Microscopic Aspects of Nonlinearity in Condensed Matter

Edited by

A. R. Bishop

V. L. Pokrovsky and

V. Tognetti

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PREFACE

The collection of articles in this volume represents the proceedings of the inaugural Conference of the Institute of Theoretical Matter Physics at the University of Florence, Italy, June 7-13, 1990. The aim of this Institute (sponsored by the Consorzio Interuniversitario di Fisica della Materia, INFM) is to foster interdisciplinary approaches to theoretical problems in Solid State, Electronics and Optics. Correspondingly, the Conference surveyed a broad scope of modern analytic techniques in nonlinear science and their application to novel liquids, solids and optical environments. The Conference was held at Centro Studi CISL, a magnificent Villa up to Florence.

The major topics addressed through survey and specialized lectures, discussions and posters were: *Complexity and Coherence* in spin glasses, associative distributive memory, low dimensional electronic materials, magnets, Josephson junction arrays, optics, semiconductors, convection cells, material science, high-temperature superconductivity; *Physics of Quantum Devices* such as nonlinear feedback oscillators, macroscopic quantum tunneling, hetero structures, quantum ballistic devices, tunneling in atomic devices; *Field Theory and Statistical Mechanics*, especially for soliton-bearing and quantum chaotic systems.

Many exciting ideas and results were presented and debated in lively fashion. Perhaps two major themes were evident throughout the workshop.

> *Firstly*, "nonlinear science" is now emerging as a combination of two complentary paradigms: (a) The possibility of chaotic or irregular dynamics even in small degree-of-freedom systems (as, e.g., in period-doubling routes to chaos in one-dimensional maps) or chaotic spatial patterns in certain static, discrete lattices (as, e.g., in commensurate-incommensurate phase transition models). The common ingredient here is the presence of competing frequency and/or length scales. Indeed synergistic mappings frequently exist between long-time attractors of time-dependent problems and ground states of time-independent problems in a higher spatial dimension; and (b) The appearance of coherent collective structures in nonlinear space-time systems - structures such as solitons, dislocations, vortices. This duality of chaos and coherence in extended nonlinear systems results in the many fascinating current examples of pattern formation and competition, low-dimensional nonlinear mode reductions and low-dimensional chaos, and so forth. Of course the collective-coordinate description of such coherent structures, of their dynamics and interactions, and of their self-organized mesoscale patterns, is essential to establishing a bridge between microscopic and macroscopic properties of materials.

Secondly, the interface between controlled condensed matter experiments and realistic phenomena in material science, optics, etc., is proving to be extremely fertile. Imaginative, laboratory scale condensed matter experiments are able to isolate essential ingredients of material science leading to complex patterns, dynamics and response. This is where interdisciplinary interactions are, and surely will continue to be, a true driving force for applications of the paradigms of nonlinear science.

Sponsorship of the Conference was generously provided by the NATO (ARW 900372) through the special program Chaos Order and Patterns (COP), the Consorzio di Fisica della Materia (INFM), the Italian National Science Council (CNR) and the Department of Physics of the University of Florence. We are also deeply appreciative of the expert secretarial support from Genny Ferasin, both during the Conference and in the preparation of manuscripts.

Firenze, January 1991

A.R. Bishop
V.L. Pokrovskii
V. Tognetti

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SPACE TIME COMPLEXITY IN QUANTUM OPTICS

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Abstract

Two recent experiments in quantum optics, namely, i) a waveguide laser supporting many transverse modes, and ii) an optical cavity with a photorefractive gain medium and a variable aperture have displayed controllable routes to space complexity. This fills the gap between the single mode dynamics and the many domain turbulent-like behavior, which so far was unreachable for a radiation field. Due to the high accuracy of optical measurements, we foresee a precise way to test many conjectures formulated for fluids or other nonlinear field problems. Thus we have called this new research area "dry hydrodynamics".

In particular I show the first experimental evidence of two phenomena, recently described theoretically, namely:

i) chaotic itinerancy = self induced switching among different slow manifolds
ii) Space-Time Chaos (STC) = high dimensional chaos, with strongly non Gaussian statistics in real space, but with a Gaussian spectral statistics, up to a critical wave number given by the reciprocal of the correlation length.

1- Introduction

Self organization evolved from a qualitative concept used by bio-mathematicians in the therties [1] to a quantitative appraisal of a physical phenomenon as soon as physicists were able to pick up a low-dimensional laboratory system displaying a satisfactory agreement between its experimental behavior and the corresponding theoretical model. While theoretical models were worked out for non-equilibrium transitions in fluids and chemical reactions, the first clear experimental evidence was provided in the middle sixties by the single mode laser. This system displayed an outstanding agreement between theory and experimental tests carried by photon counting statistics [2].

Later on, similar threshold measurements were performed at the onset of instability phenomena in fluid convection or in chemical reactions, thus opening the field of the so called "non equilibrium phase transitions" [3]. However in fluids it is straightforward to scale up the system size from small to large cells, thus making it possible to explore in many ways the passage from systems coherent (fully correlated) in space to systems made up of many uncorrelated, or weakly correlated, domains.

Crucial questions such as: i) the passage from order to chaos within a single domain and ii) possible synchronizations of time behaviors at different space domains, have been addressed in the past years, with the general idea in mind that space-time organization is what makes a large scale object complex, that

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is, richer in information than the sum of the elementary constituents, due to an additional non trivial cross- information [4].

Thus far, such an investigation was not possible in the optical field, because all coherent optics is based on Schawlow-Townes original idea of a drastic mode selection [5]. The opposite, thermodynamic limit of optics was explored early in this century [6]. A confrontation of the two limits was shown experimentally in terms of photon statistics [7] in 1965, however what is in between has not been adequately explored, at variance with hydrodynamic instabilities or unstirred chemical reactions.

Here I show very recent evidence of space-time complexity in optics. The experimental configurations which have made possible to fulfill this twenty-year long search appear so promising that we can foresee an extensive investigation of space phenomena in optics along the coming decade. Let me call this area of investigation "dry hydrodynamics".

The paper is organized as follows. In Sec. 2 I present a qualitative description of the phenomena which characterize low dimensional chaos, that is, chaotic attractors with space semplicity, then space-time chaos (STC), as well as the transition between the two regimes. In order to stimulate the theoretical interest, we try to connect these phenomena with the present theoretical understanding, even though the experimental findings were not motivated by preliminary theories. On the contrary, they have been based on powerful heuristic conjectures [8], and search for satisfactory theories is still in progress. Secs. 3 and 4 are devoted to the presentation of the experimental systems investigated by my Group and of the preliminary results available.

2 - Periodical alternance, chaotic itinerancy and STC

In this Section I anticipate and explain what we are going to see in the experiments reported in the later Sections. To appreciate the role of space coupling let me summarize the present status of affairs in quantum optics. Since all coherent phenomena take place in a cavity mainly extended in a z-direction (as e.g. the Fabry-Perot cavity), we expand the field $e(r,t)$, which obeys the wave equation

$$\Box^2 e = -\mu \ddot{p} \tag{1}$$

(where $p(r,t)$ is the induced polarization), as

$$e(\vec{r},t) = E(x,y,z,t)e^{-i(\omega t - kz)} \tag{2}$$

If the longitudinal variations are mainly accounted for by the plane wave, then we can take the envelope E as slowly varying in t and z with respect to the variation rates ω and k in the plane wave exponential. Furthermore we call P the projection of p on the plane wave. By neglecting second order envelope derivatives it is easy to approximate the operator on E as

$$\Box^2 \rightarrow 2ik\left(\partial_z + \frac{1}{c}\partial_t\right) + \partial_x^2 + \partial_y^2 \tag{3}$$

as is usually done in the eikonal approximation of wave optics. This further suggests three relevant physical situations.

2.1 (1+0) - dimensional Optics

In such a case there is only a time dependence and no space derivatives, that is, $\Box^2 \rightarrow 2i\omega d_t$. Assuming that the laser cavity is a cylinder of length L, with two mirrors of radius a at the two ends, the cavity resonance spectrum is

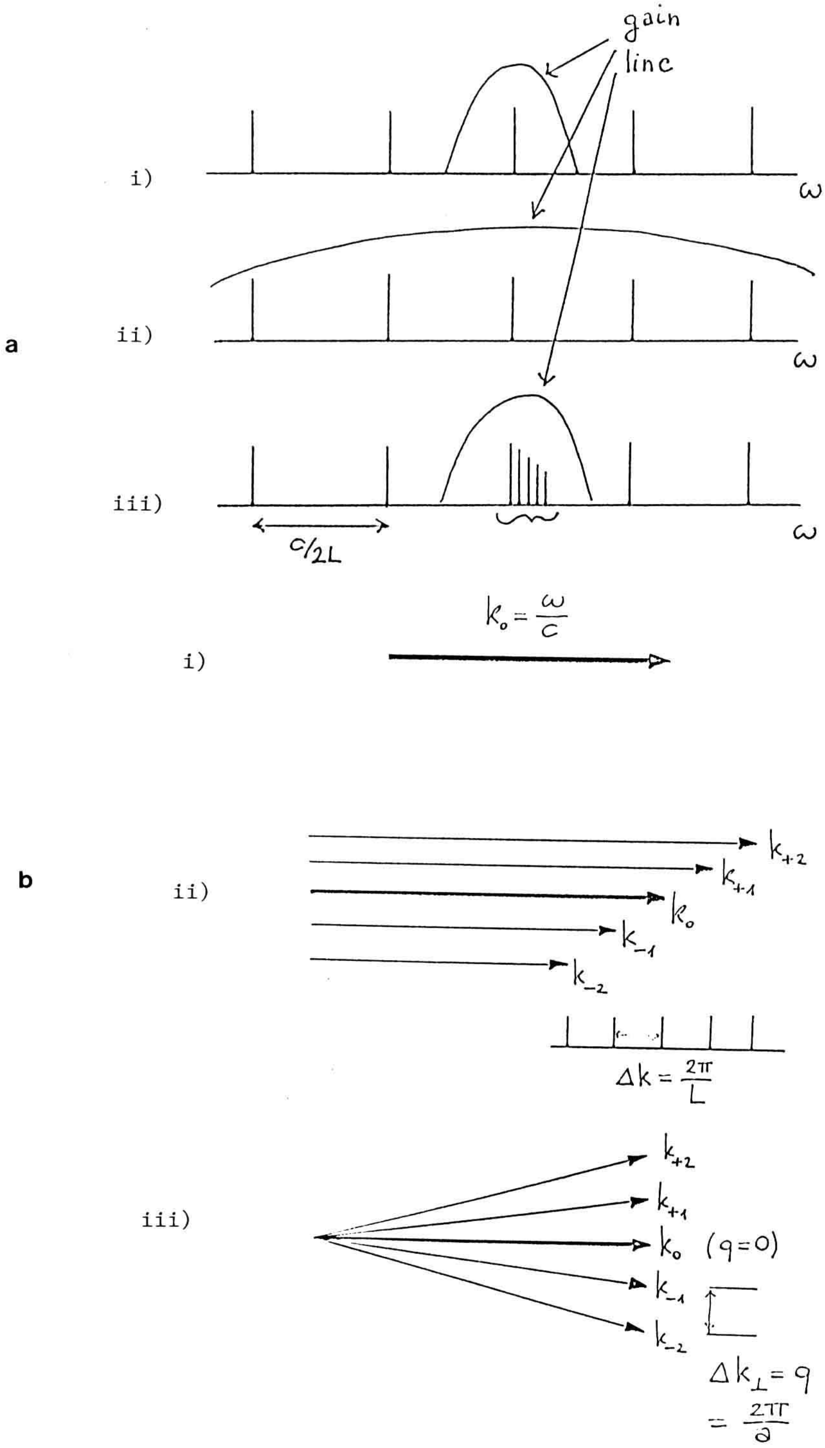

Fig. 1. ω-space (a) and K-space (b) pictures of the lasing modes in the: i) (1+0), ii) (1+1), and iii) (1+2)-dimensional cases.

made of discrete lines separated by $c/2L$ in frequency, each one corresponding to an integer number of half wavelengths contained in L, plus a crown of quasi-degenerate transverse modes at the same longitudinal wavenumber, with their propagation vectors separated from each-other by a diffraction angle λ/a (Fig. 1b).

This case corresponds to a gain line narrower than the longitudinal frequency separation (so called free spectral range) and to a Fresnel number

$$F \equiv \frac{a^2}{\lambda L} \tag{4}$$

of the order of unity, so that the first off axis mode already escapes out of the mirror. Intuitively F is the ratio between the geometric angle a/L of view of one mirror from the other and the diffractive angle λ/a.

In such a case, the resulting ODE replacing the wave PDE has to be coupled to the matter equations giving the evolution of P. In the simple case of a cavity mode resonant with the atomic line, we obtain the so called Maxwell-Bloch equations [9].

$$\begin{aligned} \dot{E} &= -\gamma_E E + gP \\ \dot{P} &= -\gamma_\perp P + gEN \\ \dot{N} &= \gamma_\| N - 2gEP + A \end{aligned} \tag{5}$$

where N is the population inversion (we have modeled the gain medium as a collection of two level atoms), γ_E, $\gamma_\perp$ and $\gamma_\|$ are the loss rates of E, P and N respectively, g is the field-matter coupling constant and A is the pump rate.

Eqs. (5) are isomorphic to Lorenz equations for a model of convective fluid instability. Being three nonlinear equations, they provide the minimal conditions for deterministic chaos. However time scale considerations can rule out some of the three dynamical variables, yielding a dissipative dynamics with only one variable (fixed point attractor) or two variables (limit cycle attractor). Only when the three damping rates γ_E, $\gamma_\perp$ and $\gamma_\|$ are of the same order of magnitude, we do have a three-equation dynamics and hence possibility of a chaotic motion (strange attractor). The above three cases have been classified as class A,B, and C lasers respectively.

A comprehensive review of experiment and theory for these single-domain, (1+0)-dimensional systems is given in the book cited in Ref. 9, covering the period 1982-87 over which these space invariant instabilities have been studied.

2.2 - (1+1) - dimensional Optics

Here, we have a cavity thin enough to reject off axis modes, but fed by a gain line wide enough to overlap many longitudinal modes. The superposition of many longitudinal modes means that one must retain the z gradient. Thus the wave equation reduces to

$$(\partial_t + c\partial_z)E = GP \tag{6}$$

where G is a scaled coupling constant.

Having a PDE, any mode expansion with reasonable wavenumber cut offs provides a large number of coupled ODE's, thus it is immaterial whether P and N are adiabatically eliminated, as in class A and B lasers, or whether they keep their dynamical character as in class C laser. Anyway, we have enough equations to see space time chaos.

Equations as (6) have been solved numerically in the sixties [10], to explain space variations on a length scale much smaller than L, as seen in regular or

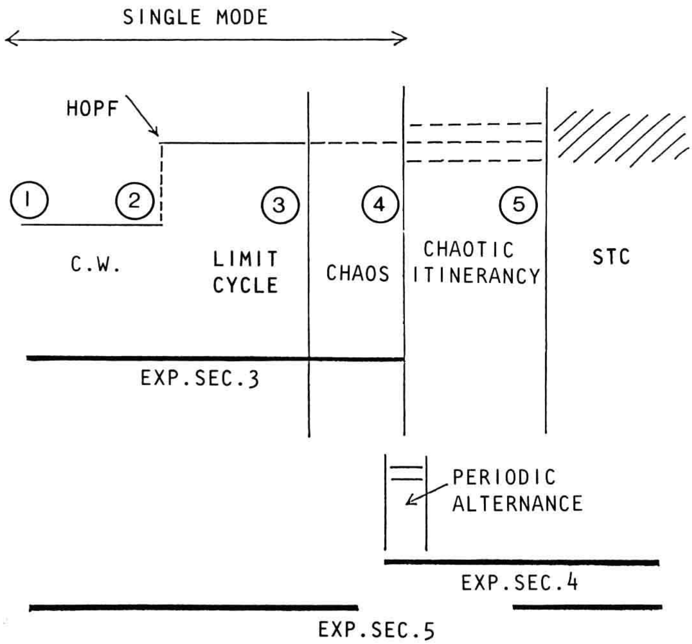

Fig. 2. Space-time complexity in optics. Qualitative plots of different behaviors observed in laboratory experiments and in numerical solutions of model equations. In Sec. 4 we report periodic alternance of modes instead of chaos in a single mode, between thresholds 3 and 4.

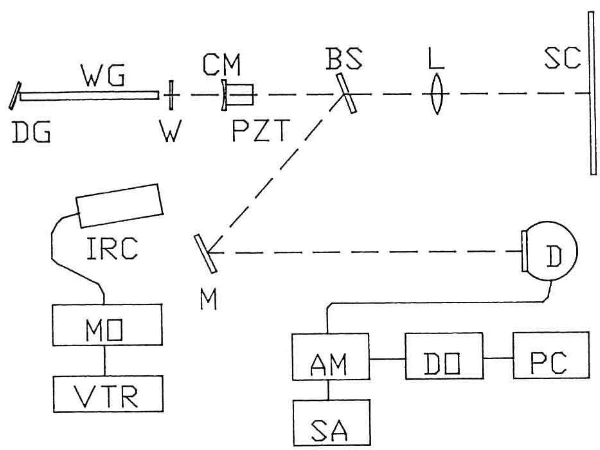

Fig. 3. Experimental set up: DG, diffraction grating: WG, Pyrex hollow cylindrical guide; W, ZnSe window; CM, coupling mirror; PZT, piezotranslator; BS, beam splitter (10%); S, screen; IRC, infrared camera; Mo, monitor; VTR, videotape recorder; D, detector; A, video-band amplifier; DO, digital oscilloscope; PC, personal computer; SA, spectrum analyzer (20-40 MHz).

erratic mode locking [11]. Fig. 2 collects a sequence of possible behaviors as one increases an intensive control parameter (the pump to loss ratio) for a cavity long enough to provide a high ratio of gain linewidth to free spectral range, or alternatively, as one increases an extensive parameter, that is, the latter ratio for a fixed pump- to-loss ratio. Since the free spectral range is given by $c/2L$, increasing the extensive parameter amounts to increasing the cavity length L.

The circled numbers 1 to 5 in Fig. 2 denote the transition points, and the solid lines denote the range of experimental observations reported in Secs. 3 to 5.

Threshold n. 1 is the usual laser threshold, whereby uncorrelated spontaneous emission selforganizes into single mode coherent laser action. Mathematically Eqs.(5) provide a pitchfork bifurcation, with critical divergence of the fluctuation amplitude and correlation time (critical slowing down). These transition phenomena have been experimentally demonstrated in the middle sixties in a series of experiments reported in Ref. 2.

An analogy with the Landau model of a single domain phase transition (no space features) stems from the trivial properties of such a bifurcation [12]. In order to consider space variations, one must couple Eq.(6) with the matter equations (second and third Eq.(5)). This was done by a mode expansion of Eq.(6), and a second threshold, n.2, which is a Hopf bifurcation toward an oscillatory regime, was introduced [13]. A more appropriate analogy with thermodynamic phase transitions, including space correlations, was developed on such a new set of equations [14].

A third threshold marks the onset of deterministic chaos in a single mode laser. In fact it is a cascade of bifurcations depending on the specific route to chaos, which is influenced by possible laboratory perturbations, as modulations or feedback [9]. The isomorphism of Eqs.(5) with Lorenz equations was first pointed out by Haken [15]. After that, a large amount of experimental and theoretical investigation has been devoted to chaos in a single mode laser.

Recent consideration of a space extended optical system [16] by a model made up of Eq.(6) and the last two Eqs.(5) has shown evidence of a further behavior, called "chaotic itinerancy". It consists in the jump from one slow manifold to another, i.e., from one quasi-attractor to another. At any time, a single mode with a chaotic behavior is present, but after a while it is replaced by another mode, and so on. Alternatively, in Sec. 4 we will show experimental evidence of a non chaotic, but periodic alternance of modes, schematized in the lower part of Fig. 2. As stressed in Sec. 4, the main indicator of chaotic itinerancy is that, while a local measurement provides a chaotic signal, measurement of the space correlations provides a highly correlated signal.

Above the threshold n. 5 we enter a new regime, called spatio- temporal chaos (STC) where a large number of modes coexist. This regime has been characterized on very general grounds by Hohenberg and Shraiman [8]. Suppose that we have a generic field $u(r,t)$ ruled by a PDE including nonlinear and gradient terms. Such is indeed the situation of our (1+1)- dimensional optical system. Let us take the field of deviations away from the local time average

$$\delta u(r,t) = u(r,t) - < u(r,t) >$$

where $< ... >$ denote time average. Under very broad assumptions, we can take the leading part of the correlation function as an exponential, that is,

$$C(r,r') = < \delta u(r,t)\delta u(r',t) > \simeq e^{-|r-r'|/\xi} \tag{7}$$

Whenever the correlation length ξ is larger than the system size $L(\xi > L)$ we have low dimensional chaos, that is, even though the system can be chaotic in time, it is coherent in space (single mode, in a suitable mode expansion). The corresponding chaotic attractor is low dimensional. In the opposite limit of $\xi << L$, a local chaotic signal is not confined in a low dimensional space. However a new outstanding feature appears. If we collect a local time series of data $\delta u(r,t)$ at a given point r, the corresponding statistical distribution $P(\delta u(r,t))$ is strongly non-Gaussian. No wander about that: after all δu stems