

TECHNOLOGY MATHEMATICS HANDBOOK

DEFINITIONS ■ FORMULAS ■ GRAPHS
SYSTEMS OF UNITS ■ PROCEDURES
CONVERSION TABLES
NUMERICAL TABLES

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*The editors for this book were Harold B. Crawford and Ruth Weine
and its production was supervised by Teresa F. Leaden.*

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TECHNOLOGY
MATHEMATICS
HANDBOOK

*To my father Joseph,
and my son Peter*

PREFACE

This Handbook presents in one volume a concise summary of the major definitions, formulas, graphs, tables, and examples of elementary and intermediate mathematics. It places emphasis on technological applications and was prepared to serve as a desk-top reference book for aeronautical, architectural, civil, mechanical, chemical, industrial, electrical, and construction technologists and the practicing engineers in these fields.

The content of the book is grouped into four parts, each related to a particular type of technical calculations.

Part I - Elementary mathematics (Chapters 1 to 5) covers arithmetic, algebra, plane geometry, space geometry, and plane trigonometry.

Part II - Intermediate mathematics (Chapters 6 to 11) presents analytical geometry, differential calculus, sequences, series, integral calculus, matrices, determinants, and vectors.

Part III - Numerical procedures (Chapter 12 and Appendix A) gives a comprehensive outline of operations with decimal and complex numbers, applications of tables of numerical constants and elementary functions, calculations of interest and annuities, and various practical approximations.

Part IV - Conversion procedures (Chapter 13 and Appendix B) introduces the systems of units of measure (FPS system, SI system) with emphasis on their definitions, classification, and conversion.

The form of presentation has many special characteristics allowing easy and rapid location of the desired information and permitting the indexing of this information.

1. Each statement in the book is a coded sentence designated by the position number and key word.
2. The related sentences form logical sequences and their lengths allow speed reading.
3. The extensive index of all key words (of all sentences) given in the last part of the book offers the possibility of using this handbook as a dictionary of technology mathematics.
4. All formulas are presented in general symbols and their applications are illustrated by examples where this is desirable.
5. The application of tables of numerical coefficients is described in step-by-step procedures in Chapter 12, with cross-references given below each table.

In the preparation of this book free use was made of the pertinent material from my earlier book, "Engineering Mathematics Handbook," McGraw-Hill, New York, 1970. Although this handbook overlaps in part the Engineering Mathematics Handbook, their respective levels and objectives are so different that instead of competing with each other, they form a complementary set which many users may find useful in their professional work.

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1

ARITHMETIC

1.01 DEFINITIONS AND NOTATIONS

(1) Definitions

(a) **Arithmetic** is the systematic study of fundamental operations with real numbers and of the use of these operations in solving practical problems.

(b) **Real numbers** are:

- (α) The *natural numbers* (Sec. 1.01–1c).
- (β) The *rational numbers* (Sec. 1.10–1d).
- (γ) The *irrational numbers* (Sec. 1.10–1e).

(c) **Natural numbers** (also called the arabic numbers or positive integers) are the symbols arrived at by counting such as 1, 2, 3, 4, ..., where the three dots mean “and so on.”

(d) **Four fundamental operations** of arithmetic are:

- (α) The *addition* (Sec. 1.02–1).
- (β) The *subtraction* (Sec. 1.02–2).
- (γ) The *multiplication* (Sec. 1.02–3).
- (δ) The *division* (Sec. 1.02–4).

(2) Symbols of Relationship

The following symbols define the relationship of two numbers:

$=$ or $::$	Equals	\neq or \neq	Does not equal
$>$	Greater than	$<$	Less than
\geq	Greater than or equal	\leq	Less than or equal
\equiv	Identical	\approx	Approximately equal
\nless	Not greater than	\nless	Not less than

(3) Symbols of Aggregation

The symbols of grouping (aggregation) are:

$()$	Parentheses	$\{ \}$	Braces
$[]$	Brackets	$—$	Vinculum

(4) Signs of Operations

The signs of operations are:

+	Plus or positive	−	Minus or negative
±	Plus or minus, positive or negative	∓	Minus or plus, negative or positive
× or ·	Multiplied by	÷ or :	Divided by
a^n	n th power of a	$\sqrt[n]{a}$	n th root of a
$\log \}$ $\log_{10} \}$	Common logarithm, or Briggs's logarithm	$\ln \}$ $\log_e \}$	Natural logarithm, or Napier's logarithm

1.02 FOUR FUNDAMENTAL OPERATIONS

(1) Addition

(a) **Addition** is the operation of finding the sum of two or more numbers.

example:

$2 + 3 + 4 = 9$

where 9 is the *sum* and 2, 3, 4 are the *terms* of the sum.

(b) **Order of terms** in addition may be changed without affecting the sum (commutative law).

examples:

$2 + 3 + 4 = 9 \quad 3 + 4 + 2 = 9 \quad 4 + 2 + 3 = 9$

(c) **Grouping of terms** in addition may be changed without affecting the sum (associative law).

examples:

$(2 + 3) + 4 = 5 + 4 = 9 \quad 2 + (3 + 4) = 2 + 7 = 9$

(2) Subtraction

(a) **Subtraction** is the operation of finding the difference of two numbers.

example:

$9 - 4 = 5$

where 5 is the *difference*, 9 is the *minuend*, and 4 is the *subtrahend*.

(b) **Difference of two equal numbers** is zero.

example:

$9 - 9 = 0$

(3) Multiplication

(a) **Multiplication** is the operation of finding the product of two or more numbers.

example:

$$2 \times 3 \times 4 = 24$$

where 24 is the *product* and 2, 3, 4 are the *factors* of the product.

(b) **Order of factors** in multiplication may be changed without affecting the product (commutative law).

examples:

$$2 \times 3 \times 4 = 24 \quad 3 \times 4 \times 2 = 24 \quad 4 \times 2 \times 3 = 24$$

(c) **Grouping of factors** in multiplication may be changed without affecting the product (associative law).

examples:

$$(2 \times 3) \times 4 = 6 \times 4 = 24 \quad 2 \times (3 \times 4) = 2 \times 12 = 24$$

(4) Division

(a) **Division** is the operation of finding the quotient of two numbers.

example:

$$24 : 8 = 3$$

where 3 is the *quotient*, 24 is the *dividend*, and 8 is the *divisor*.

(b) **Quotient of two equal numbers** is 1.

example:

$$24 : 24 = 1$$

(5) Even, Odd, and Prime Numbers

(a) **Even number** is an integer divisible by 2.

example:

2, 4, 6, ... are even numbers.

(b) **Odd number** is an integer not divisible by 2.

example:

1, 3, 5, ... are odd numbers.

(c) **Prime number** is an integer divisible only by 1 and itself.

example:

1, 2, 3, 5, 7, ... are prime numbers.

(6) Factoring

(a) **Every nonprime number** greater than 1 can be expressed as a product of prime numbers.

examples:

$$30 = 2 \times 3 \times 5 \quad 60 = 2 \times 2 \times 3 \times 5$$

(b) **Highest common factor (HCF)** of a given set of numbers is the largest number that is a factor of all the numbers.

example:

$$\begin{array}{ccccccc} 24 = 2 \times 2 \times 2 \times 3 & 60 = 2 \times 2 \times 3 \times 5 & 84 = 2 \times 2 \times 3 \times 7 \\ \hline & \text{HCF} = 2 \times 2 \times 3 = 12 \end{array}$$

where HCF is the product of the prime factors that are common to all the numbers of the set.

(c) **Lowest common multiple (LCM)** of a given set of numbers is the smallest number that has each of the given numbers as a factor.

example:

$$\begin{array}{ccccccc} 24 = 2 \times 2 \times 2 \times 3 & 60 = 2 \times 2 \times 3 \times 5 & 84 = 2 \times 2 \times 3 \times 7 \\ \hline & \text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840 \end{array}$$

where LCM is the product of all the different prime factors of the given numbers, each taken the greatest number of times that it occurs in one of the numbers.

1.03 SIGNED NUMBERS

(1) Graphical Representation

(a) **Real numbers** may be represented by points on a straight line as shown in Fig. 1.03-1, where the distance between two adjacent points is constant and equals 1.

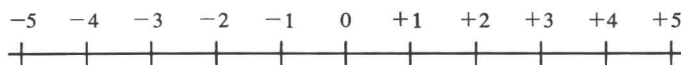


Fig. 1.03-1

(b) **Positive numbers** +1, +2, +3, ... are then associated with the points on the right side of the origin designated by 0 (zero).

(c) **Negative numbers** -1, -2, -3, ... are then associated with the points on the left side of the origin.

(d) **Positive and negative numbers** are called the *signed numbers*. Zero in arithmetic has no sign, and all unsigned numbers are assumed to be positive numbers.

(e) **Absolute value of a number** is its numerical value regardless of sign and is designated by two vertical lines surrounding the signed number.

examples:

$$|+5| = 5 \quad |-5| = 5 \quad |+5| = |-5|$$

which means the absolute value of a positive number equals the absolute value of the negative number and vice versa.