



**Brockett &
Levine**

**Statistics and
Probability
and Their Applications**

STATISTICS & PROBABILITY & THEIR APPLICATIONS

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Preface

A noncalculus statistics text of the kind that is useful to both students and practitioners in a wide variety of fields faces a number of stringent demands. The traditional material (random variables, estimation, hypothesis testing, regression, correlation, and so on) must be covered together with the appropriate notation so that a student who wishes to continue along either the mathematical road or any other route to learning statistics is in no way penalized. Moreover, the material must be written to take into account the considerable differences between readers with respect to their facility with basic algebra.

In addition, two other demands are increasingly being made by those departments that desire their students to obtain a basic foundation in statistics. First, they insist that their students *understand* the material. The applications of statistics studied by undergraduates in their special fields have become sufficiently profound that a “cookbook” course no longer suffices as background. The other demand concerns the *coverage* of the material. Generally, textbooks emphasize the analysis of data. The design of the data collection process, the checking, editing, and reduction of data generally merit little attention. By not being exposed to these methodologies, the student obtains a distorted view of statistical activities. As a result, there is an increasing demand that nonsuperficial discussions of sampling techniques and sampling instruments (such as questionnaires) be included in a first course.

We have attempted to satisfy the conditions noted above. A careful attempt has been made to explain notation and conventions clearly; we have included discussions of “data collection” techniques, including questionnaire design, survey and sampling design, experimental design, and so on, to provide a balance of coverage not available in texts of this level. The area of “data collection” is a large one; we have attempted to give the student insight into those techniques used both in scientific experiments and in surveys.

In addition, we have taken pains to include a great deal of material of importance to practitioners. We present examples and problems taken from actual situations in agronomy, business, biology, economics and finance, education,

engineering, epidemiology, law, medical research, social science, physics, and psychology. The inclusion of numerous applications should give the reader a strong sense of “applied statistics” and through this sense a better understanding of statistical theory.

We have also sought to increase this understanding by using the concept of the “mathematical model” to present efficiently the basic theory. This concept facilitates both the discussion of probability and the models underlying the statistical analyses.

The text has been designed to give the instructor considerable latitude in the organization of course material. Some independent pathways are plotted in the course outlines that follow. One can begin, for example, with probability, the preference of many instructors, rather than with the “presentation of data.” The instructor, the one most knowledgeable about the strengths and weaknesses of the students, may find none of these particular suggestions applicable. There is sufficient material to derive many other course sequences.

ORGANIZATION OF COURSES

The following are suggestions for course outlines. However, the instructor may have a time schedule or a need for emphasis totally different from that described below. For example, one of the authors had to present a two-week course for quality-control engineers at an aircraft plant in Los Angeles; this course had to be integrated with other two-week courses in physics and engineering given by different instructors. In this case, none of the outlines below were directly applicable, yet the material selected for this short statistics course was gleaned from the material in this text.

I. One-Year Course (2 semesters)

- A. Emphasis on statistics, with light coverage of probability.
 - 1. First semester: Chapters 1 to 9. Omit sections 3.8, 4.8 to 4.11, 4.14 to 4.16. Assign the easiest problems in Chapters 3, 4, and 5. If 4.15 is omitted, then sections 7.7 and 8.6 should be omitted also.
 - 2. Second semester: Chapters 10 to 15. Note that Chapters 8, 10, and 11 have sections that discuss data collection in scientific experiments whereas Chapters 13 and 14 deal mostly with survey sampling. The instructor can select those topics that are of interest to the type of student taking the course since these topics are discussed independently of each other.
- B. Emphasis on probability.
 - 1. First semester: Chapters 3 to 7. Chapter 2 may be included if desired. Explain all models including those in the problem sets of Chapters 3 and 4.

2. Second semester: Chapters 7 to 13. Sections 13.3 to 13.6 may be omitted if the instructor wishes to have time to include sections of interest from Chapters 14 and 15.
- C. A course intermediate between A and B.
1. First semester: Chapters 3 to 8 omitting those same sections listed in (A) but including the more difficult problems from the problem sets in Chapters 3 and 4.
 2. Second semester: Chapters 9 to 15, omitting 13.3 to 13.6.
- II. One-Semester Course
- A. Emphasis on statistics. Light coverage of Chapters 3 and 4 followed by 5 to 9 and 11. Only the rules of probability, conditional probability, independence, and expectation should be covered in Chapters 3 and 4 with the assignment of the easiest problems. Omit 7.4 to 7.7, 8.2; either 8.4 or 8.5, or both; 8.6; 9.5; and Chapter 11, except 11.1 and 11.2.
- B. Emphasis on probability. A one-semester course can be given from Chapters 3 to 5 alone if one goes deeply into the problem sets and all topics.

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C H A P T E R - 1

The Nature and Purposes of Statistics

“Let us sit on this log at the roadside, says I, and forget the inhumanity and the ribaldry of the poets. It is in the glorious columns of ascertained facts and legalized measures that beauty is to be found. In this very log that we sit upon, Mrs. Sampson, says I, is statistics more wonderful than any poem.”

O. Henry
The Handbook of Hymen

Statistical information plays an important part in a wide range of fields, many of which intimately affect our lives. In this chapter we study two main areas in statistics: descriptive and inferential statistics. We also explore two concepts basic to all our subsequent discussions: the “population” and the “sample.”