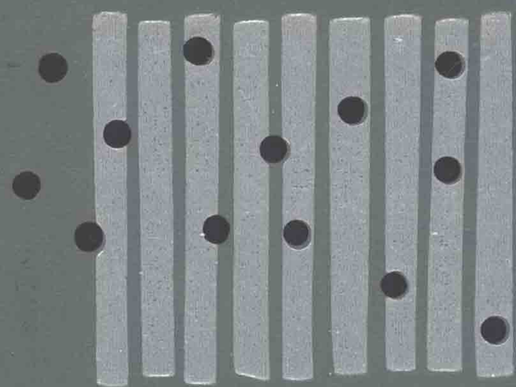
INTRODUCTION TO THE THEORY OF STATISTICS



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Introduction to the Theory of Statistics

THIRD EDITION

To HARRIET	A.M.M.
To my GRANDCHILDREN	F.A.G.
To JOAN, LISA, and KARIN	D.C.B.

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INTRODUCTION
TO THE THEORY
OF STATISTICS

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This book was set in Times Roman.
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PREFACE TO THE THIRD EDITION

The purpose of the third edition of this book is to give a sound and self-contained (in the sense that the necessary probability theory is included) introduction to classical or mainstream statistical theory. It is not a statistical-methods-cookbook, nor a compendium of statistical theories, nor is it a mathematics book. The book is intended to be a textbook, aimed for use in the traditional full-year upper-division undergraduate course in probability and statistics, or for use as a text in a course designed for first-year graduate students. The latter course is often a "service course," offered to a variety of disciplines.

No previous course in probability or statistics is needed in order to study the book. The mathematical preparation required is the conventional full-year calculus course which includes series expansion, multiple integration, and partial differentiation. Linear algebra is not required. An attempt has been made to talk to the reader. Also, we have retained the approach of presenting the theory with some connection to practical problems. The book is not mathematically rigorous. Proofs, and even exact statements of results, are often not given. Instead, we have tried to impart a "feel" for the theory.

The book is designed to be used in either the quarter system or the semester system. In a quarter system, Chaps. I through V could be covered in the first quarter, Chaps. VI through part of VIII the second quarter, and the rest of the book the third quarter. In a semester system, Chaps. I through VI could be covered the first semester and the remaining chapters the second semester. Chapter VI is a "bridging" chapter; it can be considered to be a part of "probability" or a part of "statistics." Several sections or subsections can be omitted without disrupting the continuity of presentation. For example, any of the following could be omitted: Subsec. 4.5 of Chap. II; Subsecs., 2.6, 3.5, 4.2, and 4.3 of Chap. III; Subsec. 5.3 of Chap. VI; Subsecs. 2.3, 3.4, 4.3 and Secs. 6 through 9 of Chap. VII; Secs. 5 and 6 of Chap. VIII; Secs. 6 and 7 of Chap. IX; and all or part of Chaps. X and XI. Subsection 5.3 of Chap VI on extreme-value theory is somewhat more difficult than the rest of that chapter. In Chap. VII, Subsec. 7.1 on Bayes estimation can be taught without Subsec. 3.4 on loss and risk functions but Subsec. 7.2 cannot. Parts of Sec. 8 of Chap. VII utilize matrix notation. The many problems are intended to be essential for learning the material in the book. Some of the more difficult problems have been starred.

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EXCERPTS FROM THE FIRST AND SECOND EDITION PREFACES

This book developed from a set of notes which I prepared in 1945. At that time there was no modern text available specifically designed for beginning students of mathematical statistics. Since then the situation has been relieved considerably, and had I known in advance what books were in the making it is likely that I should not have embarked on this volume. However, it seemed sufficiently different from other presentations to give prospective teachers and students a useful alternative choice.

The aforementioned notes were used as text material for three years at Iowa State College in a course offered to senior and first-year graduate students. The only prerequisite for the course was one year of calculus, and this requirement indicates the level of the book. (The calculus class at Iowa State met four hours per week and included good coverage of Taylor series, partial differentiation, and multiple integration.) No previous knowledge of statistics is assumed.

This is a statistics book, not a mathematics book, as any mathematician will readily see. Little mathematical rigor is to be found in the derivations simply because it would be boring and largely a waste of time at this level. Of course rigorous thinking is quite essential to good statistics, and I have been at some pains to make a show of rigor and to instill an appreciation for rigor by pointing out various pitfalls of loose arguments.

While this text is primarily concerned with the theory of statistics, full cognizance has been taken of those students who fear that a moment may be wasted in mathematical frivolity. All new subjects are supplied with a little scenery from practical affairs, and, more important, a serious effort has been made in the problems to illustrate the variety of ways in which the theory may be applied.

The problems are an essential part of the book. They range from simple numerical examples to theorems needed in subsequent chapters. They include important subjects which could easily take precedence over material in the text; the relegation of subjects to problems was based rather on the feasibility of such a procedure than on the priority of the subject. For example, the matter of correlation is dealt with almost entirely in the problems. It seemed to me inefficient to cover multivariate situations twice in detail, i.e., with the regression model and with the correlation model. The emphasis in the text proper is on the more general regression model.

The author of a textbook is indebted to practically everyone who has touched the field, and I here bow to all statisticians. However, in giving credit to contributors one must draw the line somewhere, and I have simplified matters by drawing it very high; only the most eminent contributors are mentioned in the book.

I am indebted to Catherine Thompson and Maxine Merrington, and to E. S. Pearson, editor of *Biometrika*, for permission to include Tables III and V, which are abridged versions of tables published in *Biometrika*. I am also indebted to Professors R. A. Fisher and Frank Yates, and to Messrs. Oliver and Boyd, Ltd., Edinburgh, for permission to reprint Table IV from their book "Statistical Tables for Use in Biological, Agricultural and Medical Research."

Since the first edition of this book was published in 1950 many new statistical techniques have been made available and many techniques that were only in the domain of the mathematical statistician are now useful and demanded by the applied statistician. To include some of this material we have had to eliminate other material, else the book would have come to resemble a compendium. The general approach of presenting the theory with some connection to practical problems apparently contributed significantly to the success of the first edition and we have tried to maintain that feature in the present edition.

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I

PROBABILITY

1 INTRODUCTION AND SUMMARY

The purpose of this chapter is to define *probability* and discuss some of its properties. Section 2 is a brief essay on some of the different meanings that have been attached to probability and may be omitted by those who are interested only in mathematical (axiomatic) probability, which is defined in Sec. 3 and used throughout the remainder of the text. Section 3 is subdivided into six subsections. The first, Subsec. 3.1, discusses the concept of probability models. It provides a real-world setting for the eventual mathematical definition of probability. A review of some of the set theoretical concepts that are relevant to probability is given in Subsec. 3.2. Sample space and event space are defined in Subsec. 3.3. Subsection 3.4 commences with a recall of the definition of a function. Such a definition is useful since many of the words to be defined in this and coming chapters (e.g., probability, random variable, distribution, etc.) are defined as particular functions. The indicator function, to be used extensively in later chapters, is defined here. The probability axioms are presented, and the probability function is defined. Several properties of this probability function are stated. The culmination of this subsection is the definition of a probability space. Subsection 3.5 is devoted to examples of probabilities defined on finite sample spaces. The related concepts of independence of events and conditional probability are discussed in the sixth and final subsection. Bayes' theorem, the multiplication rule, and the theorem of total probabilities are proved or derived, and examples of each are given.

Of the three main sections included in this chapter, only Sec. 3, which is by far the longest, is vital. The definitions of probability, probability space, conditional probability, and independence, along with familiarity with the properties of probability, conditional and unconditional and related formulas, are the essence of this chapter. This chapter is a background chapter; it introduces the language of probability to be used in developing *distribution theory*, which is the backbone of the theory of statistics.

2 KINDS OF PROBABILITY

2.1 Introduction

One of the fundamental tools of statistics is probability, which had its formal beginnings with games of chance in the seventeenth century.

Games of chance, as the name implies, include such actions as spinning a roulette wheel, throwing dice, tossing a coin, drawing a card, etc., in which the outcome of a trial is uncertain. However, it is recognized that even though the outcome of any particular trial may be uncertain, there is a *predictable* long-term outcome. It is known, for example, that in many throws of an ideal (balanced, symmetrical) coin about one-half of the trials will result in heads. It is this long-term, predictable regularity that enables gaming houses to engage in the business.

A similar type of uncertainty and long-term regularity often occurs in experimental science. For example, in the science of genetics it is uncertain whether an offspring will be male or female, but in the long run it is known approximately what percent of offspring will be male and what percent will be female. A life insurance company cannot predict which persons in the United States will die at age 50, but it can predict quite satisfactorily *how many* people in the United States will die at that age.

First we shall discuss the classical, or a priori, theory of probability; then we shall discuss the frequency theory. Development of the axiomatic approach will be deferred until Sec. 3.

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