which only one is the thing you want to know. But then by discussing this in some detail, you can show that the other two parts are unimportant at least for high pion energy. So we then are reduced essentially to the result of Chew and Low and that, of course, is a happy solution. You have to calculate the dispersion integral, Eq. 1.10, the imaginary part of which is just what I got from unitarity, while the real part is the one which gives you the position of the resonance.

Now looking at this expression, the F_α, from all the complicated argument which I have told you, behaves about as k (the momentum) at large momenta, so that the integral in Eq. 1.10 will diverge linearly with momentum. Thus you see that indeed the high momenta give you the main contribution. Because this is so, the main contribution comes from those values of x' where I know the F_α very well - it's the Chew-Low value. Then the integral itself will be close to what Chew and Low give. The result then is g, for which I get just the same thing as Chew and Low, namely

$$g_\alpha(\omega) = 1 - \omega r_1 - \frac{i}{\omega}\left[\lambda_\alpha k_\omega^{\ 3} v^2(k)\mu^{-2} + \omega^2 r_2 \right] \tag{1.11}$$

where r_1 is what Chew and Low call the effective range. Now the imaginary part in Eq. 1.11 is slightly modified for the reasons I mentioned, but the real part is much the same as Chew and Low obtained and therefore, it should go to zero just at the same point where Chew and Low go to zero, namely at the resonance energy. Therefore, r_1 should be just about one over the resonance energy. The imaginary part has a correction which is related to Π and is not very important. I started from going along the direction of real k, but now, by using the magic of complex variables, I can transform all this to an integral along real values of the energy, real values of omega, and then I am essentially back to where I started from with a slight correction.

So if you now want to calculate the behaviour of pions in nuclear matter, you assume the self-energy Π as known. From that you calculate g, g gives you h, and h gives you f, the scattered amplitude, and this in turn gives you the Π, that is the self-energy, and then you iterate until you get self-consistency. The final formula for the (3,3) scattered amplitude f_3 is

$$f_3 = C_k \frac{2\lambda_3}{\mu^2} \frac{k_\omega^{\ 2}\, v^2(k_\omega)}{\omega(1 - \omega r_1) - i[\quad]} \tag{1.12}$$

where k_ω is the value of the momentum which corresponds to a given energy omega, $[\quad]$ is the square bracket of Eq. 1.11, C_k is a number, and v(k) I haven't told you. It is a cut-off factor which was introduced by Chew and Low in order to cut off the divergence of the integrals.

Now, this is all well and good, except it leaves out two important points - one is the Pauli principle and the other is some restrictions on the energy. I told you before that the Pauli principle shouldn't really come in, but that applied only to the case of the intermediate states of the pion, and it does not apply to the final state. What we have in general is that the scattered amplitude describes a scattering in which the pion initially going in direction $\hat{\underset{\sim}{k}}$ encounters a nucleon and afterwards is scattered in another direction, let's say $\hat{\underset{\sim}{k}}_f$. When it does so, then of ccurse it gives a recoil to the nucleon, and this recoil may bring the nucleon into a state which is already occupied, and this would then be forbidden by the Pauli principle. Now I no longer have the excuse that the k of the pion is high, because the final k must be the same as the initial k or less, because the pion has lost some energy. So, therefore, I do have to take into account the Pauli principle for the states of the same energy which can be reached in such a scattering by a single nucleon. (Let me for the moment consider the nucleon as infinitely heavy).

Such problems are generally done by the Lippman-Schwinger equation

$$T = K + i\int d^3k_f\ \delta(\omega_f - \omega)\ K|k_f\rangle\langle k_f|T\ Q(k_f) \tag{1.13}$$

in which you start off by knowing the K-matrix which is the matrix not taking into account the condition that you always need to have outgoing waves; i.e. K is obtained using the <u>real</u> part of the dispersion relation. You want to get the T-matrix which is the actual scattering matrix. The relation of these is given by an integral over all directions of the pions, directions of $\hat{\underset{\sim}{k}}_f$, where you go with the T-matrix to the intermediate state and the intermediate state must be empty, that's expressed by the operator $Q(\underset{\sim}{k}_f)$, the Pauli operator, and then you go with the K matrix to the final state. The condition for Q is that the nucleon momentum after this first scattering must be outside the Fermi sea so the incident nucleon momentum plus the incident pion momentum minus the intermediate state pion momentum must be greater than the Fermi momentum. Now it is useful to introduce the sum of the incident momenta, that is the incident total momentum, which I call $\underset{\sim}{P}$, and to choose $\underset{\sim}{P}$ as an axis; then you can introduce polar coordinates Θ, Ø for $\underset{\sim}{k}_f$ and the Pauli principle simply comes down to a condition

$$\cos\Theta < \lambda \tag{1.14}$$

where λ is calculable from the given quantities

$$\lambda = \frac{P^2 + k^2 - p_F^{\,2}}{2Pk} \tag{1.15}$$

Well, this Lippmann-Schwinger equation, Eq. 1.13, which looks rather formidable, can be solved very easily and we have done so. The result is given in Eq. 1.16 in terms of the quantity λ, which I just defined. The solution for the forward scattering is just

$$T(\underset{\sim}{k},\underset{\sim}{k}) = -\pi K \left\{ \frac{3(1-\lambda^2)}{1-\frac{1}{4}ik\omega K_o(1+\lambda)(2+\lambda-\lambda^2)} + \frac{1+3\lambda^2}{1-\frac{1}{4}ik\omega K_o(1+\lambda)(2-\lambda+\lambda^2)} \right\} \tag{1.16}$$

The forward scattering amplitude contains the term

$$1 + 3\cos^2\alpha \equiv 1 + 3\lambda^2 \tag{1.17}$$

characteristic of $P_{3/2}$ scattering, as many of you will know, where α means the angle between the incident pion momentum and the incident momentum $\underset{\sim}{P}$. (Well, the main thing I wanted to show you is that it can be done, the details are not essential). What the Pauli principle will do is reduce the imaginary term in the denominator; this imaginary term I will now call damping for the sake of having a short name for it, and so the damping is not quite as big as it would be for free pions, or for an ordinary delta resonance if it were left alone.

Now the second thing that has to be taken into account is that the nucleon mass is not infinite, but is in fact very finite and is still further reduced: there is an effective mass inside the nucleus which is maybe seven tenths of tha total nuclear mass as we know from nuclear matter theory, and so that mass isn't all that much bigger than the pion mass. Now because of this the energy which may be transferred to the nucleon in the scattering may be very large, especially because there is a tendency for the pion momentum to be very large: I mentioned in the beginning that the pion momentum inside the nucleus is much larger than outside; the simple way to say it is that the pion is attracted by nuclear matter and therefore has a higher momentum inside. So