SIXTH EDITION

CALCULUS
WITH ANALYTIC GEOMETRY

DALE VARBERG
EDWIN J. PURCELL

Sixth Edition

CALCULUS

with Analytic Geometry

Dale Varberg
Hamline University

Edwin J. Purcell
University of Arizona

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To Jarod, Benjamin,
Aaron, Caleb, and Lauren

Preface

This sixth edition of *Calculus with Analytic Geometry* follows in the tradition of its predecessors. It is designed for the standard science and engineering calculus course offered at most U.S. universities and colleges. It seeks to be correct without being overly rigorous; it aims to be up-to-date without being faddish, and it claims to be written in a style that makes mathematics palatable even to students who are afraid of the subject. All the major features of previous editions have been retained: chapter opening biographies, carefully organized problem sets, a conceptual geometric emphasis, chapter review problem sets, marginal boxes with cautions and asides, a formula card, and tables. But there is much that is new too.

A lean and lively calculus. Regardless of how we view the specific recommendations of the many groups calling for a revamping of the standard calculus course, we must agree that change is needed. First, the course and the textbooks cover too much material. In making this revision, we were determined to reduce the number of topics. But because a survey of users showed little agreement on what could be left out, we have eliminated only one topic—fluid force. Although our book is leaner than most calculus books, we urge instructors to make their course even leaner than our book. Select topics; don't try to cover everything.

To encourage students to read the text and to reinforce our *conceptual emphasis*, we begin every problem set with four fill-in-the-blank items. These test mastery of the basic vocabulary, understanding of the theorems, and ability to apply the concepts in the simplest settings. A student who has read the lesson should be able to fill in these blanks quickly. We think students should respond to these items before proceeding to the later problems. We encourage this by giving immediate feedback; the correct answers are given at the end of the problem set.

Number sense distinguishes the mature mathematics student from the neophyte. All calculus students make numerical mistakes in solving problems, but the one with number sense recognizes an absurd answer and reworks the problem. To encourage and develop this important ability, we have emphasized a process we call *estimation* (introduced in Section 1.2). We suggest how to make mental estimates, how to arrive at ballpark numerical answers to questions. We do this ourselves in the text in many places, and we propose that students do this, especially in problems marked with the symbol [≈].

Use of computers. Perhaps the most significant new feature in this revision is the inclusion of a host of *computer problems.* We do not ask students to write computer programs. Rather we encourage students to use one of the commercially available calculus programs in exploring the concepts of calculus. We do this by adding computer problems to about half of the problem sets. They are always at the ends of these sets (so they can be ignored) and they are always identified with the symbol [PC]. We have worked hard to make these problems meaningful, exploratory (sometimes leading to conjectures), and reasonable. Our model has been *True BASIC Calculus* for one-variable calculus and *True BASIC MacFuntion* for multivariate calculus. These packages are inexpensive, are remarkably easy to use, and match our book exceptionally well. We have worked all the computer problems in the text using these packages, but instructors will have other favorites that can be used as well. A good share of these problems can even be worked on hand-held calculators such as the HP28S. Incidentally, we include solutions to only a few of the computer problems (since we think some instructors may want to have solutions handed in and, anyway, giving answers would spoil much of the fun); however, solutions are given in the Instructor's Manual. For those instructors who believe that computers with their wonderful graphical capabilities can enliven and enhance understanding of calculus, we suggest that computer problems be substituted for some of the more traditional calculus exercises. Other instructors may simply ignore the computer problems. After all, calculus has been taught and taught well without computers for 300 years.

We continue to label many problems with the symbol [C] to indicate that a simple scientific calculator will be useful in solving these problems.

Pedagical concern. Our 35 years of teaching calculus suggest *pacing* is very important. Our goal was to prepare sections of about equal length (1 day's lesson). But to help students over difficult hurdles, we have spread out certain concepts. For example, the introduction of the derivative and the Chain Rule are each stretched into two lessons and vectors are first introduced in two dimensions and later in three dimensions. This concern for pacing is evident in carefully constructed problem sets that gradually lead the student from routine exercises to challenging applied problems.

Our conceptual emphasis means that definitions should be given in a *consistent* way. This implies that concepts for one-variable calculus should generalize naturally to the many variable case. Note how we achieve this for the concept of limit (Sections 2.5, 13.4, and 15.3), derivative (Sections 3.2, 13.4, and 15.4), and definite integral (Sections 5.5, 16.1, 17.2, and 17.5).

Since linear algebra is now a standard course for scientists and engineers, we think the terminology of calculus should be consistent with that subject. Thus in our book, linearity is emphasized as an important idea, and vectors are written as n-tuples as well as in ijk-form.

We have been greatly concerned about *readability*. A developmental editor was employed to go over every line, making sure that concepts are clearly explained and that derivations are sufficiently detailed. To make the book more visually appealing and easier to understand, many diagrams have been added and they are now in four colors.

Supplements.
For instructors:

- *Instructor's Edition: Calculus with Analytic Geometry*, 6th Ed.
- *Instructor's Solutions Manual.* Worked-out solutions to all exercises in the text.
- *Test Item File*
- *Transparency Pack*
- $5\frac{1}{4}''$ IBM *Testmanager* and $3\frac{1}{2}''$ IBM *Testmanager*. Allows the instructor to access questions from the computerized *Test Item File* and personally prepare and print out tests. Includes an editing feature which allows questions to be added or changed. Demo software is available from College Software.
- *How to Teach Calculus Manual.* Guidelines on how to teach calculus that serve as a workshop for instructors prior to class starts-ups.
- *How to Teach Calculus Video.* Provides a 10-minute summary on teaching habits applied to mathematics and 10-minute panel discussion on teaching tips and pitfalls.

For students:

- *Student's Solutions Manual* (ISBN 0-13-118035-5). Worked-out solutions for every odd-numbered exercise in the text.
- *How to Study Calculus Booklet* (ISBN 0-13-435116-9). Contains strategies, suggestions, and hints for learning and achieving success in calculus.
- *A Calculus Companion: The Personal Computer* (ISBN 0-13-111337-2). Appropriate for self-paced courses, computer labs, and calculus courses that include coverage of computers.
- *A Calculus Companion: The Graphics Calculator* (ISBN 0-13-111345-3). This book demonstrates how programmable graphics-calculators can enhance a student's appreciation of calculus, and it enables students to graph functions.
- *Calculus Calculator Manual* (with disk) (ISBN 0-13-117441-X). A general purpose programmable graphing calculator for IBM-compatible PC's. It evaluates expressions, graphs curves and surfaces, solves equations, and integrates and differentiates functions. Students use *Calculus Calculator* to develop, explore, and test mathematical ideas.
- *Calculus Calculator Tutorial Program Pack* (ISBN 0-13-117862-8). Software disk with illustrations of computer problems in the text.
- *Interactive Experience in Calculus* (ISBN 0-13-040270-2) (MacIntosh, Wattenburg/Wattenburg). A fully-interactive software product for the MacIntosh computer. It features 20 interactive experiments that cover the major calculus topics and requires no computer literacy.
- *EPIC: Exploration Programs in Calculus.* Available as IBM $5\frac{1}{4}''$ and IBM $3\frac{1}{2}''$ floppy disks. Software is available from College Software.

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Dale Varberg

Edwin J. Purcell

CALCULUS
with Analytic Geometry

Calculus: Yesterday . . .

René Descartes
1596–1650

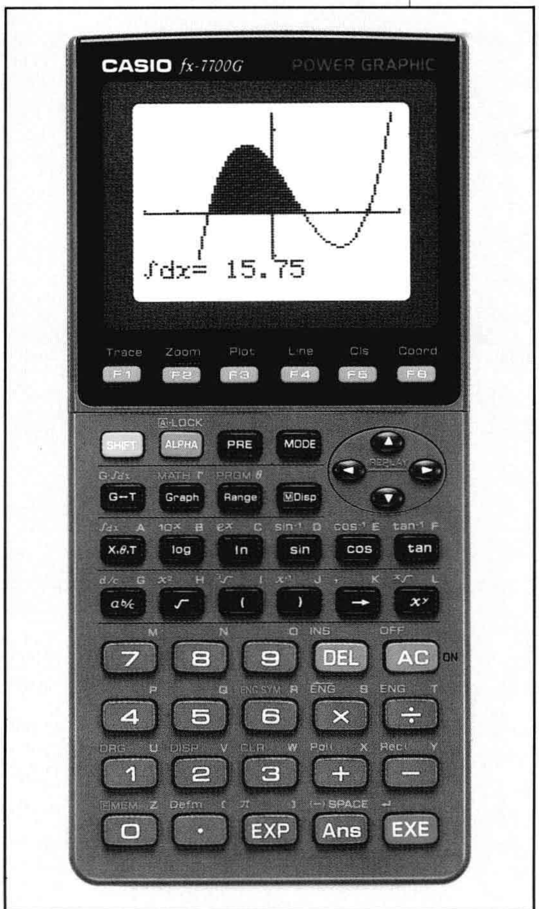
René Descartes is best known as the first great modern philosopher. He was also a founder of modern biology, a physicist, and a mathematician.

Born in Touraine, France, Descartes was the son of a moderately wealthy lawyer who sent him to a Jesuit school at the age of eight. Because of delicate health, Descartes was permitted to spend his mornings studying in bed, a practice he found so useful that he continued it throughout the rest of his life. At age 20, he obtained a law degree and thereafter lived the life of a gentleman, serving for a few years in the army and living at times in Paris, at others in the Netherlands. Invited to instruct Queen Christina, he went to Sweden in 1649, where he died of pneumonia that winter.

Descartes searched for a general method of thinking that would give coherence to knowledge and lead to truth in the sciences. The search led him to mathematics, which he concluded was the means of establishing truth in all fields. His most influential mathematical work was *La Gèomètrie*, published in 1637. In it, he attempted a unification of the ancient and venerable *geometry* with the still infant *algebra*. Together with another Frenchman, Pierre Fermat (1601–1665), he is credited with the union that we today call analytic geometry, or coordinate geometry. The full development of calculus could not have occurred without it. ■

. . . and Today

The idea of using coordinates to produce a picture (graph) of an equation is the fundamental principle exploited by the new graphing calculators.



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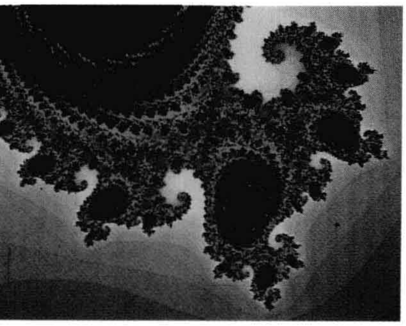

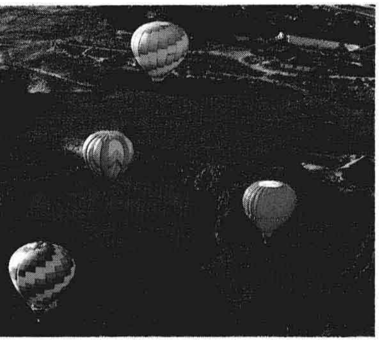

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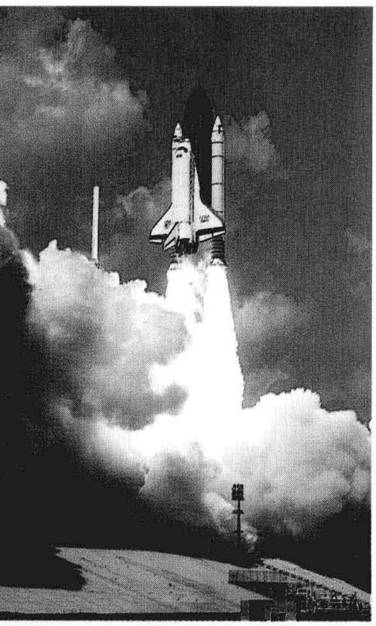

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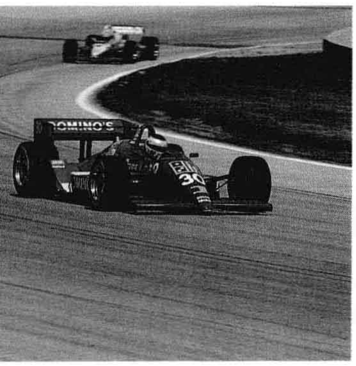

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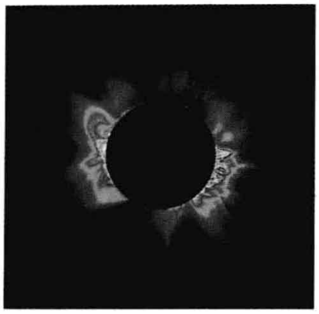

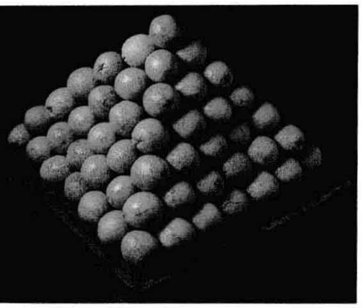

1

Preliminaries

1.1 THE REAL NUMBER SYSTEM

Calculus is based on the real number system and its properties. But what are the real numbers and what are their properties? To answer, we start with some simpler number systems.

The Integers and the Rational Numbers

The simplest numbers of all are the **natural numbers**,

$$1, 2, 3, 4, 5, 6, \ldots$$

With them we can *count*: our books, our friends, and our money. If we adjoin their negatives and zero, we obtain the **integers**:

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

When we try to *measure* length, weight, or voltage, the integers are inadequate. They are spaced too far apart to give sufficient precision. We are led to consider quotients (ratios) of integers (Figure 1), numbers such as

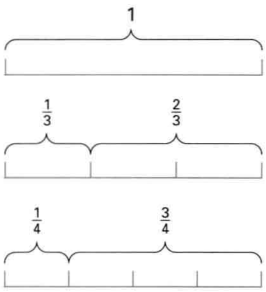

FIGURE 1

$$\frac{3}{4}, \frac{-7}{8}, \frac{21}{5}, \frac{19}{-2}, \frac{16}{2}, \quad \text{and} \quad \frac{-17}{1}$$

Note that we included $\frac{16}{2}$ and $\frac{-17}{1}$, though we would normally write them as 8 and -17, since they are equal to the latter by the ordinary meaning of division. We did not include $\frac{5}{0}$ or $\frac{-9}{0}$, since it is impossible to make sense out of these symbols (see Problem 36). In fact, let us agree once and for all to banish division by zero from this book (Figure 2). Numbers that can be written in the form m/n, where m and n are integers with $n \neq 0$, are called **rational numbers**.

FIGURE 2