Advanced Engineering Physics

Anshan

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Anshan
ANSHAN LTD

Ane Books India

First Published in 2007 by

ANSHAN LTD
6 Newlands Road
Tunbridge Wells
Kent.
TN4 9AT. UK
Tel: +44 (0) 1892 557767
Fax: +44 (0) 1892 530358
e-mail: info@anshan.co.uk
Website: www.anshan.co.uk

Published in arrangement with
Ane Books India
4821, Parwana Bhawan 1st Floor
24 Ansari Road, Darya Ganj, New Delhi -110 002, India
Tel: 91 (011) 2327 6843-44, 2324 6385
Fax: 91 (011) 2327 6863
e-mail: anebooks@vsnl.com
Website: www.anebooks.com

ISBN-10: 1 905740 03 4
ISBN-13: 978 1 905740 03 1

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

This edition not for sale in India, Pakistan, Nepal, Sri Lanka and Bangladesh

Printed at Gopsons Paper Ltd, Noida (U.P.), India

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To my parents,

Sheela and Govind

The Author is grateful to
Prof. R. Senani for his constant
encouragement and support and providing
the lab facilities. He is also grateful to
Vipin Vats for helping with the
typesetting work

PREFACE

This book is intended to serve as an advanced book for the course Engineering Physics as taught in engineering colleges all over the country and abroad at the undergraduate level as well as a reference book for applied mathematicians, theoretical physicists and electrical and mechanical engineers intending to apply signal processing techniques to problems of physics. The chapterwise description of the book is as follows:

Chapter I is about classical mechanics. The kinematical quantities are introduced and the description of their dynamical behaviour under the action of forces in accordance with the Newtonian laws of motion is formulated. The notion of work as defined by a line integral of the force field along a path is discussed. The Newtonian laws of motion are applied to a system of particles moving under their mutual interaction as well as under the influence of external forces is introduced. It is then explained how motion of a system of particles in a potential field leads to energy conservation and how in in the absence of external forces, the momentum of the system of particles is conserved. Motion under constraints is then introduced leading to the famous D'Alembert's equations of motion which can be taken as the starting point for Lagrange's variational principle in mechanics. While discussing constrained motion, we also consider the problem of a particle moving on a curved surface. Lagrange's variational principle involving the derivation of Newtonian laws by minimizing the action functional is then introduced and how this leads to the Hamilton equations of motion which is a system of coupled first order ordinary differential equations for the position and momentum variables. The Hamilton equations of motion can also be taken as a version of nonlinear state variable equations in system theory. Poisson bracket relations are then introduced explaining the fundamental bracket relations between the canonical coordinates and momenta. These relations prove to be the starting point for the formulation of the Heisenberg equations of motion in quantum mechanics as explained in Dirac's book. The motion of spinning top is then introduced as a prototype example of rigid body motion. The description of rigid body motion involves the used of the moment of inertia tensor and and the angular velocity tensor. These are discussed in detail. The description of a spinning top also involves the introduction of the Euler angles for parameterizing rotations and these ideas are introduced. Then we come to the last section, namely waves where vibrating string, vibrating membrane and interference of light is introduced. This section also contains some ideas about lasers, polarization of light and holography. A variety of study projects is introduced at the end. The projects deal with numerical integration of the Newtonian equations of motion, the Hamilton-Jacobi formulation of classical mechanics, and Lie group ideas in mechanics.

The second chapter is about fluid dynamics. We start with the equation of continuity which is about fluid mass conservation. Then, the Navier-Stokes equation for the fluid is derived. This is a field theoretic version of the Newtonian equations of motion taking into account not only the external forces like gravity but also internal pressure forces. The Navier-Stokes equation along with the equation of continuity provide a complete description of the fluid motion. The modification of the fluid equations of motion in the presence of viscous terms is then considered. Velocity potential for irrotational flows is also discussed and how the velocity potential can be used to describe all two dimensional flows. Use of the theory of functions of a complex variable to construct the velocity potential is then introduced. Bernoulli's equation which is the energy conservation equation for a fluid taking into account velocity potential and pressure is then discussed. This equation has important practical applications especially in aircraft lift problems. The diffusion equation for the vorticity is then derived. If thermal energy of a fluid is taken into account, then one can arrive at an energy conservation equation for the fluid. This is derived. The derivation of this equation combines ideas of fluid dynamics and thermodynamics. Examples of fluid flow problems are then considered like flow past a cylinder, flow past a sphere and gravity waves in an ocean. The study projects deal with boundary layers and magnetohydrodynamics which is the motion of a charged fluid in the presence of electric and magnetic field. This is particularly important in the analysis of electromagnetic wave propagation in the ionosphere. Magnetodydrodynamics involves a combination of fluid dynamics and the Maxwell field equations.

The third chapter is about electromagnetism. We start with the fundamental equations of electrostatics, namely Coulomb's law and Gauss' law, and then discuss some practical electrostatic field computation problems which includes a section on capacitance. Formulation of the equations of electrostatics in terms of Poisson's equation with a boundary condition is then considered and uniqueness of the solution proved. How the equations of electrostatics can be expressed in a more elegant form for dielectric materials using the polarization and displacement vector fields is then discussed. We then introduce the fundamental equations of magnetostatics, namely Ampere's law combined with the no monopole condition and using these two arrive at the Poisson's equation for the magnetic vector potential. The solution to these equations leads to the Biot-Savart law and this is discussed. Some practical applications of the Biot-Savart law to the magnetic field computation is then discussed. The field produced by a little bar magnet is considered and for dealing with properties of magnetic materials, we introduce the magnetization vector. Faraday's law of induction involving the generation of electric fields from time varying magnetic fields is considered and some applications of it are discussed. Next we discuss how Ampere's law violates the charge conservation principle for nonsteady currents and charge distributions and how it can be corrected by adding a displacement current correction term. The addition of this term leads to the complete set of Maxwell equations and this set predicts the existence of electromagnetic waves traveling at the speed of light. This is discussed in detail. The complete solution to the Maxwell equations in terms of retarded potentials is then discussed. This discussion is based on the Feynman lectures on physics, volume II. The study projects range over ideas like numerical integration of the wave equation, summarizing the basic equations of electrostatics, numerical computation of the electrostatic field from the charge distribution, iterative solution to the electric field when the medium has spatially varying permittivity, writing down the Maxwell equations in a curvilinear system of coordinates, summarizing all properties about electromagnetic potentials, transmission line theory involving writing down partial differential equations for the current and voltage along a transmission line, deriving the Snell's laws of reflection and refraction for an electromagnetic wave incident on the boundary of two dielectric media, formulation of the Maxwell field equations in the language of differential forms, propagation of electromagnetic waves inside a conductor, describing scattering of electromagnetic waves in a medium having spatio-temporally varying permittivity, applying finite element techniques to waveguide problems, computation of the power flow in an electromagnetic wave and many other problems.

The fourth chapter is about the special and general theories of relativity. The chapter begins by explaining how the Newtonian formulation of the laws of mechanics violates some of the basic principles of relativity. Michelson's experiment is then described explaning how it predicts the constancy of the velocity of light. The basic postulates of relativity by Einstein leading to the Lorentz transformation law replacing the Gailiean transformation law are described. The most general form of the Lorentz transformation as a product of spatial rotations and reflections and Lorentz boosts is written down. An important effect called the Thomas precession arising as one of the predictions of the Lorentz transformation equations is explained. This effect states that a particle moving along a curved trajectory appears to have its frame of reference rotated. This rotation is a purely relativistic effect. The phenomena of length contraction and time dilation starting from the Lorentz transformation equations is then explained. The modification of the Galilean law of velocity addition due to relativistic effects is also discussed. This modification guarantees that the velocity of no particle can exceed that of the speed of light. How relativity leads to the famous equation of Einstein $E = mc^2$ is then explained. This is shown to be a consequence of the variation of the mass of a particle with its velocity. The invariance of the Maxwell equations under Lorentz transformations is discussed in the light that Maxwell's equations are correct from a relativistic standpoint unlike the Newtonian equations of motion. The remaining part of the chapter discusses Einstein's celebrated general theory of relativity. We start this section by introducing the principle of equivalence which leads to the fact that the presence of a gravitational field modifies the metric of space time from Minkowskian flat space-time to Riemannian curved space-time. The motion of particles in a curved space time specified by a Riemannian metric is then derived. The basic elements of Riemannian geometry like parallel displacement, covariant derivatives, Riemann Christoffel curvature tensor, Ricci tensor and general tensor transformation laws are then introduced. The physical reason for developing tensor calculus in detail here is that the laws of physics according to the general principle of relativity must be invariant not merely under uniform motion but under any arbitrary transformation of the space-time coordinates. We then introduce the field equations of Einstein using the properties of the Ricci tensor and the energy momentum tensor of matter. These equations are second order highly nonlinear partial differential equations for the metric tensor and they prove to be a correction to the Newtonian inverse square law of gravitation. The Schwarzchild solution to the Einstein field equations for a spherical distribution of matter is then discussed and how this leads to the existence of blackholes from which light and particles cannot escape is described. The study projects develop Riemannian geometry in greater detail and also discuss the formulation of the equations of motion of particles in special relativity. Some projects also discuss general relativistic modifications of the Maxwell equations of electrodynamics which describe the interaction between gravity and electricity.

The fifth chapter is about quantum mechanics. Just as position and momentum are the fundamental kinematical quantities in classical mechanics, the wave function is the fundamental kinematical quantity in the quantum theory. The idea of a wave function for a system of particles and its probabilistic interpretation due to Max Born has been explained at the beginning. To describe more general quantum states, we need the concept of a density matrix or a density operator and this has been discussed illustrating how it can be used to describe a mixed state quantum mechanical system. Observables in quantum mechanics are Hermitian operators. This has been introduced stating how one can obtain the probability distribution for the values assumed by it in a given quantum state. The formula for the average value of an observable in a mixed quantum state has been derived. The fundamental Heisenberg uncertainty relation for observables that do not commute has been derived. This shows that two observables which do not commute cannot be simultaneously measured. The De-Broglie relation relating the momentum of a quantum particle to the wavelength of the matter wave associated with the quantum particle has been derived. The fundamental Planck relation for the frequency of a photon released when the quantum system makes a transition from a higher energy state to a lower one has been explained. Then we come to Bohr's model of the atom. Based on semiclassical arguments, Bohr was able to arrive at the correct energy spectrum of the Hydrogen atom. This argument is based on quantization of the angular momentum of the electron. Using the De-Broglie and Planck equations, one can arrive at the Schrodinger equation for a free particle and also for a particle moving in a potential field. In this equation, the kinetic energy appears as a second order partial differential operator in the spatial variables and the total energy appears as a first order temporal derivative. This equation was first derived by Schrodinger and is the starting point for modern quantum theory. This has been explained. How the Schrodinger equation can be used to obtain the energy levels of a free particle in a box, a harmonic oscillator and the Hydrogen atom has been explained. The solution to these problems are based on eigenvalue theory of second order differential equations. An alternate formulation to the Schrodinger wave equation is the Heisenberg equations of wave mechanics. These equations have been derived and their equivalence to the Schrodinger formalism has been explained. If the energy of a potential barrier exceeds the energy of a particle, then classical mechanics prevents the particle from crossing over the barrier. However quantum mechanics does not prevent such a cross over. This has been explained in the section on tunnelling. When one combines relativity with quantum mechanics, then one arrives at the Dirac relativistic wave equation for the electron. This has been derived. The wave function acquires four components. This has been derived. Spin of the electron arises as a relativistic effect in this equation. The study projects deal with numerical simulation of quantum systems, many electron and many nucleon problems in quantum mecbanics, angular momenta and their addition in the quantum theory, qubits, quantum computation and realization of quantum gates using physical systems, finite state quantum systems, Quantum motion of a charged particle in an electromagnetic field, the role of group theory in quantum mechanics, perturbation theory and simulation of quantum systems using the TMS DSP processor.

The sixth chapter is about circuit theory. The chapter begins with the description of the current-voltage relations for a resistor, capacitor and and inductor which form the basic elements of any linear circuit. These relations are developed starting from the field theoretic concepts such as flux, charge, electric field and current density. We next introduce the reader to the Kirchchoff voltage and current relations. These relations are respectively circuit theory versions of the equations of charge conservation and the fact that the line integral of the electric field around a closed loop is zero. The highlight of the chapter is the graph theoretic analysis of circuits. This technique is particularly suited to programming a computer for performing the analysis of a circuit. The graph theoretic formulation is based on expressing the Kirchchoff current relations using cutset matrices and the Kirchchoff voltage relations using fundamental loop matrices. Then follows the discussion on mutual inductance which involves the linking of magnetic field between two closely spaced coils. Different kinds of controlled sources are then introduced. These involve a circuit element along with a voltage or a current source when the voltage or the current generated by the source is proportional to the voltage or current across the element. Circuits with sinusoidal excitation are then discussed. The idea of representing the response of the circuit to a sinusoid in terms of the circuit transfer function is discussed here. Then comes the use of Laplace transforms in circuit analysis. The idea is first to represent the circuit dynamics by means of a system of coupled first order differential equations, ie, as a state variable system. Laplace transforming this system leads to a transfer function relationship between the input transform and the output transform. The initial conditions of the circuit also come into picture here. The solved problems of this chapter include computation of charge from charge density, computation of current from current density and vice versa, deriving current from charge, the dynamics of a time dependent capacitance, computing the magnetic energy stored in an inductor, two port networks involving controlled sources, writing down the circuit equations in the Laplace transform domain, computation of initial conditions in a circuit, transistor as a non-linear two port network. Some study projects are also included here. One of them is about performing a perturbation theoretic analysis of a diode resistor capacitor circuit. Another study project deals with the analysis of equipment in a basic electronics laboratory.

The seventh chapter is a small one on optics. The chapter begins with how Fourier analysis can be used to represent light of all frequencies. We then discuss the cylindrical wave equation with application to propagation of light inside an optical fibre. Geometrical optics is then considered where the Eikonal equation is derived for the phase of the light wave. The phenomenon of total internal reflection of light is explained. One of the study projects deals with writing down the Maxwell equations in the cylindrical coordinate system. This has applications to optical fibres, the other project deals with optical fibres whose permittivity is a function of the radial distance from the fibre axis.

The eighth chapter is also a small one on solid state physics. The quantum mechanical wave equation in a periodic Lattice is formulated. The idea of reciprocal lattice vectors is given. This has applications to representing periodic functions inside a crystal using Fourier series. Diffraction theory is then elucidated. This gives a relationship between the wavelength of an incident wave and that scattered by the crystal lattice in terms of the reciprocal lattice vectors. Applications of group theory to crystal structure is discussed. Crystals having permutational, rotational and reflectional symmetry are discussed in detail. The idea of a group representation is also mentioned. Vibrational waves propagating inside a crystal lattice are discussed. The Einstein-Debye theory of specific heat of a solid is also explained. The idea is to regard the solid as a collection of harmonic oscillators and to derive the specific heat from the partition function of the solid. Using the Fermi-Dirac statistics, we next derive an expression for the specific heat of a metal consisting of free electrons. The chapter includes just one study project, namely deriving the structure of some of the important matrix groups.

The ninth chapter is about basic digital signal processing. The chapter is an introduction to signal and system theory and has been included because of the importance of the methods developed by electrical engineers in the physical sciences. The chapter begins with signal classification where aspects such as discrete and continuous time, periodic and aperiodic signals, signal energy and power are introduced. Then the general theory of linear systems is developed. Examples of linear systems from electromagnetic field theory are given. The notion of time invariance is then introduced. Time invariant systems described by differential equations with constant coefficients is considered, the convolution sum and integral representation of linear time invariant systems is discussed, methods for solving linear differential equations with constant coefficients is considered, the sampling theorem for analog systems is also discussed. This theorem plays an important role in processing analog signals using digital techniques. Filters based on frequency selectivity are introduced next where the characterization of filters using the Fourier transform is discussed. The Fast Fourier transform for discrete time signals as an important digital signal processing tool is introduced next. Then we talk about random signals in discrete time and quantization noise in a digital filter. The latter plays an important role in analyzing the errors that propagate through a digital filter that uses delay elements and multipliers implemented using digital computers that have finite memory. Filter design is the next issue addressed. Then we talk about basic adaptive filter theory. This topic deals broadly with identifying the parameters of systems that vary slowly with time based on the most recent measurements. The subject of adaptive filters was initiated by Bernhard Widrow.

The tenth chapter is about probability theory. The basic tools required for probability are introduced here. These are probability spaces, distribution and density functions, moments of a random variable, characteristic function, independence of random variables and notion of random processes with examples. These are dealt with here. This chapter plays a fundamental role in applications involving problems where the signal waveforms fluctuate wildly with time and in problems where measurement errors prevent exact identifiability of system parameters. The most important application of random process theory is perhaps to the identification of a system by exciting all its degrees of freedom with random inputs and matching the moments of the output computed from the system dynamics with the estimated moments. The material discussed in this chapter will explain how this program can be carried out.

The eleventh and final chapter is one of the most important applications of probability theory to physics, namely statistical mechanics. This subject deals with systems that can occupy one of several states in accord with a probability distrbibution. The computation of thermodynamical quantities such as pressure, energy, entropy and enthalpy of a gas using the ideas of statistical mechanics is discussed here. All computations in statistical mechanics revolve around the partition function and the maximum entropy principle. These are discussed in this chapter. Some of the study projects of this chapter deal with the Boltzmann kinetic transport equation. This is the fundamental equation that governs the dynamics of a gas of ions.

Acknowledgements: The author would like to thank Mr.Vipin Vats for helping with the typsetting of the manuscript. He is also grateful to Professor Raj Senani for providing the facilities and congenial environment in the Department.

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