Elementary Mechanics of Solids

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ELEMENTARY MECHANICS OF SOLIDS

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Preface

The book deals with the fundamentals required for a first undergraduate course in the subject. It covers the course contents usually given in the second year of a four-year degree course on mechanics of solids/strength of materials. It does not deal with advanced topics, but penetrates deep into the fundamentals. It is lucidly written and is suitable for Indian and many foreign universities. There is no pre-requisite for the course except that some knowledge of mathematics and elementary statics is required. The book contains ten chapters; it has about 500 problems, out of which 90 are fully solved. Answers to all the problems are given. All the problems are in SI units. By an extensive use of footnotes, an attempt has been made to securely link the subject matter presented in the text to more advanced studies of mechanics to be undertaken by the reader later. A brief outline of the book may be given as follows.

We begin by introducing certain fundamental concepts of the subject in Chapter 1. More emphasis has been given at this stage on the physical concept of quantities such as stress and strain rather than their mathematical concept. These may provide a clear insight into the physical phenomenon through an imaginative approach and broad outlook.

In Chapter 2, we discuss the determination of stresses and strains in simple cases of loading. We start with the case of a bar in tension or compression. The concept of uni-axial loading and deformation is then gradually extended to bi-axial and tri-axial cases. Throughout this chapter, we have put emphasis on the process of constructing idealized simple mathematical models of real physical situations. In short, we have devoted this chapter to make the beginners realize as to how a logical extension of the concepts of mechanics helps us to understand the mechanical behaviour of structural members and leads us to develop rational rules for their design based on strength and stiffness.

We next present, in Chapter 3, the theory of torsion of a circular shaft. Physical arguments of symmetry and simplifying assumptions regarding the geometry of deformation have been discussed in detail. This helps in clearly understanding the problem of torsion. The concept of shear-flow due to torsion of thin-walled members has been introduced. A brief introduction to the torsion of non-circular shafts has also been given.

One-dimensional load-carrying members subjected primarily to bending are referred to as beams. They are frequently used in engineering structures, and analysis of their bending is of great importance to the designers. As such, we have devoted the next three chapters to the theory of bending of beams. In Chapter 4, after a brief description of various types of beams and their loading, we have discussed the variation of shearing force and bending moment along the length of the beam. Modern sign conventions for S.F. and B.M. have been used. The concept of singularity functions has been introduced to help set up a single equation for S.F. or B.M., valid for the entire length of the beam under arbitrary loading. We have made an attempt to prescribe methods for drawing the S.F. and B.M. diagrams accurately and speedily. In Chapter 5, we have discussed stress distribution due to bending over a cross-section of the beam. Starting with the simple case of pure bending of a symmetrical beam, we have then considered the case of transverse bending and the resulting shear stress distribution has been found out. A more generalized flexural theory for unsymmetrical beams has also been given. The chapter concludes with an approximate determination of normal stress distribution for a beam with a large initial curvature. Chapter 6 deals with the slope and deflection of the centroidal axis of a beam due to bending. Double integration method using singularity functions (sometimes also called Macaulay's method) has been discussed in greater length. However, other methods have also been discussed. Use has also been made of the deflection characteristics in the solution of statically indeterminate beams such as fixed beams, continuous beams, propped cantilevers, etc.

In Chapter 7, we examine the state of stress and strain at a point in an elastic and isotropic solid. In Part "A" of the Chapter, we have restricted ourselves to the state of plane stress and plane strain. Transformation laws for the two-dimensional stress and strain tensors have been established. These laws have been given in index notation as well, which could be easily extended to general three-dimensional cases by more serious readers. Mohr's graphical representation has been explained for a quick evaluation of stress or strain components. In view of the practical importance, the analysis of data obtained by strain rosettes has also been considered. A brief discussion of a general three dimensional stress and strain analysis finds its place in Part "B" of the chapter.

Having dealt with the four basic types of loading, namely, tension (or compression), shear, torsion and bending separately in previous chapters, we now present in Chapter 8, cases of combination of these loadings. Superposition method has been used to determine the state of stress due to combined loading. For beginners, sometimes it may be difficult to locate the critical points i.e., the points of maximum stresses or strains, etc., in a s ystem under combined loading. As such we have laid emphasis on this aspect of practical interest by taking up several cases and locating the critical section or point carefully in each case. We have concluded this chapter with an introduction to various stress and strain theories of elastic failure.

There are occasions when a member may meet the requirements of strength and stiffness but it may lack stability. An example of such a member is a long column, i.e., a long slender member subjected to compression. There are many other cases where the question of stability predominates. However, in Chapter 9, we have concerned ourselves with elastic stability of columns only. Euler's method for determining the critical (buckling) oad and its validity and limitations have been discussed. Rankine's theory practical columns of intermediate length has also been given. Eccentrily loaded columns and beam-columns find their place commensurate the elementary level of this text. Energy method for determining the itical load has been postponed for the next chapter.

So far we have used Newtonian approach and dealt with vector or tensor fields for the solution of deformation problems. An alternative treatment of similar types of problems is through what is known as Lagrangian approach using scalar fields. In Chapter 10, we have used the latter approach based on elastic strain energy concept. First of all, we have described the methods of computing the elastic strain energy in a bar under various conditions of loading. These methods have then been extended to compute the strain energy per unit volume at any point in an elastic body under various stress fields. Next, we have given the statements and proofs of various energy theorems for linear as well as non-linear systems. These theorems provide a powerful method for determining displacement of any element of an elastic system subjected to any type of loading. It has been shown as to how a complex deformation problem becomes simpler by working with this method rather than the ones previously discussed. Many practical applications have been discussed to highlight the usefulness of the energy methods. Then we present Rayleigh's energy method for obtaining an approximate value of critical load for a column under various loadings and end conditions.

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P.N.S.
P.K.J.

List of Symbols and Abbreviations

A	Area, vector quantity, matrices
a	Radius, coefficient, length, acceleration
B	Matrices
b	Coefficient, width
C	Constant, couple, curvature
c	Coefficient
D	Diameter
d	Depth, diameter
E	Young's modulus of elasticity
e	Relative displacement
F	Force
f	Function
G	Shear modulus
g	Acceleration due to gravity
H	Height
h	Height
I	Stress invariant, moment of inertia
$\vec{i}, \vec{j}, \vec{k}$	Unit vectors
J	Strain invariants, shape factor of torsion, polar moment of inertia
K	Bulk modulus
k	Stiffness
L	Length
l	Length
l, m, n	Direction cosines
M	Bending moment
m	Mass
N	Revolutions per minute (r.p.m.)
n	Constant
$\vec{n}$	Unit outer normal vector
P	Power, arbitrary point, force
Q	Force, static moment of area
q	Shear flow, intensity of loading
R	Radius of curvature, reactive force
r	Radius

$\vec{r}$	Position vector
S	Surface
$\vec{S}$	Stress vector
s	Distance
T	Twisting moment, torque, absolute temperature
t	Time, width
$\vec{t}$	Unit tangent vector
U	Strain energy
u	Displacement
U^*	Complementary energy
V	Volume, potential energy
v	Displacement
W	Work, energy weight
w	Displacement, intensity of loading
X, Y, Z	Coordinate axes
x, y, z	Coordinates of a point
α	Angle, coefficient of thermal expansion, flexibility coefficient
β	Angle, coefficient
γ	Shear strain
Δ	Differential quantity, displacement
δ	Deflection (vertically downward), extension
ε	Strain (generally normal strain)
η	Coefficient of viscosity
θ	Angle, angle of twist
λ	Lame's constant, coefficient, direction cosines
μ	Lame's constant, coefficient of friction
ν	Poisson's ratio
ρ	Mass density, radius
$\sum$	Summation
σ	Normal stress
τ	Stress (generally shear stress)
ϕ	Angle, fictitious force
ψ	Angle
ω	Angular velocity
$\exp(x)$ or e^x	exponential function of x
$\ln x$	Logarithm of x to the base e
lim	Limit
∇	Del operator

$_S\oint$	Integral over surface area S
$(\dot{\ })$	$=\frac{\partial}{\partial t}(\ )$
$\doteq$	Approximately equal to
$\because$	Since
$\therefore$	Hence

B.M.	Bending moment
lat	Lateral
long	Longitudinal
max	Maximum
min	Minimum
r.p.m.	Revolutions per minute
S.F.	Shearing force
w.r.t.	With respect to

Contents

*Indicates that it is not intended for a beginner

1

Introduction and Fundamental Concepts

1.1 Introduction

Readers are familiar with Newtonian mechanics from their studies in physics. We have seen that mechanics is the science of forces and motion. Like all other branches of science, mechanics too, with its numerous engineering applications, has been rendering valuable service to the mankind since the very beginning of our civilization. These applications have been achieved through a proper blending of the principles of mechanics with certain postulates and assumptions based on experiments and experience. Rapid developments in almost all the branches of engineering have created a tremendous technical literature on mechanics. This vast field of knowledge, which is still going through a period of intensive development, goes by the popular name of Applied Mechanics.

Mechanics of Solids is a branch of applied mechanics that deals with the mechanical behaviour of deformable solids subjected to various types of external forces (loads).* It is a field of study that goes by a variety of names, such as Strength of Materials, Mechanics of Materials and Mechanics of Deformable Solids.** The solid bodies to be considered in this book will include various components of structures or machines in the form of bars, beams, shafts, columns, plates, shells, blocks, etc. Behaviour of these components under different conditions of loading is of great concern to the engineers of almost all disciplines for a reliable and successful design of any structure or machine. The study of mechanics of solids helps a design engineer to ensure that the structural members (or machine elements) meet the following requirements with the minimum expenditure of material.

(i) Each and every component should be able to resist, without failure, the external forces to which it is subjected under service conditions. In other words, it should possess adequate strength. This requirement

*When the word 'force' (or load) is used in a general sense, as at this place, it is supposed to include a couple also.

**Some authors, however, tend to make minor distinctions between some of these.

demands that it should be of proper dimensions (shape and size) and be made of appropriate material.

(ii) Every component should be able to resist deformation (change in shape or size or both) under loading conditions. Deformations, beyond certain specified limits (not necessarily large), may cause functional damage, i.e. the component may fail to function satisfactorily because of, say, mismatch between two mating components. This requires that the component should possess adequate stiffness. Like strength, stiffness too depends upon the dimensions and material of the component. However, proper strength does not necessarily ensure proper stiffness. A component possessing high strength may have poor stiffness and vice versa. The geometry of a component is actually decided by the limiting requirements of strength and stiffness.

(iii) Some of the components may be required to withstand dynamic loads. They should, therefore, be capable of absorbing energy within certain limits, without any damage. Technically speaking, they should possess adequate toughness.* In general, toughness depends upon a suitable combination of strength and stiffness.

(iv) Every component should have a tendency to retain, under load, its original state of equilibrium. This is called the condition of stability. Lack of stability may cause an abrupt change in the shape of a part and the character of its deformation. Instability may occur at magnitudes of loads that are quite safe from the viewpoint of strength or stiffness. For example, a long thin member subjected to axial compression may become unstable; it would buckle if the load exceeds a certain limit, even though, this may be below the limiting load from strength or stiffness consideration.

Thus, we see that the principal objective of mechanics of solids is to provide us with rational rules for the design of any structural member or machine element based on the considerations of strength, stiffness, toughness and stability. In view of this broad objective, we may realize that the scope of our subject is quite vast and the applications are very wide. In this elementary book on the subject, however, we shall confine ourselves primarily to structural members of simple geometric form possessing a considerable degree of stiffness under simple loading conditions. Limitations and the scope of applications will be more evident as we proceed with the study of our subject. In addition to the relative simplicity of the methods discussed in this text, they are very useful as they apply to a large number of technically important problems.

1.2 Generalized Procedure

In mechanics of solids, a general procedure of analysing a system consists of the following steps:

*Energy absorbed (per unit volume) up to fracture, in a uniaxial test, is generally taken to be the measure of toughness of a material. However, the energy absorbed (per unit volume) up to the elastic limit is called "the modulus of resilience".

Step 1. *Setting up of an idealized model of the system*

First of all, the system of interest is selected. It is then isolated from its surroundings and its characteristics are idealized. The system so obtained is an idealized model. It should be simple enough to analyse and yet exhibit the physical phenomenon of the real system.

Any of the following may be selected and isolated as the system of interest depending upon the degree of sophistication and nature of the problem:

(a) The entire structure or machine,
(b) Sub-assembly of the structure or machine,
(c) A single component of the structure or machine,
(d) An infinitesimal element within a component.

Idealization of the system is achieved through simplification of the characteristics of the real situation. For this purpose, a distinction is clearly made between the essential and non-essential features of the actual system. Non-essential features are then neglected in the analysis. Essential features are further simplified. Some of the simplifying procedures are as follows:

(a) The geometrical form of the real system may be approximated to some simple form, e.g. rods, plates, shells, blocks, etc.

(b) Some basic assumptions are made regarding the properties of the material. These assumptions are common to all the problems dealt within this book. They are discussed in Art. 1.3.

(c) A few more assumptions are made which are appropriate only to a particular problem.

It may be realized at this stage that, in view of a large number of assumptions and simplifications, the methods of mechanics of solids are simple and approximate, and are, therefore, of more practical importance. A rigorous treatment of similar types of problems often with the help of cumbersome mathematical apparatus is generally done in *The Mathematical Theory of Elasticity*.

Step 2. *Analysis of the forces on the idealized model*

This step involves the analysis of forces, either external or internal. External forces arise due to interaction of the system with its surroundings. These forces and their general classification have been discussed in Art. 1.4. Internal forces come into play within a system to resist the action of the external forces on it. A brief account of how these forces arise has been given in Art. 1.5. The method of sections, which reveals the internal forces, has been discussed in Art. 1.6. The concept of stresses, which are the measure of the distribution of internal forces, has been introduced Art. 1.7.

In the general analysis of forces, equilibrium equations of statics are used. Cases where statics alone may fail to analyse the forces are called 'statically indeterminate.' It may be mentioned here that determination of stresses is inherently a statically indeterminate problem. In order to analyse