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semigroup theory and applications

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Semigroup Theory and Applications

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Preface

The meeting Trends in Semigroup Theory and Applications was held September 28 to October 2, 1987, at the Dipartimento di Scienze Matematiche of the University of Trieste, Trieste, Italy. The conference was devoted in particular to maximal regularity problems, interpolation spaces, multiplicative perturbations of generators, linear and nonlinear evolution equations, integrodifferential equations, dual semigroups, positive semigroups, applications to control theory, and boundary value problems.

The Organizing Committee wishes to thank all the institutions that made this meeting possible, in particular, the Consiglio Nazionale delle Ricerche and the Ministero della Pubblica Istruzione (funds MP140/MP160) of Italy, the Dipartimento di Scienze Matematiche of the University of Trieste, and the Regione Autonoma Friuli-Venezia Giulia. In addition, the Organizing Committee gratefully acknowledges the assistance and support of Professors Lucio Delcaro, Carlo Pagani, Luciano de Simon, and Giovanni Torelli.

Furthermore, with special thanks, we acknowledge the generous contribution to the administration of the meeting offered by the staff of the Dipartimento di Scienze Matematiche of Trieste, in particular, by Michela Fasanella and Giacomo Ambrosi, and by its Head, Professor Aljoso Volcic.

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Semigroup Theory and Applications

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Zygmund Classes with Boundary Conditions as Interpolation Spaces

PAOLO ACQUISTAPACE* Scuola Normale Superiore, Pisa, Italy

0. INTRODUCTION

In this paper we are concerned with the characterization of real interpolation spaces $(D_A,E)_{\alpha,\infty}$ (Lions-Peetre, 1964) and $(D_A,E)_\alpha$ (Da Prato-Grisvard, 1979), where A is an elliptic differential operator of order 2m, with general boundary conditions, and E is the Banach space of continuous functions on a bounded open set $\Omega \subset \mathbb{R}^n$; following Grisvard (1969), we denote such spaces by $D_A(\theta,\infty)$ and $D_A(\theta)$, respectively, where $\theta = 1 - \alpha$.

In an earlier paper (Acquistapace-Terreni, 1987) we studied the case $2m\theta \notin \mathbb{N}$; the purpose here is to study the "critical" cases $2m\theta = q \in \mathbb{N}$. All notations here are the same as in Acquistapace-Terreni (1987).

1. ASSUMPTIONS

Let Ω be a bounded open set of $\mathbb{R}^n$, $n \geq 1$, with C^{2m} boundary, $m \geq 1$. We introduce the differential operators

$$A(x,D) := \sum_{|\alpha|\leq 2m} a_\alpha(x)D^\alpha, \qquad x \in \bar{\Omega} \tag{1.1}$$

$$B_j(x,D) := \sum_{|\beta|\leq m_j} b_{j\beta}(x)D^\beta, \qquad x \in \partial\Omega, \quad j = 1, \ldots, m \tag{1.2}$$

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under the following assumptions:

$$a_\alpha \in C(\bar{\Omega},\mathbb{C}),\quad |\alpha| \le 2m; \qquad b_{j\beta} \in C^{2m-m_j}(\partial\Omega,\mathbb{C}),\quad |\beta| \le m_j, \tag{1.3}$$
$j = 1, \ldots, m$

(ellipticity) There exist $\eta \in [0,2\pi[$, $\nu > 0$ such that (1.4)

$$\nu(|\xi|^{2m} + t^{2m}) \le \left| \sum_{|\alpha|=2m} a_\alpha(x)\xi^\alpha - (-1)^m e^{i\eta} t^{2m} \right|, \quad \forall x \in \bar{\Omega},$$
$\forall \xi \in \mathbb{R}^n$, $\forall t \in \mathbb{R}$

(root condition) If $x \in \partial\Omega$, $\xi \in \mathbb{R}^n$, $t \in \mathbb{R}$, and $|\xi| + |t| > 0$, $(\xi|\nu(x)) = 0$, then the polynomial (1.5)

$$\zeta \mapsto \sum_{|\alpha|=2m} a_\alpha(x)[\xi + \zeta\nu(x)]^\alpha - (-1)^m e^{i\eta} t^{2m}$$

has exactly m roots $\zeta_j^+(x,\xi,t)$ with positive imaginary part.

(complementing condition) If $x \in \partial\Omega$, $\xi \in \mathbb{R}^n$, $t \in \mathbb{R}$, and $|\xi| + |t| > 0$, $(\xi|\nu(x)) = 0$, then the m polynomials (1.6)

$$\zeta \mapsto \sum_{|\beta|=m_j} b_{j\beta}(x)[\xi + \zeta\nu(x)]^\beta$$

are linearly independent modulo the polynomial

$$\zeta \mapsto \prod_{j=1}^{m} [\zeta - \zeta_j^+(x,\xi,t)].$$

$$0 \le m_j \le m_i \le 2m - 1 \qquad \text{if } 1 \le j < i \le m. \tag{1.7}$$

REMARK 1.1 Condition (1.7) replaces the normality condition assumed in Acquistapace-Terreni (1987).

Indeed, it is easily seen that the transversality condition [(1.9) of that paper], that is,

$$\sum_{|\beta|=m_j} b_{j\beta}(x)\nu(x)^\beta \ne 0, \qquad x \in \partial\Omega,\quad j = 1, \ldots, m,$$

is implied by the complementing condition (1.6); thus the only difference here with respect to Acquistapace-Terreni (1987) is that we just require the orders of the boundary operators to be less than 2m, without forcing them to be different from one another.