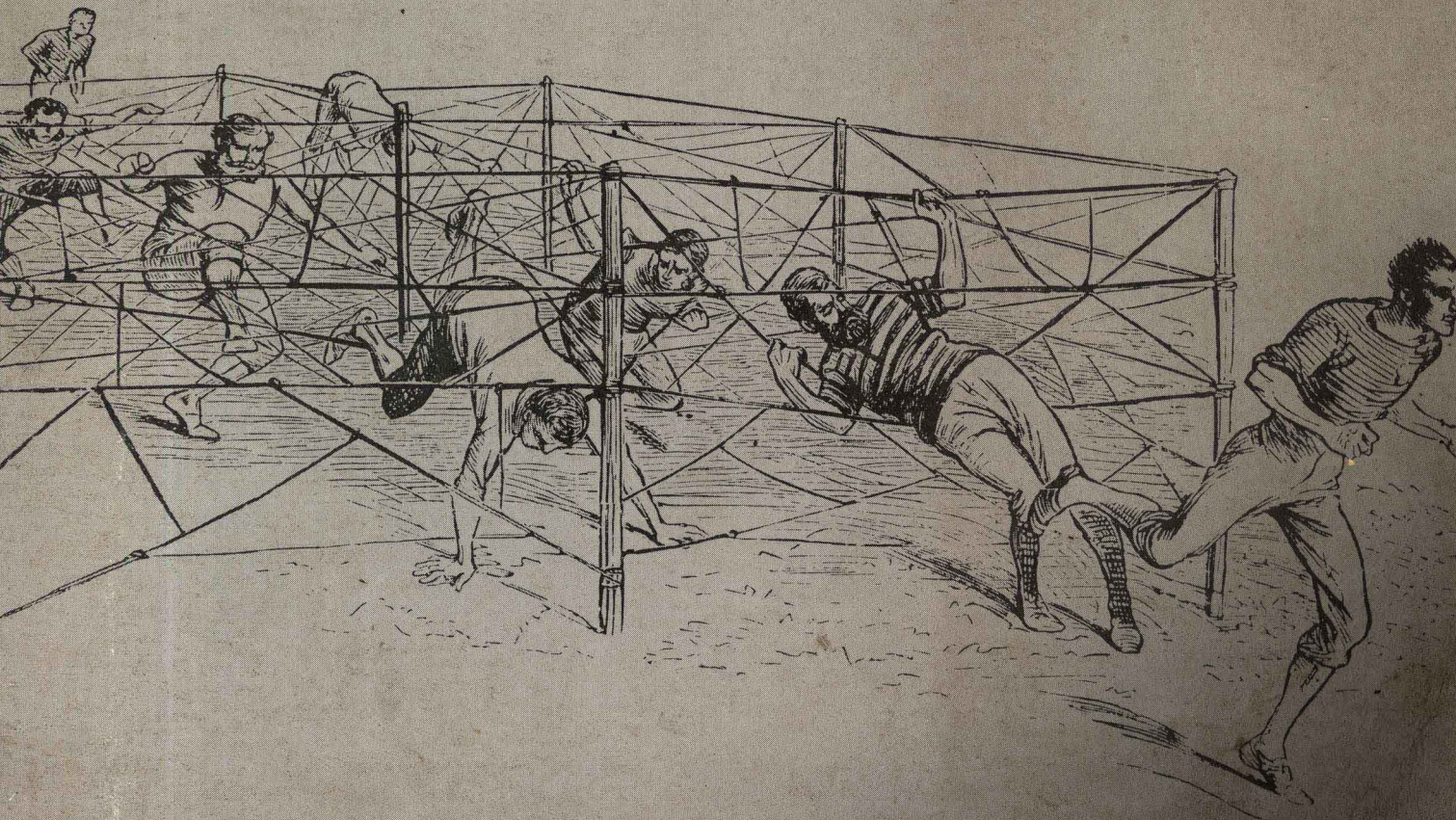


# ESSENTIAL ALGEBRA

Johnston & Willis





# **ESSENTIAL ALGEBRA**

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1 2 3 4 5 6 7 8 9 10—79 78 77 76 75

# SYMBOLS USED IN THIS BOOK

<u>Symbol</u>	<u>Meaning</u>	<u>Page</u>
$=$	read <i>is equal to</i>	4
$\neq$	read <i>is not equal to</i>	4
$\approx$	read <i>is approximately equal to</i>	406
$<$	read <i>is less than</i>	3
$\nless$	read <i>is not less than</i>	4
$\leq$	read <i>is less than or equal to</i>	231
$>$	read <i>is greater than</i>	3
$\ngtr$	read <i>is not greater than</i>	4
$\geq$	read <i>is greater than or equal to</i>	231
$\sqrt{\quad}$	read <i>square root</i>	30
$\sqrt[3]{\quad}$	read <i>cube root</i>	33
$3^4 = 81$	exponent	28
	4th power of 3	28
	base	28
$3 \times 5 = 3(5) = (3)5$ $= (3)(5) = 3 \cdot 5$	product of 3 and 5	16
$\Rightarrow$	read <i>implies</i>	319
$(\quad)$	parentheses	44
$[\quad]$	brackets	44
$\{\quad\}$	braces; used to show a set	44, 388
$\overline{\quad}$	bar	44
$\{1, 2, 3, \dots\}$	set of natural (counting) numbers	2
$\{0, 1, 2, \dots\}$	set of whole numbers	3
$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$	set of digits	4
$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	set of integers	7
$4 \in N$	4 is an element of set $N$	389
$9 \notin A$	9 is not an element of set $A$	389
$n(A)$	cardinal number of set $A$	389
$\emptyset = \{\quad\}$	empty set	389
$U$	universal set	390
$A \subseteq B$	set $A$ is a subset of set $B$	392
$P \subset Q$	set $P$ is a proper subset of set $Q$	392
$R \not\subset T$	set $R$ is not a subset of set $T$	392
$A \cup B$	union of set $A$ and set $B$	394
$A \cap B$	intersection of set $A$ and set $B$	394
@	at	445
$\pm$	plus or minus	375



# ESSENTIAL ALGEBRA



## Preface

In this book we use simple and direct language to present a working background of the fundamentals of algebra for students who have had no previous training in algebra or who feel the need for an intensive review.

Some of the major features of this book are:

1. The contents are arranged in small sections, each with its own examples and exercises. We use a one-step, one-concept-at-a-time approach.
2. After careful and detailed topic explanation, we use many concrete examples that lead up to general algebraic principles which can then be used to work the exercises.
3. Important concepts and algorithms are enclosed in boxes for easy identification and reference.
4. The complete solutions for all odd-numbered exercises are included in the back of the book together with the answers for all even-numbered exercises. In most cases the even-numbered exercises provide practice on problems analogous to the odd-numbered exercises. An instructor can assign even-numbered exercises for homework, and the student can use the solutions to the odd-numbered exercises as a study aid. If an instructor wishes to assign exercises for which no answers are available to students, he can make copies of Chapter Examinations found in the Instructor's Manual. (See Item 5.)
5. An Instructor's Manual contains four different tests for each chapter that may be easily removed and duplicated for class use. These tests are prepared with adequate space for students to work the problems. Answer keys for these tests are provided in the manual.
6. A Diagnostic Test follows each chapter. Complete solutions to all problems in these Diagnostic Tests, together with section references, appear in the answer section.
7. A comprehensive summary is included at the end of each chapter.
8. In word problems our approach is to deal with fewer types of problems, and to provide a more detailed method for changing a word statement into an algebraic equation.
9. Special attention is given to the operations with zero in a single section of Chapter One which includes a discussion of the common errors students make with zero. References are then made to this section throughout the book.
10. Liberal use is made of visual aids such as the number line, shading, and other graphics.
11. In special "Words of Caution" major and common algebraic errors are identified. Attention is drawn to avoiding mistakes that are commonly made by the inexperienced student.



12. The importance of checking solutions is stressed throughout the book.
13. A brief summary of arithmetic is included in the Appendix to help students who have trouble with arithmetic in algebra.

This book can be used in three types of instructional programs:

1. The conventional lecture course. This book is particularly easy to fit into a program of regular assignments because it is divided into many small self-contained units. Examinations that can be given for each chapter are provided in the Instructor's Manual. Following each chapter in the book is a Diagnostic Test which students may use for review and diagnostic purposes. This book has been class-tested by the authors and several other instructors in the conventional lecture course program.
2. The learning laboratory class. Because of the format of explanation, example, and exercise carried on in each section of the book, together with the Diagnostic Tests and solutions for each chapter, we feel that a wide degree of latitude in the pace at which a student progresses is possible when using this book in the learning laboratory class.
3. Self-study. This book lends itself to self-study because (a) each new topic is short enough to be mastered before continuing and (b) almost 800 examples are solved and over 1,600 complete solutions are given for the odd-numbered exercises, together with all the answers for the even-numbered exercises. In using this book for self-study, a student should begin by taking the Diagnostic Test for Chapter One, and then checking answers against the solutions at the back of the book. The solutions section will direct the student to specific sections of the book that explain the particular problems done incorrectly. The student can continue in this manner at his own pace throughout the book.

We wish to thank Don Dellen, Mathematics Editor, and members of the production staff of the Wadsworth Publishing Company for their help and encouragement in the development of this book.

This book is dedicated to our students, who inspired us to do our best to produce a book worthy of their time.

C. L. Johnston  
Alden T. Willis



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# ONE

Operations with  
Signed Numbers



*Arithmetic* is calculation with numbers using fundamental operations such as addition, subtraction, multiplication, and division. (See the Appendix for a brief review.) *Algebra* deals with the same fundamental operations with numbers, but uses letters to represent some of the numbers.

Before beginning the study of algebra we review for your benefit a few basic definitions relating to numbers.

# 101

## Basic Definitions

NATURAL NUMBERS. The numbers

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and so on,

are called the *natural numbers* (or *counting numbers*). These were probably the first numbers invented to enable people to count their possessions, such as sheep, goats, etc.

NUMBER LINE. Natural numbers can be represented by numbered points equally spaced along a straight line (Figure 101A). Such a line is called a *number line*.



Figure 101A

The arrowhead shows the direction in which numbers get larger, and also shows that the numbers continue on. Numbers that follow one another (without interruption) are called *consecutive numbers*. This means that 6 follows 5; 7 follows 6. Therefore, 5, 6, and 7 are consecutive numbers. Later we will discuss other kinds of numbers, such as fractions, which can be placed on the number line.

The smallest natural number is 1. The largest natural number can never be found because no matter how far we count there are always larger natural numbers. Since it is impossible to write all the natural numbers, it is customary to represent them as follows:

$\{1, 2, 3, 4, \dots\}$

↖ read "and so on"

The three dots to the right of the number 4 indicate that the remaining numbers are to be found by counting in the same way we have begun: namely, to add 1 to the preceding number to find the next number. We call the set of natural numbers  $N$ . (Sets are covered in Chapter 15.) So

$$N = \{1, 2, 3, 4, \dots\}$$



WHOLE NUMBERS. When 0 is included with the natural numbers, we have the set of numbers known as *whole numbers* (Figure 101B).

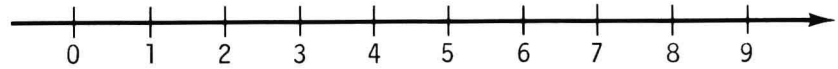


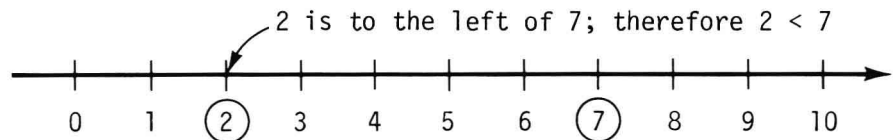
Figure 101B

We call the set of whole numbers  $W$ . So

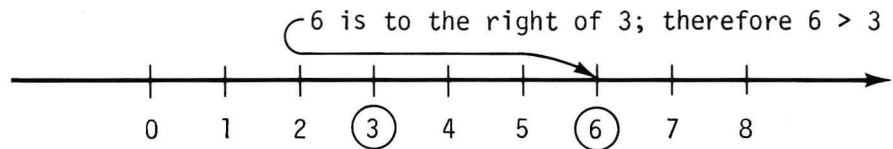
$$W = \{0, 1, 2, 3, \dots\}$$

INEQUALITY SYMBOLS. The symbols  $>$  and  $<$  are called *inequality symbols*. Let  $X$  be any number on the number line. Then numbers to the right of  $X$  on the number line are said to be *greater than*  $X$ , written " $>X$ ". Numbers to the left of  $X$  on the number line are said to be *less than*  $X$ , written " $<X$ ".

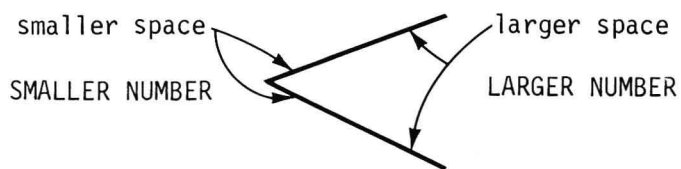
Example 1



Example 2



An easy way to remember the meaning of the symbol is to notice that the wide part of the symbol is next to the larger number.



Some people like to think of the symbols  $>$  and  $<$  as arrowheads that point toward the smaller number.



### Example 3

- (a)  $7 > 6$  is read "7 is greater than 6"
- (b)  $7 > 1$  is read "7 is greater than 1"
- (c)  $5 < 10$  is read "5 is less than 10"
- (d)  $3 < 4 < 5$  is read "3 is less than 4 and  
4 is less than 5"

Note that  $7 > 6$  and  $6 < 7$  give the same information even though they are read differently.

Another inequality symbol is  $\neq$ . A slash line drawn through a symbol puts a *not* in the meaning of the symbol.

### Example 4. Showing the use of the slash line

- (a)  $=$  is read "is equal to"
- $\neq$  is read "is *not* equal to"
- (b)  $<$  is read "is less than"
- $\nless$  is read "is *not* less than"
- (c)  $>$  is read "is greater than"
- $\ngtr$  is read "is *not* greater than"
- (d)  $4 \neq 5$  is read "4 is not equal to 5"
- (e)  $3 \nless 2$  is read "3 is not less than 2"
- (f)  $5 \ngtr 6$  is read "5 is not greater than 6"

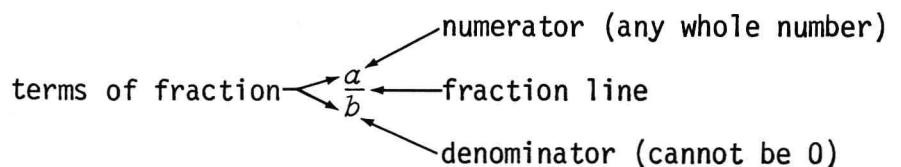
DIGITS. In our number system a digit is any one of the first ten whole numbers, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. They are shown on the number line in Figure 101B. Probably because people began counting on their ten fingers (Latin *digitus*, finger), the first ten whole numbers were called *digits*.

Numbers are often referred to as *one-digit* numbers, *two-digit* numbers, *three-digit* numbers, and so on.

### Example 5

- (a) 35 is a two-digit number
- (b) 7 is a one-digit number
- (c) 275 is a three-digit number
- (d) The first digit of 785 is 7
- (e) The second digit of 785 is 8
- (f) The third digit of 785 is 5

FRACTIONS. A *fraction* is part of a whole. It is written  $\frac{a}{b}$ . We call  $a$  and  $b$  the *terms* of the fraction. We call  $b$  the *denominator*. It tells how many parts the whole was divided into. We call  $a$  the *numerator*. It tells the number of those equal parts used.





MIXED NUMBERS. A *mixed number* is made up of both a whole number part and a fraction part.

Example 6. Mixed numbers

$$2\frac{1}{2}, \quad 3\frac{5}{8}, \quad 5\frac{1}{4}, \quad 12\frac{3}{16}$$

DECIMAL FRACTIONS. A *decimal fraction* is a fraction whose denominator is 10, 100, 1000, etc.

Example 7. Decimal fractions

- (a)  $\frac{4}{10} = 0.4$  and is read "four-tenths"
- (b)  $\frac{5}{100} = 0.05$  and is read "five-hundredths"
- (c)  $\frac{6}{1,000} = 0.006$  and is read "six-thousandths"
- (d)  $\frac{23}{10} = 2.3$  and is read "two and three-tenths"

Decimal Places. The number of decimal places in a number is the number of digits written to the right of the decimal point.

75.14 (2 decimal places)

1.086 (3 decimal places)

REAL NUMBERS. All the numbers that can be represented by points on the number line are called *real numbers*. We have already shown that any natural number or any whole number can be represented by a point on the number line. Fractions, decimals, and mixed numbers are also part of the real number system because any fraction, decimal, or mixed number can be represented by a point on the number line. The points representing some fractions, decimals, and mixed numbers are shown in Figure 101C.

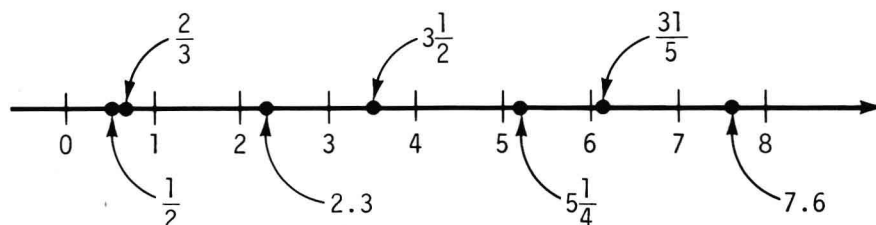


Figure 101C

In Section 102 we introduce other kinds of numbers which are also part of the real number system.

## EXERCISES 101

1. What is the second digit of the number 159?
2. What is the smallest natural number?
3. What is the smallest digit?
4. What is the smallest whole number?
5. What is the fourth digit of the number 1,975?
6. What is the smallest two-digit natural number?
7. What is the smallest three-digit whole number?
8. What is the largest one-digit number?
9. What is the largest two-digit number?
10. Write two different symbols that tell the number of days in a week.
11. Is 12 a digit?
12. Is 12 a natural number?
13. Is 12 a whole number?
14. What is the largest natural number?
15. What is the largest digit?
16. Write all the whole numbers  $< 4$ .
17. Write all the digits  $> 5$ .
18. Write the consecutive natural numbers  $> 14$  and  $< 17$ .
19. Write in consecutive order all the digits  $< 5$ .

In Exercises 20-25 determine which of the two symbols,  $>$  or  $<$ , should be used to make each statement true.

20.  $8 \underline{\quad ? \quad} 7$
21.  $0 \underline{\quad ? \quad} 1$
22.  $1 \underline{\quad ? \quad} 0$
23.  $5 \underline{\quad ? \quad} 8$
24. The weight of a man is  $\underline{\quad ? \quad}$  the weight of a baby.
25. The distance to the moon is  $\underline{\quad ? \quad}$  the distance to a star.
26. Is 2.3 a real number?
27. Is  $\frac{31}{5}$  a real number?

# 102

## Negative Numbers

In Section 101 we showed how whole numbers could be represented by equally spaced points along the number line. We now extend the number line to the left and continue with the set of equally spaced points.

Numbers used to name the points to the left of 0 on the number line are called *negative numbers*. Numbers used to name the points to the right of 0 on the number line are called *positive numbers*. Zero itself is neither positive nor negative. The positive and negative numbers are referred to as *signed numbers* (Figure 102A).

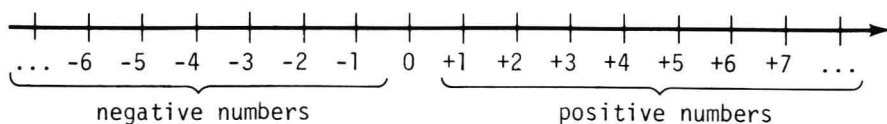


Figure 102A

