Fifth Edition

ADVANCED MECHANICS OF MATERIALS

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PREFACE

This fifth edition represents a major revision of the fourth edition. However, as in previous editions, the blend of analysis, qualified approximations, and judgements based on practical experience is maintained. Each topic is developed from basic principles so that the applicability and limitations of the methods employed are clear. Introductory statements in each chapter serve as guidelines for the reader to the topics that are discussed. The topics are divided into three major parts: Part I—Fundamental Concepts; Part II—Classical Topics in Advanced Mechanics; and Part III—Selected Advanced Topics.

Part I, Chapters 1–5, includes topics from elasticity, plasticity, and energy methods that are important in the remainder of the book. In Chapter 1, the role and the limits of design are discussed. Basic concepts of one-dimensional load-stress, load-deflection, and stress-strain diagrams are introduced. A discussion of the tension test and associated material properties is presented, followed by an introduction to failure theories. These concepts are followed, in Chapter 2, by the theories of stress and strain, and by strain measurements (strain rosettes) and, in Chapter 3, by the theory of linear stress-strain-temperature relations. The discussion of anisotropic materials has been expanded, and example problems on orthotropic material behavior are given. Student problems for anisotropic materials are also included. Chapter 4 contains much new material related to inelastic (nonlinear) behavior and a broader treatment of yield criteria, including elastic-plastic behavior of beams, strain-hardening effects in bars, and residual stresses in elastic-plastic bars after unloading. The application of energy methods, Chapter 5, is expanded to include an in-depth discussion of the dummy-load method used by structural engineers and its relation to the Castigliano method. Additional worked examples and many new problems have been added. (In this edition, problems have been placed at the end of each Chapter, rather than at the end of each section.)

Part II, Chapters 6–12, treats some classical topics of advanced mechanics. Torsion is treated in Chapter 6, including new examples and problems. In addition, a finite difference solution of the rectangular cross section bar is presented. An example of limit analysis and residual stresses in a circular cross section shaft is also included. In Chapters 7 to 9, the three topics of unsymmetrical bending, shear center, and curved beams are examined on a rigorous basis, and limitations on existing analyses are indicated. A presentation of beams on elastic foundations, plus new problems and references, is given in Chapter 10. Some minor clarifications for the thick-wall cylinder and many new student problems are given in Chapter 11. In Chapter 12, the topic of stability of columns is expanded considerably, and a wide range of practical example problems and student exercises is included.

Part III, Chapters 13–19, presents the more advanced topics of flat plates, stress concentrations, fracture mechanics, fatigue, creep (time-dependent deformations), contact stresses, and the finite element method. The linear theory of flat plates is given in Chapter 13, including some illustrative problems and a collection of student exercises. The level is appropriate as an introduction for master-level students and for practicing engineers. Chapter 14 collects, in an integrated manner, material on stress concentrations previously presented in parts of Chapters 3, 12, and 13 of the fourth edition. New examples and exercise problems have been added, as well as some new charts of stress concentration factors for rectangular cross section beams. The topic of fracture mechanics is introduced in Chapter 15; it includes material previously given in Chapters 3 and 12 of the fourth edition and a brief discussion of other factors, such as elastic-plastic fracture, crack-growth analysis, load spectra and stress history, testing, and experimental data interpretation. A number of up-to-date books and papers are referenced. Progressive fracture (fatigue) is discussed in Chapter 16, including additional problems and references. An extended discussion of creep is presented in Chapter 17, including creep of metals and nonmetals (concrete, asphalt, and wood). Chapter 18, contact stresses, is essentially unchanged from Chapter 14 of the fourth edition. Chapter 19, the finite element method, is a completely rewritten treatment of Chapter 15 of the fourth edition. It includes discussions of the constant strain triangular element, the bilinear rectangular element, the linear isoparametric quadrilateral element, and the plane frame element. Example problems and exercise problems are included.

As a result of the new material and problems that have been added, this edition is larger than its predecessors. Consequently, it provides a greater choice of topics for study. It also has the advantage that the book can be used over a lifetime of practice, as a reference to topics of lasting importance in engineering. The book contains more material than can be covered in a one-quarter or a one-semester course. It is, however, with the proper selection of topics, suitable for a one-semester (one-quarter) course at either the senior level or the first-semester graduate level, for a two-semester (two- or three-quarter) course sequence, or as a reference work in several courses in mechanics.

The computer program listings in the fourth edition have been omitted from the current edition. However, revised versions of the programs from the fourth edition and new programs for applications in this edition are available on request from one of the authors (R. J. Schmidt, Department of Civil and Architectural Engineering, Box 3295, University of Wyoming, Laramie, WY 82071).

We thank Charity Robey, Wiley engineering editor, for her expert help and advice during the development of this edition. We also greatly appreciate the help of Suzanne Ingrao, with the difficult task of galley and page proof editing. We thank the reviewers of the preliminary format and content of the fifth edition for their constructive criticism and suggestions for improving the fourth edition. These reviewers are Stanley Chen, Arizona State University; Donald DaDeppo, University of Arizona; D. W. Haines, Manhattan College; Loren D. Lutes, Texas A. & M. University; Esmet M. Kamil, Pratt Institute; and Thomas A. Lenox, U.S. Military Academy, West Point, NY. We thank especially the reviewers of the draft manuscript for their helpful suggestions. These reviewers are J. A. M. Boulet, University of Tennessee-Knoxville; Ray W. James, Texas A. & M. University; A. P. Moser, Utah State University; William A. Nash, University of Massachusetts-Amherst; and Sam Y. Zamrik, Pennsylvania State University. We also acknowledge the contribution of Travis Finch for the artwork in Figs. 12.12 and 18.1.

Finally, we welcome comments, suggestions, questions, and corrections from the reader. They may be sent to Arthur P. Boresi, Department of Civil and Architectural Engineering, Box 3295, University of Wyoming, Laramie, WY 82071.

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PART I

FUNDAMENTAL CONCEPTS

In Part I of this book, Chapters 1 to 5, we introduce and develop fundamental topics that are important in the remainder of the book. In Chapter 1, we emphasize basic material properties and their use in design. Theories of stress and strain are presented in Chapter 2, and linear stress-strain-temperature relations are introduced in Chapter 3. Inelastic material behavior is discussed in Chapter 4 and finally, energy methods are treated in Chapter 5.

1

INTRODUCTION

In this chapter, we present general concepts and definitions that are fundamental to many of the topics discussed in this book. The chapter serves also as a brief guide and introduction to the remainder of the book. The reader may find it fruitful to refer to this chapter, from time to time, in conjunction with the study of topics in other chapters.

1.1

THE ROLE OF DESIGN

This book emphasizes the methods of mechanics of materials and applications to the analysis and design of components of structural/machine systems. As such, it is directed to aeronautical, civil, mechanical, and nuclear engineers, as well as to specialists in the field of theoretical and applied mechanics. As engineers, we are problem solvers. The problems that we solve encompass practically all fields of human activity. We solve problems related to buildings, transportation (including automotive, rail, water, air and outer-space travel), water systems (e.g., dams and pipelines), manufacturing, specialized medical equipment, communication systems, computers, hazardous wastes, etc. These problems are generally encountered in the design, manufacture, and construction of engineering systems. Ordinarily, these systems are not built or manufactured before the design process is completed. The design process usually involves the development of many drawings and/or CAD files to describe the final system. One of the major purposes of the design process is to analyze or evaluate various design alternatives before a final design is selected. One of the simplest objectives of the analysis is to ensure that all components of the system will fit together and function properly. More complicated analysis involves the evaluation of forces in the proposed design to ensure that each component of the system functions properly (for instance, safely withstands loads or does not undergo excessive displacements). This analysis is essential in the process of refining the design to meet required conditions such as adequate strength, minimum weight, and minimum cost of production.

The process of refining the design can be very complicated and extremely time-consuming. For example, consider the design of a space vehicle, such as the shuttle. After the shuttle's mission or use has been established, the designer must decide on the shape of the vehicle and the materials to be used. The designer must analyze the vehicle's structure to determine if it is strong and stiff enough to withstand the aerodynamic and thermal loads to which it will be subjected. The designer must analyze the skin and individual component parts of the structure to determine how these loads will be carried and safely transmitted from part to part. This first analysis usually reveals evidence that a redesign of some members in the structure may provide a more efficient and safer distribution of load and perhaps a more cost-effective design. Unfortunately, the designer may also discover that improvements in one part of the system may require changes in another part and possible problems in still other parts. Thus, the designer may be faced with one or more iterations between analysis and design to ensure that the entire system will function properly. This type of iteration is a common feature of design (Cross, 1989; de Neufville, 1990).

Considerations other than resistance to and transfer of loads, such as those of form or appearance, cost, ease of manufacturing, time constraints, etc., may influence or even control the design. Indeed, these factors may not only govern the design of an individual component but also may have a strong influence on the design of a more general engineering system, such as an office building. However, considerations of this kind are secondary to the topics treated in this book.

The term *design* as used in this book is not limited to the detailed calculations required to determine the proper dimensions of a member; rather, this term is used in a broader sense that emphasizes the relation of the methods of mechanics of materials to the concepts and philosophy of a rational design code or specification. In particular, emphasis is placed on the development of equations, formulas, or methods by which detailed analyses can be performed. Thus, this text provides an analytical foundation that is fundamental to the design process. Readers interested in the general concepts and methods of design may refer to the books by Cross (1989) and de Neufville (1990).

1.2 TOPICS TREATED IN THIS BOOK

This book is intended for advanced undergraduate and graduate engineering students, as well as practicing engineers. The topics treated are separated into three groups: Part I, Fundamental Concepts; Part II, Classical Topics in Advanced Mechanics of Materials; Part III, Selected Advanced Topics. Part I treats general concepts that pertain to the entire book, theories of stress and strain, linear stress-strain-temperature relations, yield criteria for multiaxial stress states, and energy methods. These topics are intended to be read sequentially, more or less. However, depending on the background of the reader, some of these topics may be bypassed. Part II presents several chapters on classical applications of the methods of mechanics of materials, namely, torsion, nonsymmetrical bending of beams, shear center for thin-wall beam cross sections, curved beams, beams on elastic foundations, thick-wall cylinders, and buckling of columns. These chapters may be treated in any order, except that the chapter on shear centers should be studied after the chapters on torsion and nonsymmetrical bending of beams. Part III introduces chapters on selected advanced topics, namely, flat plates, stress concentration factors, contact stresses, fracture mechanics, high cycle fatigue, time-dependent deformation/creep, and finite element methods. Each of these chapters may be treated independently, more or less.