A. Fidlin

Nonlinear Oscillations in Mechanical Engineering

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With 150 Figures

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Preface

General

Oscillations are extremely important in all areas of human activities, for all sciences, technologies and industrial applications. Any development, any change can be interpreted as motion. Any motion is deeply connected with one of the most fundamental properties of nature – its ability to react with oscillations at any internal change or external influence.

Sometimes these oscillations are harmless, often they can be noticed as noise or cause wear. Vibrations, if they are not desired, can be dangerous. But sensibly organized and controlled vibrations may be pleasant (think of all kinds of music) or vitally important (heartbeat). If the oscillations are sufficiently small and the considered dynamical system is smooth, it can be very helpful to linearize it in the vicinity of some static or dynamic solution and use the corresponding powerful analytical methods. But in many practical cases the oscillations are either not small or the system is not smooth. In these cases a non-linear analysis becomes essential to understand physics of the system or of the process.

"To understand"- is the key word here. The main objective of any analysis is comprehension. The only systematic way to solve any problem or to improve any system is to understand its nature and foresee its behavior. There are different ways to obtain the knowledge of the object one is working with. The first and still the most important way is the experiment. It is the only way to get any objective information about nature. But this way is not a simple one. The problem is not only that it is often very difficult and expensive to perform real experiments. Even more important is the fact, that the measurement itself never helps a scientist (or an engineer) to understand anything, before he tries to interpret it. And the only way to interpret anything is modeling. However the experiment always remains the last proof of each theory or model. So like two eyes are necessary for spatial vision, both modeling and experiment are inevitable for physical understanding.

Different types of modeling are often used in modern fundamental and applied sciences. The main rule says: Models must be as simple as possible and as accurate as necessary. In practice it means, a simple analytic model is often better than complex numeric one, if it is sufficiently accurate. On the other hand, if simple models are not able to describe significant properties of a natural object, the numeric or even the natural modeling (for example in hydro- and aerodynamics) are the only efficient way to achieve comprehension.

The same statement can be applied not only to natural sciences, but even to mathematics. However, the experiments here are mostly numeric. The greatest mathematician of the late 19th and early 20th century, Henri Poincaré, wrote: "Usually an equation was considered to be solved if the solution had been expressed in a finite number of known functions. But this is only possible in one case out of hundred. What we always could do, or better should do, is to solve the problem qualitatively. This means trying to find the general form of the curve which tracks the unknown function." (Henri Poincaré, Science et Méthode).

This idea of Poincaré gave the main direction to the development of applied mathematics and mechanics in the 20th century, and it did not loose its actuality even now. His methods to find periodic solutions of the perturbed ordinary differential equations and to analyze their stability [98], caused a mental revolution both in the theory of differential equations and in the nonlinear mechanics. It was the starting shot for the development of the perturbation methods, being now one of the most powerful groups of analytic methods in the theory of nonlinear oscillations.

It is interesting that the Poincaré's statement is also valid for applications. Engineers are seldom interested in particular solutions. Usually the parameters of the system are known with some grade of uncertainty and the initial conditions are very difficult to control. Hence an engineer is interested first of all in general tendencies in the behavior and evolutions of a system, he is working on. To explain the main qualitative phenomena and to predict the qualitative influence of specific parameters is the task of an analyst and the base for design. Thus approximate analytic methods are both necessary and useful.

Nonlinear Oscillations

Dozens of brilliant books were written on different aspects of vibrations both from theoretical and practical points of view. At least the classical works by Lord Rayleigh [108], Poincaré [98] and Timoshenko et al. [126] should be mentioned here. Even these three titles demonstrate the main difficulty for students and practical engineers to find an appropriate balance between sophisticated mathematical methods and problems, which are relevant for applications.

There are excellent textbooks and monographs devoted to different mathematical aspects of nonlinear oscillations. Bogoliubov and Mitropolskii [23], Bolotin [25], Nayfeh [79], Arnold [8], Sanders and Verhulst [114], Guckenheimer and Holmes [44], Mitropolskii and Nguyen [75], Nayfeh and Mook [80], Volosov and Morgunov [134] and Verhulst [131] can be mentioned here. Being significant contributions to the development of mechanics and approximate methods in the theory of ordinary differential equations and dynamical systems, these books remain almost unavailable for the majority of mechanical engineers, first of all, because of their mathematical language.

Books intended for applied scientists and practical engineers are very seldom. Most of them are concentrated on qualitative discussion of practically important particular effects without addressing generality and mathematical rigor of the used methods. The most versatile book of this type is no doubt Panovko and Gubanova [85]. But it is also necessary to point to excellent specialized monographs on oscillations in systems with collisions [10, 15, 58, 62, 77], systems with friction [49, 104, 130], machines with high frequency vibrational excitation [15, 17, 20, 78], autoparametric resonance [129] and chaotic vibrations [76].

"Nonlinear Oscillations in Mechanical Engineering"

The recent book is concentrated on the effects connected with the nonlinearities usual in mechanical engineering. These nonlinearities are caused, first of all, by contacts between different mechanical parts. So the main part of this book is devoted to oscillations in mechanical systems with discontinuities caused by dry friction and collisions. Another important source of nonlinearity is caused by rotating unbalanced parts usual in various machines and variable inertias occurring in all kinds of crank mechanisms, for example in combustion engines.

This book is devoted to nonlinear oscillations and is written for advanced undergraduate and postgraduate students, but it may be also helpful and interesting for both theoreticians and practitioners working in the area of mechanical engineering at universities, in research labs or institutes and first of all in the development departments of industrial corporations.

Mathematical Methods in This Book

Perturbation methods, especially averaging and multiple scales are the basic approach used in this book. The objective here is to adopt these methods to the described class of problems and make their use convenient for mechanical engineers without loosing mathematical correctness.

It was also Poincaré who justified the use of divergent series and introduced the concept of asymptotic analysis. The next important step was done in the middle of the 20th century in Russia. Krylov, Bogoliubov, Mitropolskii and their colleagues [23, 24, 66, 74, 75, 111, 134] suggested and justified the averaging method. This method allows the asymptotic analysis not only of stationary or periodic solutions, but it is also suitable for transient processes. However, the first successful attempts to link perturbation methods with classical variation of parameters were done by van der Pol [99] and corresponding ideas can be found even earlier in the works of Lagrange [68]. The book by Bogoliubov and Mitropolskii [23] gave a strong impulse for the further development of asymptotic methods both due to clear mathematical proof of the approach and due to the great variety of considered applications. It is still an interesting and inspiring source for scientists.

Fast and Slow Motions

Averaging method clearly displays the main feature of different phenomena and processes of fundamental and applied interest taking place in mechanical systems, which allows developing and using asymptotic methods in nonlinear oscillations. Very often motions of these systems can be split into some slow evolution and overlaying "fast" vibrations of a high frequency at a small amplitude. These slow motions describing the evolution of the system are, as a rule, of the main interest for the researcher.

Several scientists have used this property as the fundamental background for the development of further methods and approaches. Two of them should be mentioned here. The first one is the method of multiple scales. It was developed by Nayfeh and his colleagues [79, 80]. This method is substantially very close to the averaging method, but due to its straight forward formulation is very popular in physical applications. First of all, the use of multiple scales is very simple for differential equations with partial derivatives, even if its accuracy is not always proved.

The same can be said about the "method of the direct separation of motions". It was originated in the works of Kapitsa [55, 56]. This method was most generally formulated by Blekhman [16 – 21], who also gave numerous examples of its use for different problems in mechanics and physics. The method of the direct separation of motions is even easier to use than the multiple scales. Its accuracy is proved for the most typical cases and it was successfully used for systems described by both ordinary and partial differential equations [19, 20, 53,119 – 125].

The main uncertainty connected with both multiple scales and direct separation of motions is that if the considered problem is even a little bit aside the typical applications the user never knows if the results are correct or not and the method itself does not give any instrument to control its accuracy. Averaging to the contrary contains a clear way for its mathematical validation including sensible accuracy control. If the requirements of the corresponding theorems are not fulfilled in a practical problem, there is always a chance to expand the method's applicability by formulating and proving new theorems. This area was and remains the mathematicians' domain. The author would be very pleased if he could attract their attention to numerous and diversified engineering problems, even though only an infinitesimally small subset of this inexhaustible field is touched in the present book.

Structure

The book is structured as follows. A short **introduction** to the problems under consideration is given in the first chapter. It includes the standard perturbation techniques alongside the usual simplest descriptions of dry friction and collisions between rigid bodies.

Chapter 2 is devoted to vibrations in systems with dry friction. The discussion encloses both the self excited oscillations due to the negative friction gradient and the friction caused flutter alongside the vibration caused transportation.

Oscillations in systems with almost elastic collisions are discussed in **Chapter 3**. The analysis is based on the ideas of the unfolding transformations, which give a clear and transparent framework for analysis of one dimensional systems restricted from one side (for example an oscillator near a wall) and from both sides (for example the same oscillator in a clearance).

Systems with strong energy dissipation are discussed in **Chapter 4**. The main idea here is to separate the dissipative subsystem, which moves in many important cases as a slave of the almost conservative subsystem (at least in the first order approximation). This approach is consequently applied both to systems with strong linear damping and to systems with inelastic collisions.

Chapter 5 is devoted to the problem of the significantly nonlinear resonance, which occurs always, when the power of an exciter is comparable with the energy demand of the machine. It is actually the case in all real machines otherwise their drive would be too powerful and expensive. So the practical importance of the nonlinear resonance can be hardly overestimated.

The basic ideas of analysis and elementary effects in systems subjected to strong high frequency excitation are discussed in **Chapter 6**. Stiffening, softening, biasing alongside smoothing of dry friction are the main effects illustrated by simple examples. Misbehavior of the "optimally" controlled pendulum under the influence of the HF excitation is the advanced example combining several approaches introduced in the previous chapters.

Further development and general analysis of systems subjected to high frequency excitation is given in **Chapter 7**. Especially results concerning systems excited due to oscillating inertial coefficients are relevant for applications.

All the analysis in the present book is based on the appropriately modified averaging. The relevant theorems are formulated and explained qualitatively. Mathematical proofs are given in the **Appendixes**, which can be omitted by readers interested in applications, but are strongly recommended for those interested in the development of theoretical approaches.

Acknowledgements

The book summarizes and supplements the investigations of the author, published in different scientific journals in different languages – firstly in Russian, then in English.

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Alexander Fidlin

Contents

1. Introduction

1.1. Usual Sources of Nonlinearity in Mechanical Engineering

The world around us and we ourselves are inherently nonlinear. The simplest experiment illustrating this statement is an attempt to bend a wooden beam. As long as the load is small, the deflection of the beam is approximately proportional to the applied force. But at some sufficiently large level the beam will simply break. This strong and definitely irreversible change is an elementary example of nonlinear behavior illustrating an important feature enforcing us to formulate the first statement more precisely. The world is nonlinear, but in many cases, if we consider only small influences and changes, the linear approximation is often sufficient to understand, predict and control its behavior.

Nonlinearities and their consequences in the physical and technical world are highly diversified and the development of the corresponding theoretical framework and mathematical language is still in its infancy. We would like to start with several examples demonstrating the most usual sources of nonlinearity in mechanical engineering.

1.1.1 Geometrical Nonlinearities

The first and the simplest one are geometrical nonlinearities arising form pure kinematics. The first example shows the pendulum (*cf.* Fig. 1.1), whose dynamics is governed by the following equation:

$$\ddot{\varphi}+\frac{g}{l}\sin\varphi=0 \tag{1.1}$$

For small oscillations around the down pointing equilibrium $\varphi=0$ this equation can be linearized, but if one is interested in large oscillations or even in the rotational motions of the pendulum its nonlinearity becomes significant.

Another example is based on the crank mechanism usual in all kinds of machines (Fig. 1.2). It consists of a rotating rod, which is attached to a fixed point by a spring with stiffness C and free length x_0. We assume that the spring is linear (the last statement means that the deflection of this spring is proportional to the applied force, irrespective of its magnitude.) The governing equation for this system has the following form:

$$\ddot{\varphi} = \frac{CL\left(\sqrt{L^2 + l^2 - 2Ll\cos\varphi} - x_0\right)}{ml\sqrt{L^2 + l^2 - 2Ll\cos\varphi}}\sin\varphi \tag{1.2}$$

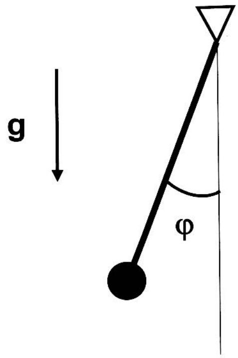

Fig. 1.1. The mathematical pendulum is one of simplest examples of geometrically nonlinear systems

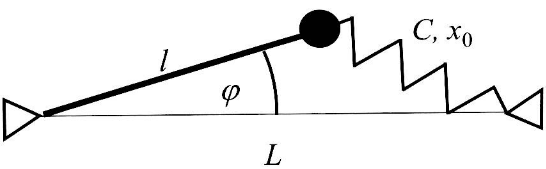

Fig. 1.2. The geometrically nonlinear crank

1.1.2 Physical Nonlinearities

The assumption concerning the linearity of the spring is also correct only for small deflections. Both rubber (Fig. 1.3) and steel (Fig. 1.4) demonstrate nonlinear relationships between stress and strain if the applied load is sufficiently large.