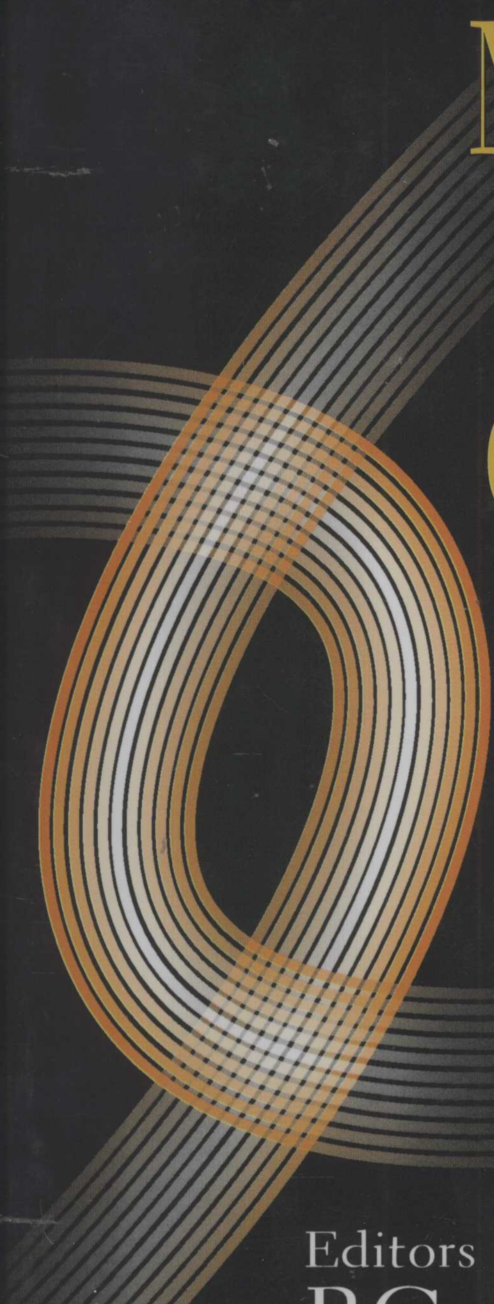
Mathematical Modelling, Optimization and their Applications



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P.C. Jha
M.N. Hoda

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P.C. Jha
Department of Operations Research
Faculty of Mathematical Sciences
University of Delhi
Delhi

M.N. Hoda
Director
Bhartiya Vidyapeeth
Institute of Computer Application and Management
New Delhi

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P.C. Jha

M.N. Hoda

Contents

1

Profit Maximization Policy for Manufacturers in a Supply Chain Management System

Madhu Jain, Vidushi Sharma and Kriti Priya
Department of Mathematics, Institute of Basic Science
Dr. B.R. Ambedkar University, Agra-282002, India

ABSTRACT

Outsourcing the product from the suppliers has become the strategic decision to help the firm meet customer's demand and also focus on their core competencies. In this paper we develop a pricing model, where in-house manufacturing and outsourcing processes occur simultaneously for different fractions of demand so as to increase the capacity. We have calculated the optimum quantity to be procured from outsourced supplier, optimum quantity to be manufactured in-house, retailer price, maximum profit of the supplier taking in consideration the price elasticity of the product.

Keywords: Supply chain management, Outsourcing, Shortages, Non-linear demand.

1. INTRODUCTION

Supply chain is centered around managing the flow of logistics and inventory of supply chain. Hence the key concern area of supply chain management in a manufacturing firm is coordination between the suppliers and the manufacturing organizations for the smooth functioning of the production activity while minimizing the cost and providing the right quantity of product at the right time. Meixell and Gargeya [2005] gave the global supply chain design of the firms. Supply chain network has become very important and is considered a dynamic source of business, managing the decision making at various levels. (cf. Hameer and Paatela, 2005). Li et al. [2006] studied the impact of supply chain management practices on the competitive advantage and performance of the organizations. Cost allocation strategies were studied by Gaballa [1974]. Major cost is involved in purchasing raw materials and holding the inventory at the manufacturer's premises. Kok [2000] studied the capacity allocation and outsourcing in a process industry. Lee et al. [2002] developed an advanced planning and scheduling model that requires due date specification with outsourcing within a manufacturing supply chain.

In this paper we have considered outsourcing decision, based on the manufacturing capacity of the manufacturer. Pricing and outsourcing are jointly considered and the conditions are specified under which the outsourcing makes sense.

2. MODEL DESCRIPTION

A simple supply chain model is considered in which the manufacturer is linked to a supplier and a retailer. The manufacturer meets the demand 'D' of the product by in-house manufacturing and/ or by outsourcing from the supplier. Manufacturer produces the product at a unit price 'c' and provides the product to the retailer at the price 'r' and in case the product is outsourced, he procures the product from the supplier at the price 'm' ($m > c$). It is assumed that 'f' fraction of the total demand is met by in-house manufacturing and the remaining $(1 - f)$ fraction of demand is met by the supplier. Also, with increasing value of f, the manufacturer would have to incur a cost of capacity enhancement which can be taken as zf^2 where z is the cost incurred by the manufacturer for producing increasing number of units in-house. The demand is assumed to be an exponential function of price i.e.

$$D = ae^{-br} \qquad \ldots (1)$$

where a is the scaling factor and b is price elasticity. To make the model more realistic, we have also considered the possibility of shortages. Let 's' be the shortage cost per unit of time per unit quantity and 'h' be the holding cost per unit of time per unit quantity. The total time period is divided into equal parts of interval 't'. Further this time interval 't' is divided into two parts 't_1' and 't_2' such that $t = t_1 + t_2$. During time t_1 the product is supplied to the retailer and during t_2, orders for the product are being accumulated but not filled. When the amount Q of the product is produced (or delivered form the supplier), it is divided into two parts Q_1 and Q_2 such that $Q = Q_1 + Q_2$. The quantity Q_1 goes in the inventory and the quantity Q_2 is immediately taken to satisfy the unfilled demand.

3. THE ANALYSIS

Total inventory holding cost = $h\left(\frac{1}{2}Q_1t_1\right)/t$ and annual shortage cost = $s\left(\frac{1}{2}Q_2t_2\right)/t$.

Also, $\frac{t_1}{t} = \frac{Q_1}{Q}$ and $\frac{t_2}{t} = \frac{Q_2}{2}$

Making use of the above relationships, we get the total inventory holding cost = $\frac{1}{2}h\frac{Q_1^2}{Q}$

and the annual shortage cost = $\frac{1}{2}s\frac{Q_2^2}{Q} = \frac{1}{2}s\frac{(Q - Q_1)^2}{Q}$

Also the annual set up cost = $C_s\frac{D}{Q}$, where C_s is the set up cost per order. Now, the man2ufacturer's profit function can be written as

$$P = (r - m)\,(1 - f)D + (r - c)fD - \left(\frac{hQ_1^2}{2Q}\right) - \left(\frac{s(Q - Q_1)^2}{2Q}\right) - C_s\frac{D}{Q} - zf^2$$

$$= (r - m)\,(1 - f)ae^{-br} + (r - c)fae^{-br} - \left(\frac{hQ_1^2}{2Q}\right) - \left(\frac{s(Q - Q_1)^2}{2Q}\right) - C_s\frac{ae^{-br}}{Q} = zf^2 \quad \text{... (2)}$$

For maximizing the profit, we partially differentiate eq. (2) with respect to f, Q, Q_1 and r and obtain the respective optimal values as

$$f^* = \frac{D(m - c)}{2z} \quad \text{... (3)}$$

$$Q^* = \sqrt{\frac{2C_s D(s + h)}{hs}} \quad \text{... (4)}$$

$$Q_1^* = \sqrt{\frac{2C_s Ds}{(h + s)h}} \quad \text{... (5)}$$

$$r^* = m + 1/b + f^*(c - m) + C_s/Q^*$$

$$= m + 1/b - \frac{ae^{-br}(m - c)^2}{2z} + C_s\sqrt{\frac{hs}{2C_s ae^{-br}(s + h)}} \quad \text{... (6)}$$

Also Q_2^* can be obtained as $Q_2^* = Q^* - Q_1^*$

4. NUMERICAL ILLUSTRATIONS

In this section we present numerical illustrations for the model presented in the previous section. For illustration purpose, we assume the scaling factor $a = 100$ and the set up cost $C_s = 10$. For computing the values of r^*, eq. (6) is solved by using the 'fsolve' function of Matlab 6. Table 1 depicts the effect of change in the manufacturing cost c on r, f, P, Q, Q_1 and Q_2. It can be noticed that as the manufacturing cost increases the price r at which the retailers get the product also increases. Also, the fraction f of the demand to be manufactured in-house decreases with the increase in the manufacturing cost The manufacturer can thus produce f fraction in-house and outsource the rest $(1 - f)$ fraction of the demand from the supplier. Table 1 further depicts the values of Q^*, Q_1^* and Q_2^*, which are again important decision variables for the production manager in the manufacturing concern. In table 2, the sensitivity analysis of various performance parameters is depicted for various values of m. The effect of m on the in-house production quantity fD and

Table 1 Effect of cost on model parameters

c	r*	D	f*	P	Q*	Q_1*	Q_2*
2	59.51	30.42	0.10	1500.39	35.06	34.71	0.3471
4	59.85	30.21	0.07	1495.24	34.93	34.59	0.3459
6	60.09	30.06	0.04	1491.61	34.85	34.51	0.3451
8	60.24	29.98	0.02	1489.45	34.80	34.46	0.3446
10	60.29	29.95	0	1488.72	34.78	34.44	0.3444

Table 2 Effect of m on model parameters

m	r^*	D	f^*	f^*d	$(1-f)D$	P
4	54.27	33.78	0	0	33.78	1679.67
6	56.22	32.48	0.03	0.84	31.64	1614.29
8	58.08	31.30	0.05	1.57	29.73	1552.96
10	59.85	30.21	0.07	2.19	28.02	1495.24
12	61.54	29.20	0.09	2.73	26.47	1440.77

outsource production quantity $(1-f)D$ can be seen. It is clear that with the increasing values of m, fD increases and $(1-f)D$ decreases which signifies that if the outsourcing cost m is higher then the manufacturer should fulfill the demand more by in-house manufacturing rather than by outsourcing. Table 3 depicts the effect of price sensitivity index or price elasticity b. We can observe that for the product with higher price sensitivity, the demand decreases drastically. Fig. 1 presents the effect of the shortage cost on the optimal quantity kept for the fulfillment of back orders for

Table 3 Effect of b on model parameters

b	r^*	D	P
0.01	108.05	33.94	3381.31
0.03	41.44	28.84	950.37
0.05	28.16	24.46	479.53
0.07	22.50	20.70	287.22
0.09	19.38	17.49	186.73

different values of holding cost. As the value of s increases Q_2^* decreases, which means that lesser quantity is kept aside for backorder fulfillment so as to optimize the profitability of manufacturer.

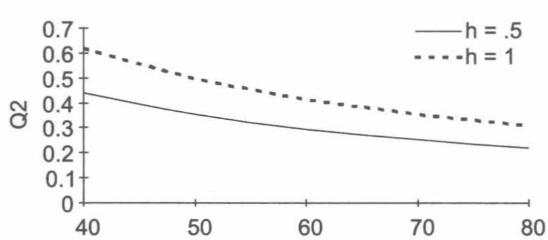

Fig. 1 Effect on $Q2$ with varying s for $h = .5$ and $h = 1$

Fig. 2 depicts that as the capacity enhancement cost z increases then f decreases resulting into decrease in the in-house production. Fig. 3 exhibits the effect of capacity enhancement cost z on the manufacturer's profit P for different values of c. Clearly, P shows a decreasing trend with z and c both which is because increase in z and c increases the overall cost of the manufacturer and hence decreases the profit.

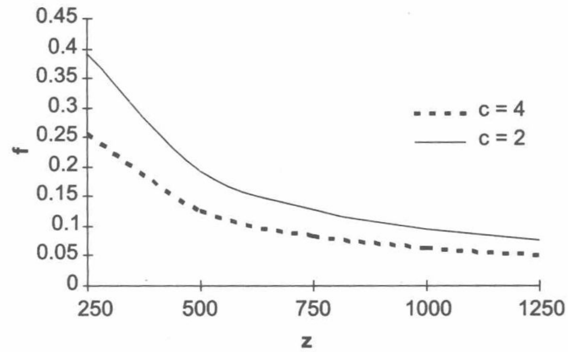

Fig. 2 Effect on P with varying values of z with $c = 4$ and $c = 2$

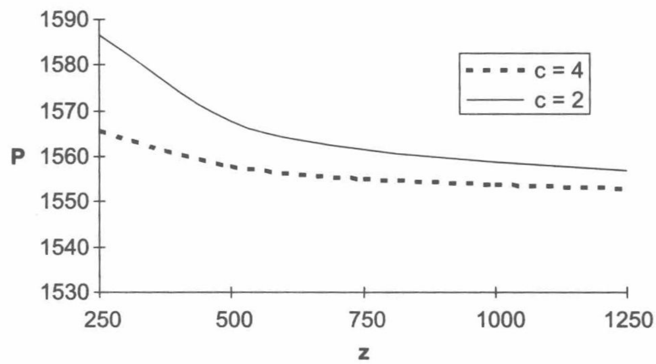

Fig. 3 Effect on f with varying values of z with $c = 4$ and $c = 2$

5. CONCLUSION

In this paper we have considered a model of supply chain management in which the outsourcing decision is also considered along with in-house production to meet the demand and optimize the profit of the manufacturer keeping in mind the capacity enhancement cost. Numerical illustrations reveal that the model is quite realistic and can help manufacturing industries to take strategic decision on the quantity to be outsourced and manufactured in-house so as to maximize their profit. This model is also beneficial for the production manager and the marketing manager to take strategic decisions on the optimum production lot size and to set up the end price of the product in the market by developing the retailer price.

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2

Optimal Release Policy for a Discrete Flexible Model Incorporating the Effect of Fault Removal Efficiency under Fuzzy Environment

Ompal Singh, Indumati and P.K. Kapur

Department of Operational Research, Faculty of Mathematical Sciences, University of Delhi.

ABSTRACT

Recent advances in software technologies have promoted the growth of computer-related applications to a great extent. As software systems have become more and more complex, the importance of effective, well planned testing has increased many folds. An important issue to the software developers related to testing is to determine the optimal time at which software testing is stopped and released to the users known as "Software Release Time Optimization Problems". Classical release time optimization problems require well defined and precise definition of various objectives, constraints set by the management and activity constant coefficients. However due to intense competition in the global market, varying requirements of the client, rapid evolution of information technology, conflicting evidence, ambiguous interpretations, poor data base, soft information, the management can only make ambiguous estimates on the available resources and their requirements bringing uncertainty (fuzziness) in the problem formulation. Further the model constraints depends on many non-deterministic factors such as testing strategies, skills and efficiency of testing team bringing uncertainty in their estimates. Fuzzy models offer the opportunity to define subjective imaginations of the decision makers/ computations of the model constants as precisely as the decision maker is able to describe. In this paper, we have formulated a release time problem minimizing the total software development testing cost subject to reliability constraint under fuzzy environment and discussed the fuzzy optimization technique to solve the problem. A discrete software reliability growth model (SRGM) incorporating the effect of fault removal efficiency during software fault removal phenomenon with logistic fault removal rate is used to describe the failure phenomenon of the testing process. Results are illustrated numerically.

Keywords: Release time, Software reliability growth model (SRGM), fuzzy optimization, fuzzy goal decision, uncertainty, cost, membership function.

1. INTRODUCTION

Quality and reliability measures are requisite quantitative parameters of estimating the performance of software to ensure its successful operation for which it is designed and built for. Software reliability measurements are essential to produce reliable and quality software efficiently and effectively. But due to human imperfection it is not possible to produce bug-free software. These bugs or faults in the software manifest themselves in terms of failures when the software runs. Testing phase in the software development process aims at detecting and removing these faults and making the software more reliable when the software is run and making the software more reliable. In particular, based on software error data analysis, it is very important to evaluate software reliability during the testing phase. Several software reliability models have been developed to describe a software error detection phenomenon during the testing phase and to measure software reliability. Models concerned with the relationship between the cumulative number of errors detected by software testing (or the time interval between software failures) and the time span of software testing is called Software Reliability Growth Model (SRGM). These models enable us to estimate software reliability measures such as mean initial error content, mean time interval between failures at any arbitrary time and the software reliability/intensity function. Most of these models use calendar time or CPU time as the unit of software error detection/removal period. However, the number of test runs (cases) can be a more appropriate unit of software fault detection/ removal period. Such an SRGM is called a discrete SRGM and relates the number of faults detected/removed to the number of test runs (cases) during the testing phase. A test case can be a single computer test run executed in an hour, day, week or even month. Therefore, it includes the computer test run and length of time spent to visually inspect the software source code. Most of these SRGMs are NHPP based and utilize historical failure data collected during the testing phase to evaluate the quality of the software. The models which use the test cases as a unit of fault detection\removal period are called discrete time models, since the unit of software fault removal period is countable.

The utility of discrete SRGMs cannot be under estimated in spite of difficulties in terms of mathematical complexity involved. Discrete models are proposed regularly, as the software failure data sets are discrete, and these models many times provide better fit than their continuous time counterparts. Yamada et al. (1985, 1986) proposed exponential and modified exponential discrete SRGM assuming that the expected cumulative number of faults removed between the nth and the $(n + 1)$ th test cases is proportional to the number of faults remaining after the execution of the nth test run. They are also termed as homogeneous and non-homogeneous fault detection rate models. Kapur et al. (1992) proposed a delayed S-shaped SRGM in which the testing phase is assumed to have two different processes, namely, fault isolation and fault removal processes, the model is extended to the case when software contains several types of faults.

Most of the models proposed above assumed the fault removal process (debugging) to be perfect i.e., every detected fault is removed with certainty. This assumption, however, seems to be a bit unrealistic. Due to the analytical nature of testing phase, manpower is mainly involved and hence there is a possibility that the testing team is not able to remove the fault perfectly on the detection of the failure and the original fault is remained or replaced by another fault. While the first phenomenon is known as imperfect debugging, the second is called fault generation. Correspondingly discrete exponential and modified exponential SRGMs were proposed by Kapur et al. (1994a, 2006) incorporating the concept of imperfect debugging. We have used a discrete SRGM incorporating the effect of imperfect debugging and error generation with logistic fault removal rate proposed by Kapur et al. (2006) in this paper.

One of the important applications of software reliability models is to estimate quantitatively the software release time. However no software can be tested indefinitely in order to make it bug free owing to the prevailing paradox that software users requirement are conflicting with the developers. Software clients want faster deliveries; cheaper software as well as quality product whereas software developers want to minimize the development cost, maximize the profit margins and meet the competitive requirements. Hence an imperative decision problem is to determine when to stop testing and release the software as per client's requirements in the market. If the release of the software is unduly delayed, the software developer may suffer in terms of penalties and revenue loss, while a premature release may cost heavily in terms of fixes (removals) to be done after release, which consequently might harm developer's reputation. Release time problems have become a prime field of study for many eminent researchers [Okumoto and Goel (1980), Yamada et al (1987), Kapur et al (1989, 1993, 2009; Pham (1996)] and companies. In this paper we are determining the optimal release time of software launch in the market. The estimation of optimal software release time is an optimization problem, which can be analyzed by different criteria's. Optimization techniques such as method of calculus, Mathematical Programming etc. are adopted to solve these problems. Here we have opted for fuzzy optimization technique to carry out our analysis. The optimization problem of determining the optimal time of software release can be formulated based on goals set by the management in terms of cost, reliability and failure intensity etc. subject to the system constraints. Okumoto and Goel (1980) were the first one to discuss an unconstrained release time optimization problem with either cost minimization or reliability maximization objective. Yamada and Osaki (1987) discussed release time problems with cost minimization objective under reliability aspiration constraint and reliability maximization objective under cost constraint.

In the existing research related to the software release time decision it is assumed that all the parameters of the problem are known precisely. Various objectives and restrictions are set by the management and cost coefficients involved in the cost function are determined based on past experience and previous data base available. This makes it difficult for the management to provide precise values of the various cost coefficients and objectives to be met by the release time. Moreover due to changing customer specifications, lack of experience of testing team or novelty, changing testing environment, complexity in the project involved, emerging factors unknowable at the start of the project adds imprecision and ambiguity to above mentioned definitions. It may also be possible that the management itself doesn't set precise values in order to provide some tolerance on these parameters due to competitive considerations. All this leads to uncertainty (fuzziness) in the problem formulation. Crisp mathematical programming approaches provide no such mechanism to quantify these uncertainties. Fuzzy optimization approach introduced by Bellman and Zadeh (1973) is a flexible approach that permits a more adequate solution of such type of problems in the presence of vague information.

Rest of the paper is organized as follows: In section 2.1 we discussed the SRGM used to describe the functional relationship between failure phenomenon and time. In section 2.2 cost model used for the formulation are defined and then the problem is formulated in section 2.3. In section 3.1 we have discussed the basic concepts of fuzzy sets and present an algorithm to solve the