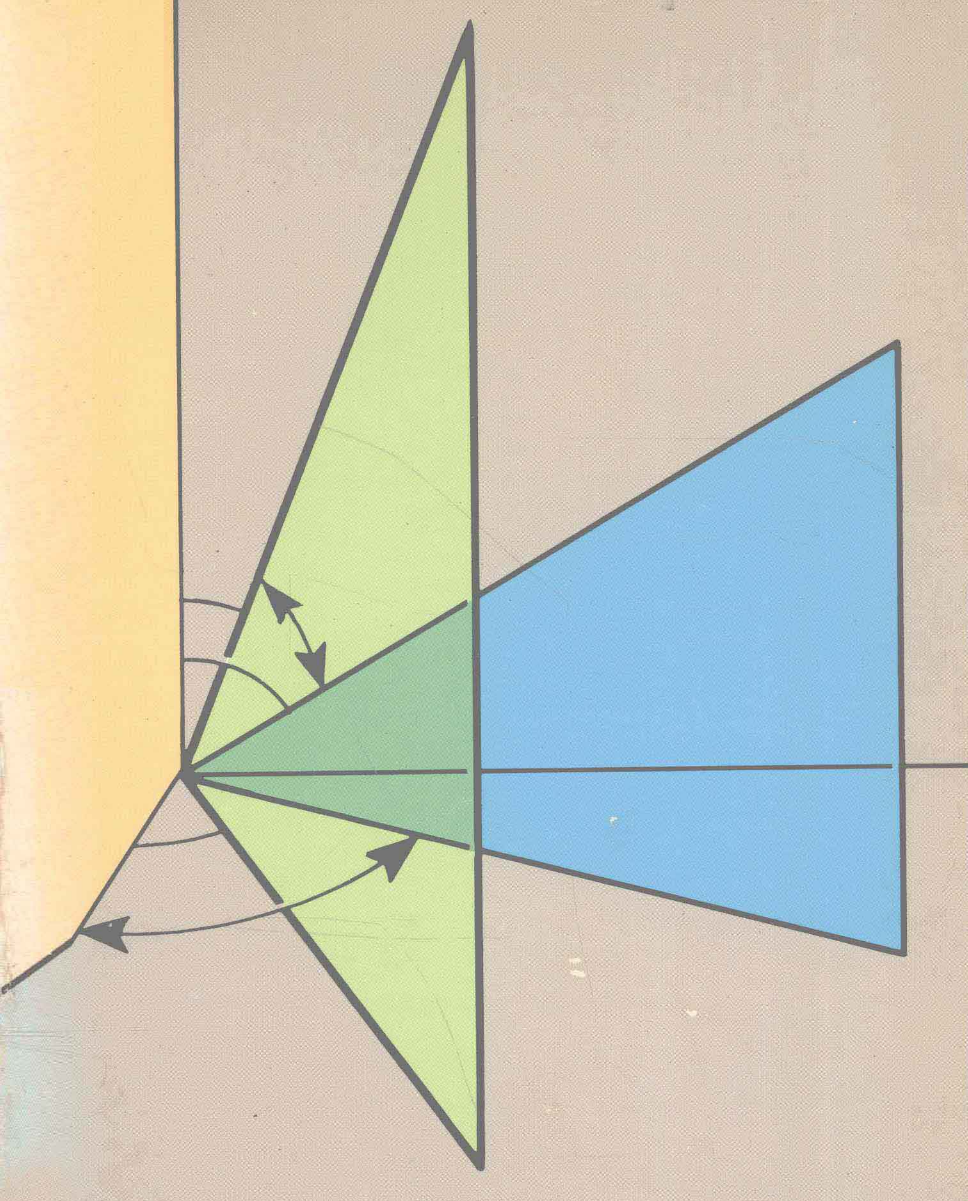

MATHEMATICAL ANALYSIS OF PHYSICAL PROBLEMS

Philip R. Wallace

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McGill University

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To my wife Jean, for her patience and understanding.

And to the memory of two men who brought both dignity and excitement to the study of physics: Leopold Infeld and Georges Placzek.

PREFACE

"After having spent years trying to be accurate, we must spend as many more in discovering when and how to be inaccurate"

Ambrose Bierce

In writing a book on mathematical physics, the author is from the beginning faced with the fact that he has not chosen a uniquely defined subject. The task would have been easier in the nineteenth century, when physics was somewhat more stable and mathematics and physics lived in closer association with each other. Today, the words "mathematical physics" have, by fairly general consent, in North America at least, taken on a special and rather conventional meaning, though there is still some fuzziness at the boundaries with other areas. As exemplified by the American Physical Society's journal of that name, the phrase signifies the investigation of mathematical techniques current in physics. The emphasis is on methodology rather than on physical content. In other places and other times, however, the separation between form and content is, and has been, less marked.

It is important, first, to understand the difference in purpose of a book such as this and a book on mathematics *per se*. Modern mathematics is primarily concerned with axiomatic systems and formal deductions made within these systems. In this sense, it consists of games: The axiomatic system defines the rules of the game, and one then plays according to these rules. The rules need not have any obvious relation to the world in which we live.

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The physicist, on the other hand, is concerned with understanding and describing the physical aspects of the world in which he lives. It is this which determines and governs his "mathematics." For him mathematics is a language, a shorthand, for *describing* and coordinating his comprehension of that world. The mathematician is concerned with rigor and with the completeness and logical consistency of his systems. The "rules of the game" must be fully defined and adhered to. The criterion which the physicist applies to his "mathematics" is its conformity to nature. Mathematical symbols represent the magnitudes of physical quantities, and "mathematical" formulas express the relations between these quantities. His systems are, and can be, only as complete as his comprehension. As for consistency, it is assured by the correspondence between his mathematical models and physical reality; he is prepared to assume the internal consistency of nature! Thus, as has been pointed out by Landau (and many others), mathematical rigor has no relevance to physics.

This must be the major feature distinguishing a book on mathematics from one on mathematical physics. The one proceeds axiomatically, governed by the requirements of rigor. The other tries to construct workable (and necessarily approximate and incomplete) models of aspects of physical reality. To attempt to judge either by the standards of the other is therefore inappropriate and mistaken.

When, in the present book, mathematical structures are described (as in chapter 2 on Linear Vector Spaces), they should be considered as the description of a common framework within which theories can be constructed of a variety of different sorts of physical phenomenon. The important questions are, whether the mathematical framework serves as an effective description of the physics, and whether it provides us with answers to physical problems which can be tested by physical observation. No greater mathematical generality is relevant than is required to describe the phenomena with which we are concerned.

Given this general definition of our aims, a few words should be said about the particular material which we have chosen to include in this book. Clearly, any book of this sort could not be "complete" without being encyclopaedic; it must represent an arbitrary selection of topics for discussion. Our selection has been chosen to link classical and modern physics through common techniques and concepts. The first chapter is on vibrating strings, in other words, on problems of one-dimensional wave propagation. Aside from recognizing the wide importance of wave phenomena in physics, this subject provides us with a testing ground for a wide range of concepts and methods with wider relevance, and does so in a simple and familiar physical context. Thus, when similar problems are met in newer and less familiar physical contexts, it will not be necessary to cope with technical difficulties as well as conceptual ones.

Chapter 2, on linear vector spaces, provides the thread which ties together most of the rest of the book. This is, of course, the basic conceptual framework of quantum mechanics; it also unifies our treatment of the problems of classical physics and provides us with some essential mathematical tools.

In chapter 3 we introduce the problems of potentials and the Laplace and Poisson differential equations. Since these are three-dimensional problems, we are led to introduce the method of separation of variables. An important feature is the introduction of spherical harmonics, which reappear later in other problems, including that of angular momentum in quantum mechanics. We have also tried, in this chapter, to illustrate how physical considerations can be used to provide the motivation for the development of mathematical techniques.

The fourth chapter is again primarily a "mathematical" one. It is concerned with the methods of Laplace and Fourier transforms and the relation between them. The methods are illustrated with a selection of useful examples.

Chapters 5 and 6 deal with important classes of problems in classical physics. Chapter 5 is concerned with problems of wave propagation in three dimensions, and chapter 6 with problems primarily of a diffusive character. Problems of the propagation of electromagnetic waves are considered at some length. Our intention is not to replace a more extensive and detailed treatment such as given in Jackson's book; chapter 5 *does* (along with chapter 3) provide a somewhat more streamlined account of some of the basic problems of the field.

Chapter 7 is concerned with probabilistic methods in physics. Some basic methods and concepts are introduced, and the basis is provided of a more fundamental description of some of the diffusive processes dealt with in chapter 6.

The final three chapters are devoted to a discussion of the fundamentals of quantum mechanics. Any treatment of methods of mathematical physics which was arbitrarily confined to "classical" physics would be artificial, as well as foregoing the pedagogical advantage of exploiting the technical similarities of many classical and quantum problems. Our emphasis is on useful techniques in quantum mechanics. In chapter 8 we deal with problems of a quite general nature, including a fairly extensive treatment of the time evolution of quantum systems, and of perturbation theory. In chapter 9 we deal with the mathematical theory of some standard problems: the hydrogen atom, the harmonic oscillator in one, two, and three dimensionals, and angular momentum theory. Extensive use is made of ladder (or creation and annihilation) operators, which provide the framework for the introduction of elementary field theoretical methods in the last chapter. Chapter 10 provides an introduction to many-body problems using the occupation number representation, and introduces a number of fundamental problems: the Hartree–Fock method, the electron gas, and the theory of density matrices and linear response.

As for omissions, perhaps the most striking is that of group theory. This is partly arbitrary, since any such book must be selective in its coverage; an additional justification is, however, that it is a difficult subject in which to give an adequate and self-contained treatment of modest length, while for longer and more extensive treatments, there have appeared in recent years a very large number of excellent books on group-theoretical methods in physics.

It is evident that, given the rapid evolution of physics curricula at the present time, and the great variety of ways in which the sort of mathematical physics incorporated in this book is distributed among course units in different universities, it is almost impossible to write a "text book" which will correspond simultaneously to the patterns of a very large number of universities. It seems to the author, however, that the very notion of a course textbook at this level is unrealistic, and that the advanced undergraduate or beginning graduate student should use a number of books in any course. My purpose is, therefore, to produce a generally useful book; one which will be a valuable addition to the personal library of students of physics. If it fits the needs of particular courses, so much the better.

A final word about problems. We have departed from the usual practice of providing collections of problems at the ends of chapters, but have instead interspersed them throughout the text. Some of these, in fact, form an integral part of the text, and will serve to test the student's grasp of what he has read, and his ability to extend it. Others are exercises in the techniques dealt with in the text. An effort has been made to avoid "problems for problems' sake." On the other hand, it is impossible to overemphasize the importance of the student "doing" for himself, and not merely learning theory by memory. I have tried, in many of the problems, to raise the sort of question which a good student might ask himself. If he *is* encouraged to ask further questions, and to seek their answers, their purpose will have been served.

I should like to express my thanks to Dr. Robert Heck for carefully checking the manuscript.

P. R. Wallace

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