

# Low-Speed Aerodynamics

SECOND EDITION



Joseph Katz

Allen Plotkin



# CAMBRIDGE AEROSPACE SERIES

**L**OW-SPEED AERODYNAMICS is important in the design and operation of aircraft flying at low Mach number, and ground and marine vehicles. This book offers a modern treatment of the subject, both the theory of inviscid, incompressible, and irrotational aerodynamics and the computational techniques now available to solve complex problems.

A unique feature of the text is that the computational approach (from a single vortex element to a three-dimensional panel formulation) is interwoven throughout. Thus, the reader can learn about classical methods of the past, while also learning how to use numerical methods to solve real-world aerodynamic problems. This second edition updates the first edition with a new chapter on the laminar boundary layer, the latest versions of computational techniques, and additional coverage of interaction problems. It includes a systematic treatment of two-dimensional panel methods and a detailed presentation of computational techniques for three-dimensional and unsteady flows. With extensive illustrations and examples, this book will be useful for senior and beginning graduate-level courses, as well as a helpful reference tool for practicing engineers.

## PRAISE FOR THE FIRST EDITION:

"The book has aimed to be fairly comprehensive and brings together topics that have naturally been scattered throughout the literature. Explanations and theoretical material have been put forward in a succinct systematic manner with numerous clear line diagrams....The reviewer concurs with one of the objectives of the authors that the work may be viewed as a textbook for advanced senior or first-year graduate course. Problems for academic use have been given....The book is also recommended to practicing aerodynamicists.

—R. K. Nangia, *Aeronautical Journal*

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## ***Second Edition***

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# Preface

Our goal in writing this Second Edition of *Low-Speed Aerodynamics* remains the same, to present a comprehensive and up-to-date treatment of the subject of inviscid, incompressible, and irrotational aerodynamics. It is still true that for most practical aerodynamic and hydrodynamic problems, the classical model of a thin viscous boundary layer along a body's surface, surrounded by a mainly inviscid flowfield, has produced important engineering results. This approach requires first the solution of the inviscid flow to obtain the pressure field and consequently the forces such as lift and induced drag. Then, a solution of the viscous flow in the thin boundary layer allows for the calculation of the skin friction effects.

The First Edition provides the theory and related computational methods for the solution of the inviscid flow problem. This material is complemented in the Second Edition with a new Chapter 14, "The Laminar Boundary Layer," whose goal is to provide a modern discussion of the coupling of the inviscid outer flow with the viscous boundary layer. First, an introduction to the classical boundary-layer theory of Prandtl is presented. The need for an interactive approach (to replace the classical sequential one) to the coupling is discussed and a viscous-inviscid interaction method is presented. Examples for extending this approach, which include transition to turbulence, are provided in the final Chapter 15.

In addition, updated versions of the computational methods are presented and several topics are improved and updated throughout the text. For example, more coverage is given of aerodynamic interaction problems such as multiple wings, ground effect, wall corrections, and the presence of a free surface.

We would like to thank Turgut Sarpkaya of the Naval Postgraduate School and H. K. Cheng of USC for their input in Chapter 14 and particularly Mark Drela of MIT who provided a detailed description of his solution technique, which formed the basis for the material in Sections 14.7 and 14.8. Finally, we would like to acknowledge the continuing love and support of our wives, Hilda Katz and Selena Plotkin.

## *Preface to the First Edition*

Our goal in writing this book is to present a comprehensive and up-to-date treatment of the subject of inviscid, incompressible, and irrotational aerodynamics. Over the last several years there has been a widespread use of computational (surface singularity) methods for the solution of problems of concern to the low-speed aerodynamicist and a need has developed for a text to provide the theoretical basis for these methods as well as to provide a smooth transition from the classical small-disturbance methods of the past to the computational methods of the present. This book was written in response to this need. A unique feature of this book is that the computational approach (from a single vortex element to a three-dimensional panel formulation) is interwoven throughout so that it serves as a teaching tool in the understanding of the classical methods as well as a vehicle for the reader to obtain solutions to complex problems that previously could not be dealt with in the context of a textbook. The reader will be introduced to different levels of complexity in the numerical modeling of an aerodynamic problem and will be able to assemble codes to implement a solution.

We have purposely limited our scope to inviscid, incompressible, and irrotational aerodynamics so that we can present a truly comprehensive coverage of the material. The book brings together topics currently scattered throughout the literature. It provides a detailed presentation of computational techniques for three-dimensional and unsteady flows. It includes a systematic and detailed (including computer programs) treatment of two-dimensional panel methods with variations in singularity type, order of singularity, Neumann or Dirichlet boundary conditions, and velocity or potential-based approaches.

This book is divided into three main parts. In the first, Chapters 1–3, the basic theory is developed. In the second part, Chapters 4–8, an analytical approach to the solution of the problem is taken. Chapters 4, 5, and 8 deal with the small-disturbance version of the problem and the classical methods of thin-airfoil theory, lifting line theory, slender wing theory, and slender body theory. In this part exact solutions via complex variable theory and perturbation methods for obtaining higher-order small disturbance approximations are also included. The third part, Chapters 9–14, presents a systematic treatment of the surface singularity distribution technique for obtaining numerical solutions for incompressible potential flows. A general methodology for assembling a numerical solution is developed and applied to a series of increasingly complex aerodynamic elements (two-dimensional, three-dimensional, and unsteady problems are treated).

The book is designed to be used as a textbook for a course in low-speed aerodynamics at either the advanced senior or first-year graduate levels. The complete text can be covered in a one-year course and a one-quarter or one-semester course can be constructed by choosing the topics that the instructor would like to emphasize. For example, a senior elective course which concentrated on two-dimensional steady aerodynamics might include Chapters 1–3, 4, 5, 9, 11, 8, 12, and 14. A traditional graduate course which emphasized an analytical treatment of the subject might include Chapters 1–3, 4, 5–7, 8, 9, and 13 and a course which emphasized a numerical approach (panel methods) might include Chapters 1–3 and 9–14 and a treatment of pre- and postprocessors. It has been assumed that the reader has taken



a first course in fluid mechanics and has a mathematical background which includes an exposure to vector calculus, partial differential equations, and complex variables.

We believe that the topics covered by this text are needed by the fluid dynamicist because of the complex nature of the fluid dynamic equations which has led to a mainly experimental approach for dealing with most engineering research and development programs. In a wider sense, such an approach uses tools such as wind tunnels or large computer codes where the engineer/user is experimenting and testing ideas with some trial and error logic in mind. Therefore, even in the era of supercomputers and sophisticated experimental tools, there is a need for simplified models that allow for an easy grasp of the dominant physical effects (e.g., having a simple lifting vortex in mind, one can immediately tell that the first wing in a tandem formation has the larger lift).

For most practical aerodynamic and hydrodynamic problems, the classical model of a thin viscous boundary layer along a body's surface, surrounded by a mainly inviscid flowfield, has produced important engineering results. This approach requires first the solution of the inviscid flow to obtain the pressure field and consequently the forces such as lift and induced drag. Then, a solution of the viscous flow in the thin boundary layer allows for the calculation of the skin friction effects. This methodology has been used successfully throughout the twentieth century for most airplane and marine vessel designs. Recently, due to developments in computer capacity and speed, the inviscid flowfield over complex and detailed geometries (such as airplanes, cars, etc.) can be computed by this approach (panel methods). Thus, for the near future, since these methods are the main tools of low-speed aerodynamicists all over the world, a need exists for a clear and systematic explanation of how and why (and for which cases) these methods work. This book is one attempt to respond to this need.

We would like to thank graduate students Lindsey Browne and especially Steven Yon who developed the two-dimensional panel codes in Chapter 11 and checked the integrals in Chapter 10. Allen Plotkin would like to thank his teachers Richard Skalak, Krishnamurthy Karamcheti, Milton Van Dyke, and Irmgard Flugge-Lotz, his parents Claire and Oscar for their love and support, and his children Jennifer Anne and Samantha Rose and especially his wife Selena for their love, support, and patience. Joseph Katz would like to thank his parents Janka and Jeno, his children Shirley, Ronny, and Danny, and his wife Hilda for their love, support, and patience. The support of the Low-Speed Aerodynamic Branch at NASA Ames is acknowledged by Joseph Katz for their inspiration that initiated this project and for their help during past years in the various stages of developing the methods presented in this book.

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## ***Introduction and Background***

The differential equations that are generally used in the solution of problems relevant to low-speed aerodynamics are a simplified version of the governing equations of fluid dynamics. Also, most engineers when faced with finding a solution to a practical aerodynamic problem, find themselves operating large computer codes rather than developing simple analytical models to guide them in their analysis. For this reason, it is important to start with a brief development of the principles upon which the general fluid dynamic equations are based. Then we will be in a position to consider the physical reasoning behind the assumptions introduced to generate simplified versions of the equations that still correctly model the aerodynamic phenomena being studied. It is hoped that this approach will give the engineer the ability to appreciate both the power and the limitations of the techniques that will be presented in this text. In this chapter we will derive the conservation of mass and momentum balance equations and show how they are reduced to obtain the equations that will be used in the rest of the text to model flows of interest to the low-speed aerodynamicist.

### **1.1 Description of Fluid Motion**

The fluid being studied here is modeled as a continuum, and infinitesimally small regions of the fluid (with a fixed mass) are called fluid elements or fluid particles. The motion of the fluid can be described by two different methods. One adopts the particle point of view and follows the motion of the individual particles. The other adopts the field point of view and provides the flow variables as functions of position in space and time.

The particle point of view, which uses the approach of classical mechanics, is called the *Lagrangian method*. To trace the motion of each fluid particle, it is convenient to introduce a Cartesian coordinate system with the coordinates  $x$ ,  $y$ , and  $z$ . The position of any fluid particle  $P$  (see Fig. 1.1) is then given by

$$\begin{aligned}x &= x_P(x_0, y_0, z_0, t) \\y &= y_P(x_0, y_0, z_0, t) \\z &= z_P(x_0, y_0, z_0, t)\end{aligned}\tag{1.1}$$

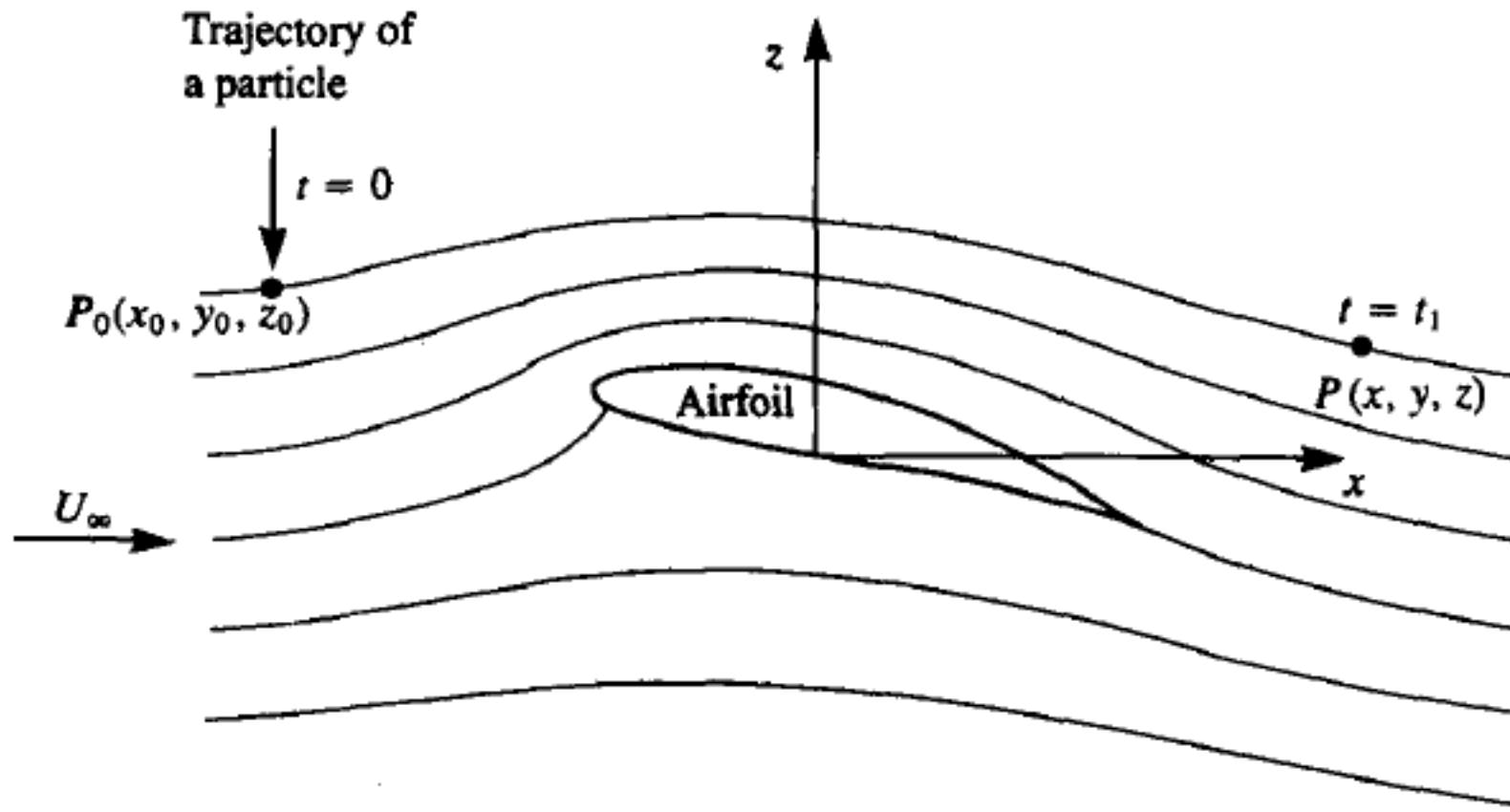
where  $(x_0, y_0, z_0)$  is the position of  $P$  at some initial time  $t = 0$ . (Note that the quantity  $(x_0, y_0, z_0)$  represents the vector with components  $x_0$ ,  $y_0$ , and  $z_0$ .) The components of the velocity of this particle are then given by

$$\begin{aligned}u &= \partial x / \partial t \\v &= \partial y / \partial t \\w &= \partial z / \partial t\end{aligned}\tag{1.2}$$

and those of the acceleration by

$$\begin{aligned}a_x &= \partial^2 x / \partial t^2 \\a_y &= \partial^2 y / \partial t^2 \\a_z &= \partial^2 z / \partial t^2\end{aligned}\tag{1.3}$$





**Figure 1.1** Particle trajectory lines in a steady-state flow over an airfoil as viewed from a body-fixed coordinate system.

The Lagrangian formulation requires the evaluation of the motion of each fluid particle. For most practical applications this abundance of information is neither necessary nor useful and the analysis is cumbersome.

The field point of view, called the *Eulerian method*, provides the spatial distribution of flow variables at each instant during the motion. For example, if a Cartesian coordinate system is used, the components of the fluid velocity are given by

$$\begin{aligned} u &= u(x, y, z, t) \\ v &= v(x, y, z, t) \\ w &= w(x, y, z, t) \end{aligned} \tag{1.4}$$

The Eulerian approach provides information about the fluid variables that is consistent with the information supplied by most experimental techniques and that is in a form appropriate for most practical applications. For these reasons the Eulerian description of fluid motion is the most widely used.

## 1.2 Choice of Coordinate System

For the following chapters, when possible, primarily a Cartesian coordinate system will be used. Other coordinate systems such as curvilinear, cylindrical, spherical, etc. will be introduced and used if necessary, mainly to simplify the treatment of certain problems. Also, from the kinematic point of view, a careful choice of a coordinate system can considerably simplify the solution of a problem. As an example, consider the forward motion of an airfoil, with a constant speed  $U_\infty$ , in a fluid that is otherwise at rest – as shown in Fig. 1.1. Here, the origin of the coordinate system is attached to the moving airfoil and the trajectory of a fluid particle inserted at point  $P_0$  at  $t = 0$  is shown in the figure. By following the trajectories of several particles a more complete description of the flowfield is obtained in the figure. It is important to observe that for a constant-velocity forward motion of the airfoil, in this frame of reference, these trajectory lines become independent of time. That is, if various particles are introduced at the same point in space, then they will follow the same trajectory.

Now let us examine the same flow, but from a coordinate system that is fixed relative to the undisturbed fluid. At  $t = 0$ , the airfoil was at the right side of Fig. 1.2 and as a result of its constant-velocity forward motion (with a speed  $U_\infty$  toward the left side of the page), later at  $t = t_1$  it has moved to the new position indicated in the figure. A typical particle's

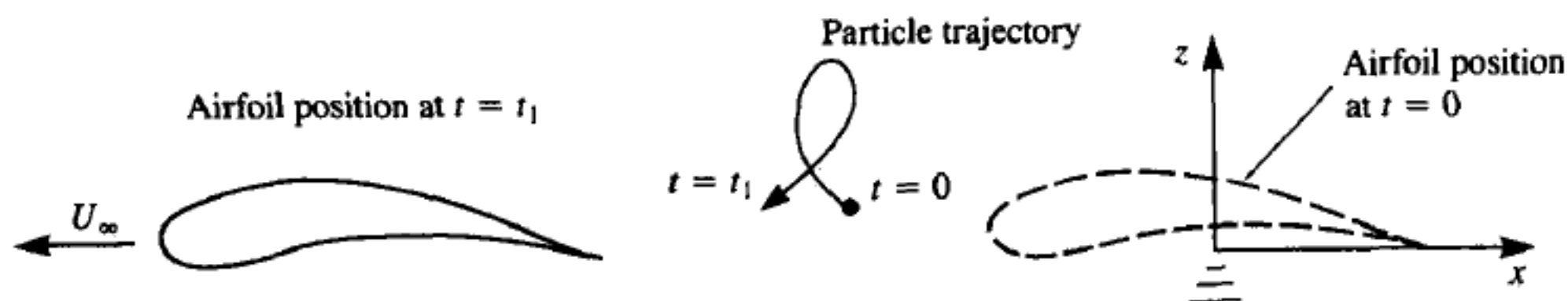


Figure 1.2 Particle trajectory line for the airfoil of Fig. 1.1 as viewed from a stationary inertial frame.

trajectory line between  $t = 0$  and  $t = t_1$ , for this case, is shown in Fig. 1.2. The particle's motion now depends on time, and a new trajectory has to be established for each particle.

This simple example depicts the importance of good coordinate system selection. For many problems where a constant velocity and a fixed geometry (with time) are present, the use of a body-fixed frame of reference will result in a steady or time-independent flow.

### 1.3 Pathlines, Streak Lines, and Streamlines

Three sets of curves are normally associated with providing a pictorial description of a fluid motion: pathlines, streak lines, and streamlines.

**Pathlines:** A curve describing the trajectory of a fluid element is called a pathline or a particle path. Pathlines are obtained in the Lagrangian approach by an integration of the equations of dynamics for each fluid particle. If the velocity field of a fluid motion is given in the Eulerian framework by Eq. (1.4) in a body-fixed frame, the pathline for a particle at  $P_0$  in Fig. 1.1 can be obtained by an integration of the velocity. For steady flows the pathlines in the body-fixed frame become independent of time and can be drawn as in the case of flow over the airfoil shown in Fig. 1.1.

**Streak Lines:** In many cases of experimental flow visualization, particles (e.g., dye or smoke) are introduced into the flow at a fixed point in space. The line connecting all of these particles is called a streak line. To construct streak lines using the Lagrangian approach, draw a series of pathlines for particles passing through a given point in space and, at a particular instant in time, connect the ends of these pathlines.

**Streamlines:** Another set of curves can be obtained (at a given time) by lines that are parallel to the local velocity vector. To express analytically the equation of a streamline at a certain instant of time, at any point  $P$  in the fluid, the velocity<sup>1</sup>  $\mathbf{q}$  must be parallel to the streamline element  $d\mathbf{l}$  (Fig. 1.3). Therefore, on a streamline:

$$\mathbf{q} \times d\mathbf{l} = 0 \quad (1.5)$$

If the velocity vector is  $\mathbf{q} = (u, v, w)$ , then the vector equation (Eq. (1.5)) reduces to the following scalar equations:

$$\begin{aligned} w \, dy - v \, dz &= 0 \\ u \, dz - w \, dx &= 0 \\ v \, dx - u \, dy &= 0 \end{aligned} \quad (1.6)$$

or in a differential equation form:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \quad (1.6a)$$

<sup>1</sup> Bold letters in this book represent vectors.



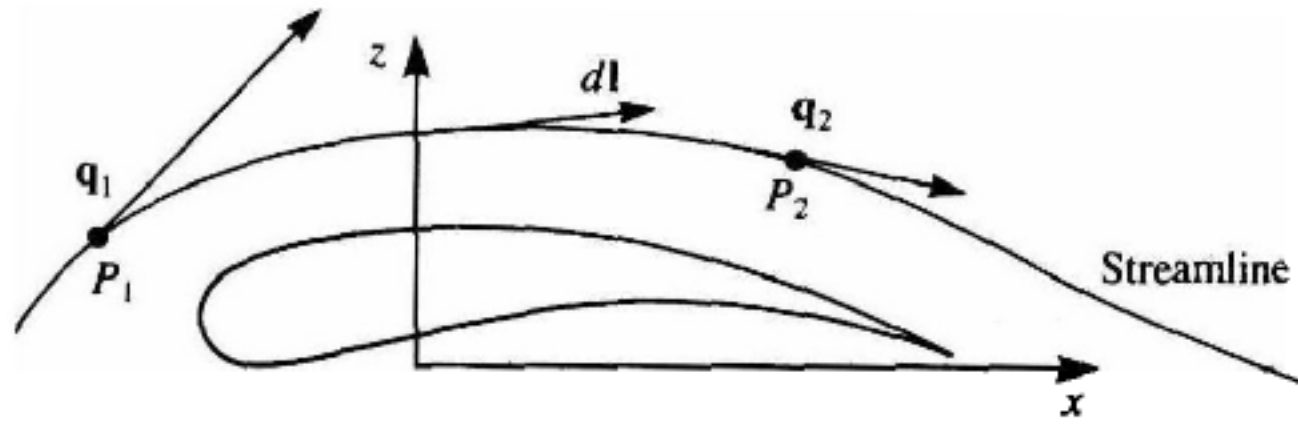


Figure 1.3 Description of a streamline.

In Eq. (1.6a), the velocity  $(u, v, w)$  is a function of the coordinates and of time. However, for steady flows the streamlines are independent of time and streamlines, pathlines, and streak lines become identical, as shown in Fig. 1.1.

#### 1.4 Forces in a Fluid

Prior to discussing the dynamics of fluid motion, the types of forces that act on a fluid element should be identified. Here, we consider forces such as body forces per unit mass  $\mathbf{f}$  and surface forces resulting from the stress vector  $\mathbf{t}$ . The body forces are independent of any contact with the fluid, as in the case of gravitational or magnetic forces, and their magnitude is proportional to the local mass.

To define the stress vector  $\mathbf{t}$  at a point, consider the force  $\mathbf{F}$  acting on a planar area  $S$  (shown in Fig. 1.4) with  $\mathbf{n}$  being an outward normal to  $S$ . Then

$$\mathbf{t} = \lim_{S \rightarrow 0} \left( \frac{\mathbf{F}}{S} \right)$$

To obtain the components of the stress vector consider the force equilibrium on an infinitesimal tetrahedral fluid element, shown in Fig. 1.5. According to Batchelor<sup>1.1</sup> (p. 10) this equilibrium yields the components in the  $x_1$ ,  $x_2$ , and  $x_3$  directions:

$$t_i = \sum_{j=1}^3 \tau_{ij} n_j, \quad i = 1, 2, 3 \quad (1.7)$$

where the subscripts 1, 2, and 3 denote the three coordinate directions. A similar treatment of the moment equilibrium results in the symmetry of the stress vector components so that  $\tau_{ij} = \tau_{ji}$ .

These stress components  $\tau_{ij}$  are shown schematically on a cubical element in Fig. 1.6. Note that  $\tau_{ij}$  acts in the  $x_i$  direction on a surface whose outward normal points in the  $x_j$  direction. This indicial notation allows a simpler presentation of the equations, and the

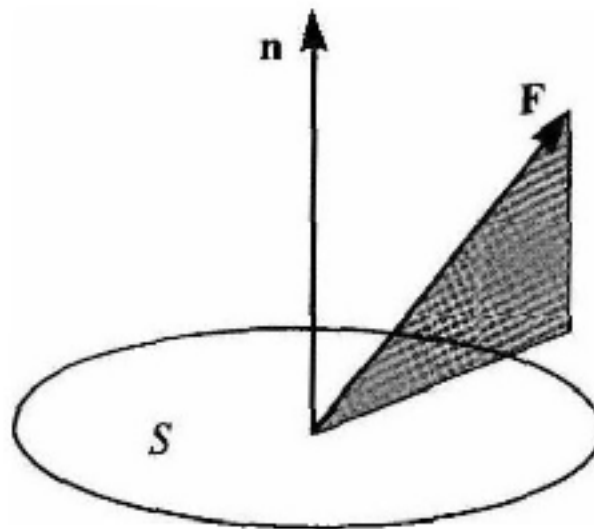


Figure 1.4 Force  $\mathbf{F}$  acting on a surface  $S$ .