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SOLID MECHANICS

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SOLID MECHANICS

Foreword

I HAVE great pleasure in writing this foreword to the textbook on "Solid Mechanics" by Dr. S. M. A. Kazimi, Assistant Professor at the IIT, Delhi.

It is a new effort to present the subject. General concepts precede the engineering theory of strength of materials. It will help students to gain a more than superficial knowledge of the subject. Experimental aspects have been interwoven into the text. The numerous illustrative examples enhance the value of the text and make it easy for the beginner to thoroughly grasp the presentation of the theory.

The young author is to be commended on his efforts in authoring such a comprehensive textbook. Teachers as well as students will find it of considerable interest in presenting and learning solid mechanics.

CSIR Campus, Adyar
Madras
10-9-1974

G. S. RAMASWAMY

Preface

THIS textbook is an outgrowth of my class lectures, delivered to various groups of students, at the third and fourth year level, in my seven years at the IIT, Delhi. Most of the topics presented here were discussed in the first course on Solid Mechanics, which was given as a core course to various disciplines like civil engineering, mechanical engineering and textile engineering. Free discussions with the outgoing students and informal opinion polls provided the feedback and have helped to improve the book. In its present form there is obviously more material than can be covered in a 6-hour, 4-credit course, and enough topics can be skipped over in the first course, to form the basis of a second course at the fourth or fifth year level.

A unified approach has been adopted and wherever possible a synthesis of the strength of material and theory of elasticity has been affected. No attempt has been made to classify the subject matter into less advanced and more advanced topics. It is left to the teacher to design a suitable course according to his specific interest and the level of the students. A careful study of the table of contents should make his task less difficult.

M.K.S. units have been adopted in this edition to make the book immediately useful to students. Provision for future conversion to SI units is kept for the next edition, and a detailed note on SI units has been added as appendix I, for ready reference.

I wish to thank my students who have helped to improve the book in some way; either by an awkward question or a brilliant remark. My thanks are also to my colleagues and seniors at the IIT, Delhi, particularly Prof. B. Karunes and Prof. K. Seetharamulu for their comments and discussions. I am grateful to Prof. G. S. Ramaswamy, Director of the Structural Engineering Research Centre, Madras, for looking through the manuscript and writing a precise foreword. Finally I thank the National Book Trust for

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PART—B (Particular Applications)

Introductory

0.1 Mechanics of Solids—Definition and Scope

The term 'Solid Mechanics' is comparatively new and may not have been known as such in the first quarter of the present century. However, most of the topics covered under this general heading were known to us earlier, and were covered under various subjects like strength of materials, mechanics of materials, structural mechanics, theory of elasticity, theory of plasticity, visco-elasticity, etc. Thus, the present trend towards recognizing Solid Mechanics as a distinct field, is mainly an organizational effort, whereby we rearrange the existing knowledge in a systematic form and try to fill in the gaps, if any, in this overall pattern. Under the new concept, the field of solid mechanics covers the study of mechanical properties of engineering materials and the behaviour of structures made thereof, with the following objects in view :

(1) To gain a clear insight into the actual phenomena.
(2) To find the optimum number of simplifying assumptions.
(3) To bring the calculations to simple numerical results.

The types of problems to be tackled are many and varied. However, in general, we are required to design a machine part or structural element so that it has enough strength, stiffness and stability and is economical in weight and overall cost. To gain this design object, we first analyse the structure to find the forces and stresses at various points. Then we select a suitable material of known deformation behaviour to withstand these loads without too much deformation, and check for stability. In fact, all these operations are interlinked and may have to be done side by side.

It may be noted that the study of solid mechanics is essential for many branches of engineering like civil, mechanical, aeronautical, chemical, agricultural and textile engineering, and its applications are found in almost every branch of engineering. Thus, it has an inter-disciplinary character and may be studied as a general course at the undergraduate level. However, at higher levels, it is subdivided into various specialized branches, like, strength of materials, theory of elasticity, theory of plasticity, visco-elasticity, visco-plasticity, thermo-elasticity, fracture mechanics, etc. and may be studied at post-graduate to post-doctorate level. In the present book, we shall confine ourselves to the basic undergraduate course that is useful to all the branches of engineering and which can be grasped with a basic knowledge of statics and mechanics.

0.2 Strength of Materials and Theory of Elasticity

Both these branches of solid mechanics are very old and have developed, almost, side by side. However, a clear distinction was made earlier between the two, and the pioneers of strength of materials were generally practising engineers and experimentallists while those of elasticity were physicists and mathematicians. In fact, for quite some time mathematical theory of elasticity developed as a branch of applied mathematics, and engineers' interest in it is of a comparatively recent origin.

The types of problems tackled by both are similar but the main difference lies in the approach. Under strength of material, we deal with a particular problem on an ad-hoc basis and find particular solution after making many simplifying assumptions. The approach is generally very simple and the emphasis is on finding a practical solution to the particular problem, rather than on understanding the general phenomenon. On the other hand, we try to formulate the problem in most general terms under the theory of elasticity and obtain a general solution, if possible. Obviously, we have to study the problem in greater depth, but the solutions are not easy in most of the cases and a deeper knowledge of mathematics is necessary. Quite often, the problems solved are not of a practical nature and highly idealised loading and boundary conditions are assumed to obtain a solution. This may be a reason why engineers did not take enough interest in it earlier.

In the present treatment, a happy compromise is reached between these two conflicting demands. As the exposition of the problems, specially the state of stress and strain at every point of a body, is definitely superior under theory of elasticity, we adopt this approach in the first part of the book, under General Analysis. This helps us to understand the problem in greater depth and the implications of various simplifying assumptions, necessary under the strength of material approach. Later on, in the second part, we try to solve practical engineering problems which obviously require a lot of simplifying assumptions. However, as we have already studied the exact theories earlier, the truth is never hidden from us, under a mass of assumptions and approximations. This helps to develop a clear insight into the problem coupled with a knack for bringing the problem to simple algebraic results by cutting down the unnecessary details. This is the real aim of a design engineer.

0.3 Structural Analysis and Design Scheme

Once the general theories of stress and strain (Part I) are learnt and their particular application (Part II) understood, the next step is the design and analysis of a real structure. A mechanical or structural system generally has a large number of components and we have to focus our attention on the machine part or structural element. Other components may be replaced by simple forces or restraints as the case may be. As an example, suppose we have to design a lift system. It consists of a number of parts and let us suppose that we are interested only in the design of ropes. Hence we may conveniently replace the lift cage by a rigid body of known mass and completely disregard its deformation properties. However, if we have to design the lift cage, we have to consider its structural properties, though now, the cables can be replaced by simple forces of known magnitude. These 'isolation' techniques are an essential part of a complete design scheme, and have to be carefully adopted. All this is schematically shown in Fig. 1 (a), (b) and (c).

0.4 Types of Forces

All the forces to be encountered by a deformable body (or rigid body for that purpose) may be studied under two systems of classification, i.e. (a) External and internal forces and (b) Surface and body forces. The forces acting on the extremities or exposed surface of a body are known as external forces, whereas those acting inside the body between the grains or molecules are known as internal forces. External forces are sometimes known as boundary forces as well.

The distinction between surface and body forces is not so easy. Sufficient to say that surface forces are acting on an area or surface, whether external or internal. Some examples could be, external loads, contact forces (produced by one body pressing against another) or forces produced by thermal expansion or contraction. Body forces are always acting on every grain of a body and are proportional to volume. Thus gravity force, inertia force and magnetic force are body forces. These are produced by a force field, in general, and could be constant or varying from point to point (vide example 1).

It is possible that a body force on one body may produce a surface force on another body or the exposed boundary of the same body. Suppose a piece of mild steel is resting on a wooden table and a strong magnet is placed below it, to create a strong magnetic field. The magnetic field

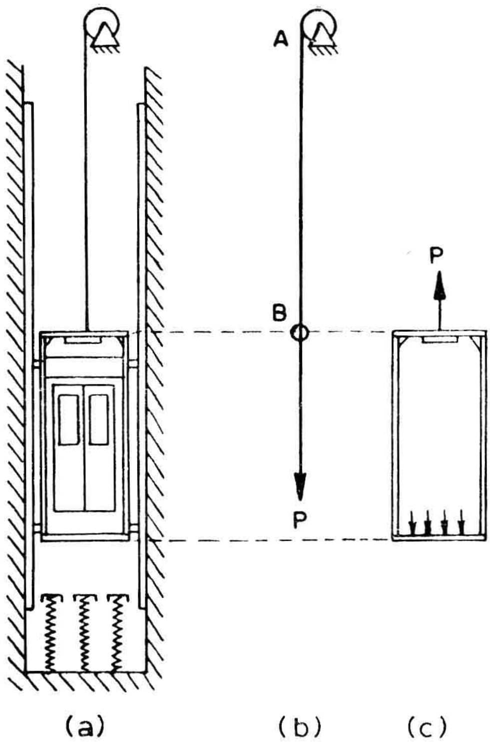

Fig. 0.1 Design scheme for a lift rope and cage
a. Lift system
b. Design scheme for lift rope
c. Design scheme for lift cage (rigid frame)

will produce a body force distribution on the steel, but as far as the wooden table is concerned, it will experience only a surface force at the surface of contact, equal to the total magnetic pull. Further, according to Newton's third law, the piece of steel will also experience an equal and opposite surface force at the surface of contact.

Example 1

Fig. 2(a) shows a solid of cubical shape, $10 \times 5 \times 8$ meters in size. If the density is uniformly increasing from 2000 kg/m^3 at the bottom face to 4000 kg/m^3 at the top face, find the intensity of the body force field, and the total body force, under a field of $4.9\,i + 9.8\,j$ m/sec^2.

Solution

If we assume the origin at the bottom surface of the solid, the equation for density will be, $\rho = 2000\,(1 + 0{\cdot}125\,y)$ kg/m^3 and field $B = 4{\cdot}9\,i + 9{\cdot}8\,j$ m/sec^2.