

NEW TRENDS IN DYNAMIC SYSTEM THEORY AND ECONOMICS

EDITED BY

Masanao Aoki

Angelo Marzollo

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AND
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PREFACE

Modern developments in economic analysis are characterized by at least two features: A Walrasian-type equilibrium is no longer taken as the "normal" state of the economy; and the assumption of perfect information for economic agents is no longer made as a matter of course. Consequently, modern analysis is focused on behavior characterized by less than perfect information or nonidentical information for economic agents, such as search, trading out of equilibrium, diverse expectations, and so forth. Control and system theory can and has contributed to this general shift of emphasis of economic analysis by adding new techniques and concepts to aid dynamic analysis of an economy.

With the objectives of exploring the current and potential areas of interactions of control and system theory with economics and of fostering active exchange of ideas among people with different backgrounds, a two-week seminar was organized with the theme "New trends in dynamic system theory and economics." We record here some of the results of fruitful discussions among the participants of the seminar. Many topics were discussed, and we have selected some representative papers for inclusion in this volume.

The seminar was held at the International Center for Mechanical Sciences (CISM) at Udine, Italy, September 12–23, 1977. We take this opportunity to thank Dr. Vinicio Turello, the president of CISM, for his generous hospitality and the CISM staff for their able assistance. We are also indebted to UNESCO and the Italian Research Council for financial assistance. In particular, the former made possible the participation of several researchers from developing countries.

The theme of the seminar was purposely broad to permit interactions among people of different backgrounds. The wide range of interests of participants is reflected by the topics covered in the papers. It is difficult to neatly classify the papers into one or two categories. Some papers could have been classified in several ways. We have however attempted to give coherence to the organization of the papers and grouped them into three parts.

In Part I we present papers dealing with information patterns and uncertainty. Decision problems of agents with different or imperfect information or under uncertainty have been discussed by Başar, Aoki, Leitman and Wan, Liu and Sutinen, Auloge, and Rustem and Velupillai. In Part II, recent advances in optimal control theory and application of control theory are represented by Blaquiere, Haurie, Schiavoni *et al.*, Wiese, Rempala, Idzik, and Wieczorek. Contributions in Part III are on various aspects of disequilibrium analysis. Disequilibrium analysis of a mac-

roeconomic model is represented by Honkapohja. Disequilibrium analysis of a different sort (reflecting the influence of the approach to the analysis of dynamic processes and global analysis by Smale) is discussed in contributions by Cornet, Marzollo *et al.*, Fogelman *et al.*, St. Pierre, Galeotti, and Birchenhall. The last three papers may be said to be on "modern" approaches to tâtonnement processes.

We turn now to a brief summary of the papers. Başar gives a detailed analysis of the properties of Nash and Stackelberg equilibria in dynamic games under several different information patterns, with emphasis on the role of the information patterns. Aoki discusses schemes for performance improvements by agents with nonidentical information patterns, which are analogous to stochastic approximation schemes in a setting of interacting decision processes. In contrast to these two microdecision problems of individual agents, Leitman and Wan discuss macroeconomic stabilization of an uncertain dynamic economy. Auloge focuses on the uncertainty of parameter values and treats a more specific problem of estimating elements of the input-output matrix. Imperfect information is an important factor in the paper by Liu and Sutinen, who develop two models for the process of exploitation of exhaustible natural resources.

Impulsive control problems arise in diverse application areas such as differential games and inventory control problems. So far rigorous treatment of impulsive control problems has been lacking. Blaquiere develops necessary and sufficient conditions for optimal impulsive controls, and Haurie treats an infinite-horizon optimal control problem. Schiavoni *et al.* apply periodic control theory as developed by them to the decision problem of optimal maintenance of machinery in a firm, while Rempala discusses a two-commodity inventory model using dynamic programming. Idzik generalizes the Farkas lemma and applies it to a nonlinear von Neuman model to discuss existence of equilibria. Rustem and Velupillai describe an interactive bargaining process in which each side modifies its own cost functions as bargaining proceeds, based on limited information on the cost functions of the other side. Wiese treats games with coalitions and "diplomacy," a term he uses to denote evolution of coalitions with respect to the state of the game which is governed by a set of differential equations.

Wieczorek proves the existence of a competitive equilibrium of an exchange economy by reformulating it as a game.

Honkapohja extends the earlier works of Barro and Grossman, Malinvaud, and others on dynamics of disequilibria of a macroeconomic model with flexible wages and prices. Marzollo *et al.* use the notion of cone optimality, a generalization of Pareto optimality, to discuss the deep connection between the optimality and stability problems in a general setting. Galeotti and Gori build upon the works of Smale and Friedman and discuss stability and convergence of exchange processes with money. St. Pierre studies a multivalued evolution equation associated with a tâtonnement process. Cornet considers an abstract monotone planning procedure. Fogelman *et al.* treat dynamic aspects of the optimal taxation problem. Birchenhall discusses the Walrasian tâtonnement process as a gradient process, in the spirit of Smale.

CONTENTS

CONTRIBUTORS	ix
PREFACE	xi

Part I **Information and Uncertainty**

Information Structures and Equilibria in Dynamic Games Tamer Başar	3
Interaction among Economic Agents under Imperfect Information: An Example M. Aoki	57
Macro-Economic Stabilization Policy for an Uncertain Dynamic Economy G. Leitmann and H. Y. Wan, Jr.	105
Pareto Optimal Leasing and Investment Policies for a Publicly Owned Exhaustible Resource P. T. Liu and J. G. Sutinén	137
Identification of an Input-Output Leontief Model J. Y. Auloge	151
A New Approach to the Bargaining Problem Berc Rustem and Kumaraswamy Velupillai	167

Part II

Control Theory and Applications

Necessary and Sufficient Conditions for Optimal Strategies in Impulsive Control and Applications A. Blaquiere	183
Optimal Control on an Infinite Time Horizon with Applications to a Class of Economic Systems Alain Haurie	215
Optimal Periodic Maintenance of a Capital Good N. Schiavoni, G. Guardabassi, and C. Brasca	245
Broad Equilibria in N -Player Games K. E. Wiese	257
A Dynamic Programming Problem for a Two-Commodity Inventory Model Ryszarda Rempała	269
Farkas Lemma for Concave–Convex Functions with an Application to the Nonlinear von Neumann Model Adam Idzik	281
Equilibria in an Exchange Economy with Many Agents: A Game-Theoretic Method for Proving Existence Andrzej Wieczorek	291

Part III

Disequilibrium Analysis

On the Dynamics of Disequilibria in a Macro Model with Flexible Wages and Prices Seppo Honkapohja	303
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Monotone Planning Procedures and Accessibility of Pareto Optima Bernard Cornet	337
Differential Techniques for Cone Optimality and Stability A. Marzollo, A. Pascoletti and P. Serafini	351
A Stable Path to Optimal Taxation F. Fogelman, R. Guesnerie and M. Quinzii	365
Approximation of Solutions in Multivalued Evolution Equation Issued from Tâtonnement Process Patrick Saint-Pierre	377
Some Notes on a Dynamical Approach to Money Mediated Exchange Marcello Galeotti and Franco Gori	391
A Note on the Nature and Significance of Catastrophes in the Walrasian Tâtonnement C. R. Birchenhall	405

PART I

INFORMATION AND UNCERTAINTY

INFORMATION STRUCTURES AND
EQUILIBRIA IN DYNAMIC GAMES

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In this paper, we investigate the properties of Nash and Stackelberg equilibria in deterministic and stochastic dynamic games under several different information patterns. Emphasis is placed on deterministic and partially stochastic dynamic games and static stochastic games. In each case, first general results are given, and then they are applied on illustrative examples. A discussion of certain open but promising problems in this area is also included.

I. INTRODUCTION

In the modeling and determination of equilibria of several phenomena in economics, the nonzero-sum dynamic game theory plays a crucial role, and when seeking equilibria in a noncooperative atmosphere the Nash and the Stackelberg equilibrium concepts are mostly appropriate. In this paper, we discuss some of the important recent results found in the

literature in this area within a general framework, and also we present certain new results on the properties of Nash and Stackelberg equilibria in dynamic games.

In the next section, we first define what is meant by a (deterministic or stochastic) dynamic game under a general information pattern, and then introduce the Nash equilibrium solution concept and three Stackelberg equilibrium solution concepts. In section III, we deal mostly with deterministic dynamic games under perfect state observation, and first introduce and elaborate on the role "representation" of a strategy plays in the equilibria of such game problems. It is claimed that a purely deterministic formulation is not an appropriate modeling in many situations since, as it is verified in the paper, such a modeling results in rather ambiguous equilibria. In order to resolve this ambiguity, we propose a partially stochastic model, and we obtain necessary and sufficient conditions that Nash and Stackelberg equilibria of such dynamic games should satisfy. A two-stage dynamic game example illustrates several of the results obtained in this section.

In section IV, we first deal with static stochastic games with imperfect observations. We derive the necessary and sufficient conditions that Nash and Stackelberg equilibria should satisfy, and then specialize to the case of linear-quadratic-gaussian models to obtain explicit closed-form unique expressions for the equilibrium solution in each case. This section is concluded with a short discussion of some aspects of the equilibria of stochastic dynamic games under imperfect information and suggestions for future research. The paper ends with a conclusion section.

II. A GENERAL FORMULATION FOR DYNAMIC GAMES

In order to formulate a dynamic game properly in general mathematical terms and within a decision theoretic framework, the following information must be provided :

(i) The number of players.

(ii) A mathematical description of the interaction of the players within the system and among themselves, i.e. specification of the functional relationships (deterministic or stochastic) between the state of the game and the decisions of the players at every stage of the decision process.

(iii) Information structure for each player, which characterizes the precise information gained and recalled by that player at every stage of the game.

(iv) Decision (strategy value) spaces for each player.

(v) Permissible strategies (decision laws) for each player, defined as mappings from information spaces into decision spaces.

(vi) An objective functional for each player, that summarizes (mathematically) the preference ordering of each player among different alternatives and for each fixed permissible strategy of the remaining players.

(vii) A mutually consistent and rational equilibrium solution concept that takes into account (a) the relative influence (power) of each player on the joint decisions, (b) the permissible information exchanges among the players, and (c) the hierarchical structure of the

decision process.

A salient feature of dynamic games that is not shared in common with static games is that every player acts more than once and each time under an updated information concerning the state of the game and/or the past actions of the other players. In this paper, we will assume a finite number of players ($M \geq 2$) and a finite number of stages (N). We let X denote the state space of the game and U^m denote the decision space of the m 'th player. Both X and U^m will in general be subsets of appropriate finite dimensional spaces. If X has a finite number of points, then we say that the game is a *finite state game*, and otherwise it is an *infinite state game*.

At each stage n , a dynamic game is described mathematically through a "relation" f_n

$$X \times U^1 \times U^2 \times \dots \times U^M \xrightarrow{f_n} X \quad (1)$$

which maps the state of the game together with the decisions of the players at stage n into a permissible state at stage $n+1$. This relation is sometimes a real function for each $n=0, \dots, N-1$, and we then write it as

$$x_{n+1} = f_n(x_n, u_n^1, \dots, u_n^M), \quad (2)$$

where $x_n \in X$ ($x_{n+1} \in X$) is the state of the game at stage n ($n+1$), and $u_n^m \in U^m$ denotes the decision of player m at stage n . A dynamic game whose evolution is governed by such a relation (2) and whose starting (initial) state x_0 is known a priori will be called a *deterministic dynamic game*.

However, more often, the mathematical description (1) is such that f_n is a point-to-set mapping, and transition to any state or region under any particular relation f_n is governed by a probability law. In such a

situation, (2) is replaced by

$$x_{n+1} = F_n(x_n, u_n^1, \dots, u_n^M, \theta_n), \quad (3)$$

where θ_n is a random variable taking values in Θ , and it characterizes the uncertainty in the mathematical description, and F_n is again a real function :

$$X \times U^1 \times \dots \times U^M \times \Theta \xrightarrow{F_n} X. \quad (4)$$

Furthermore, the joint probability distribution function of

$\{x_0, \theta_0, \dots, \theta_{N-1}\}$ is usually specified. Such a dynamic game whose evolution is governed by the stochastic difference equation (3) will be called a *stochastic dynamic game*.

The *information structure* of a dynamic game characterizes, roughly speaking, what is known to each player concerning the present and/or past values of the state and/or past decision of the other players. Precise delineation of the information structure in the formulation of the problem is a very important issue in dynamic games since, as it will be verified in subsequent sections, the structure of equilibria in dynamic games is very sensitive to changes in the nature of the information available to each player. Let us denote by

$$y_n^m = h_n^m(x_n, \theta_n^m) \quad (5)$$

the observation of player m at stage n concerning the state of the game, x_n . Here h_n^m is a real-valued function and θ_n^m is a random variable taking values in Θ . If the probability distribution of θ_n^m is one-point, then this implies that player m makes a deterministic state observation at stage n ; moreover, if h_n^m is one-to-one and onto, then the corresponding observation is perfect.