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Preface

This book is written for an interdisciplinary readership of graduate students and researchers interested in nonlinear dynamics, stochastic processes, statistical mechanics on the one hand and high energy physics, quantum field theory, string theory on the other. In fact, one of the goals that I had in mind when writing this book was to make particle physicists become interested in nonlinear dynamics, and nonlinear physicists become interested in particle physics. Why that? Didn't so far these two subjects evolve quite independently from each other? So what is this book about?

Mathematically, the subject of the book are coupled map lattices exhibiting spatio-temporal chaotic behaviour. Physically, the subject is a topic that lies at the heart of elementary particle physics: There are about 25 free parameters in the standard model of electroweak and strong interactions, namely the coupling strengths of the three interactions, the fermion and boson masses, and various mass mixing angles. These parameters are not fixed at all by the standard model itself, they are just measured in experiments, and a natural question is why these free parameters take on the numerical values that we observe in nature and not some other values. It will turn out that the answer is closely related to certain distinguished types of coupled map lattices that we will consider in this book as suitable models of vacuum fluctuations. These dynamical systems, called 'chaotic strings' in the following, are observed to have minimum vacuum energy for the observed standard model parameters. They yield an extension of ordinary quantization schemes which can account for the free parameters.

In this sense this book deals with both, nonlinear dynamics and high energy physics. So far only very few original papers have been published on this very new subject. With the current book I hope to make these important new applications for coupled chaotic dynamical systems accessible to a broad readership.

The book consists of 12 chapters. The first few chapters will mainly concentrate onto the theory of the relevant class of coupled map lattices, their use for second quantization purposes, and their physical interpretation in terms of vacuum fluctuations. In the later chapters concrete numerical results are presented and these are then related to standard model phenomenology. Sections marked with an asterisk can be omitted at a first reading, these sections deal with interesting side issues which, however, are not necessary for the logical development of the following chapters. In view of the fact that (unfortunately!) many readers may not have the time to read this book from the beginning to the end, I included a very detailed summary as a self-contained chapter 12. This summary contains the most important concepts and results of this book and is written in a self-consistent way, i.e. no knowledge of previous chapters is required.

The research described in this book developed over a longer period of time at various places. I started to work on the relevant types of coupled map lattices during my stay at the Niels Bohr Institute, Copenhagen, in 1992 and continued during a stay at the University of Maryland in 1993. Some important numerical results, now described in section 7.2 and 8.5, were obtained at the RWTH Aachen in 1994 as well as during a visit to the Max Planck Institute for Physics of Complex Systems, Dresden, in 1996. The main part of the work was done at my home institute, the School of Mathematical Sciences at Queen Mary, University of London, as well as during long-term research visits to the Institute for Theoretical Physics at the University of California at Santa Barbara in 2000 and to the Newton Institute for Mathematical Sciences at Cambridge in 2001. The hospitality that I enjoyed during these visits was very pleasant, and the nice research atmosphere was really inspiring.

The number of people from which I learned during the past years and who thus indirectly contributed to this book is extremely large— too large to list all these individuals separately here! So at this point let me just thank all of them in one go.

London, February 2002
Christian Beck

Introduction

This book deals with new applications for coupled map lattices in quantum field theories and elementary particle physics. We will introduce appropriate classes of coupled map lattices (so-called 'chaotic strings') as suitable spatio-temporal chaotic models of vacuum fluctuations.

From a mathematical point of view, coupled map lattices are high-dimensional nonlinear dynamical systems with discrete space, discrete time and continuous state variables. They were for the first time introduced by Kaneko in 1984 [Kaneko (1984)]. The dynamics is generated by local maps that are situated at the sites of a lattice. There can be various types of couplings between the maps at the lattice sites, for example global coupling, exponentially decreasing coupling or diffusive coupling. For globally coupled systems, typically each lattice site is connected to all others with the same coupling strength. In the exponentially decreasing case the coupling strength decays exponentially with distance. For diffusively coupled map lattices there is just nearest-neighbor coupling, corresponding to a discrete version of the Laplacian. The latter one is the most relevant coupling form for applications in quantum field theories. Very complicated periodic, quasi-periodic or spatio-temporal chaotic behaviour is possible in all these cases (see the color plates in chapter 2 and 4 for some illustrations).

Generally, the spectrum of possibilities of spatio-temporal structures that can be generated by coupled map lattices is extremely rich and has been extensively studied in the literature, the emphasis being on the bifurcation structure [Bunimovich et al. (1996); Just (1995); Amritkar et al. (1993); Gade et al. (1993); Amritkar et al. (1991); Pikovsky et al. (1991)], Liapunov exponents [Yang et al. (1996); Torcini et al. (1997b);

Kaneko (1986b); Isola et al. (1990)], traveling waves [Carretero-González (1997); He et al. (1997)], phase transition-like phenomena [Grassberger et al. (1991); Cuche et al. (1997); Blank (1997); Marcq et al. (1996); Boldrighini et al. (1995); Keller et al. (1992a); Houlrik et al. (1992); Miller et al. (1993); Gielis et al. (2000)], the existence of smooth invariant measures [Baladi et al. (1998); Jiang et al. (1998a); Chaté et al. (1997); Mackey et al. (1995)], synchronization [Lemaitre et al. (1999); Bagnoli et al. (1999); de San Roman et al. (1998); Jiang et al. (1998b); Wang et al. (1998); Ding et al. (1997)], control [Gade (1998); Egolf et al. (1998); Parekh et al. (1998); Mondragon et al. (1997); Ohishi et al. (1995)] and many other properties. Applications for coupled map systems have been pointed out for various subjects, among them hydrodynamic turbulence [Beck (1994); Hilgers et al. (1997b); Hilgers et al. (1999a); Bottin et al. (1998)], chemical waves [Kapral (1993)], financial markets [Hilgers et al. (1997a)], biological systems [Bevers et al. (1999); Losson et al. (1995); Martinezmekler et al. (1992); Dens et al. (2000)] and, at a much more fundamental level, for quantum field theories [Beck (1998); Beck (1995c)]. In this book we will concentrate on the quantum field theoretical applications.

A possible way of embedding coupled map lattices into a general quantum field theoretical context is via the Parisi-Wu approach of stochastic quantization [Parisi et al. (1981); Damgaard et al. (1988); Damgaard et al. (1984); Gozzi (1983); Namiki et al. (1983); Batrouni et al. (1985); Rumpf (1986); Ryang et al. (1985); Breit et al. (1984); Albeverio (1997)]. In this approach a quantized field is described by a stochastic differential equation evolving in a fictitious time coordinate. Essentially, spatio-temporal Gaussian white noise is added to the classical field equation in order to second quantize it. The fictitious time is different from the physical time; it is an additional parameter that is a useful tool for the quantization of classical fields. Quantum mechanical expectations can be calculated as expectations with respect to the realizations of the stochastic process. It is now possible to generate the spatio-temporal Gaussian white noise of the Parisi-Wu approach by a weakly coupled chaotic dynamics on a very small scale. In particular, if we choose e.g. Tchebyscheff maps to locally generate the 'chaotic noise', the convergence to Gaussian white noise under rescaling can be proved rigorously [Beck et al. (1987); Billingsley (1968); Chernov (1995); Beck (1990b); Beck (1995a); Chew et al. (2002); Zygmund (1959)]. If we quantize by means of such a chaotic dynamics, no difference occurs on large (standard-model) scales, since on large scales the chaotic behavior of the maps is very well approximated by Gaussian white noise, leading to ordinary quantum field theoretical behavior. However, on very small scales (e.g. the Planck scale or below) there are interesting differences and new remarkable features. The view that the ultimate theory underlying quantum mechanical behaviour on a small scale is a deterministic one exhibiting complex behaviour has also been advocated by t'Hooft [t'Hooft et al. (1992); t'Hooft (1997a); t'Hooft (1997b)].

How can a discrete chaotic noise dynamics arise from an ordinary field theory? How can there be a dynamical origin of the noise? We will show that ordinary continuum field theories with formally infinitely large self interaction directly and intrinsically lead to diffusively coupled map lattices exhibiting spatio-temporal chaos. This limit of large couplings stands in certain analogy to the anti-integrable limit of Frenkel-Kontorova-like models [Aubry et al. (1990); Baesens et al. (1993)]. One of our main examples is a self-interacting scalar field of ϕ^4-type, which leads to diffusively coupled cubic maps in the anti-integrable limit. A discrete dynamics with strongest possible chaotic properties can then be obtained, which can be used for stochastic quantization. One can then consider coupled string-like objects in the noise space, which, to have a name in the following, will be called 'chaotic strings'. We will use this model and some related ones as dynamical models of vacuum fluctuations. The chaotic dynamics will be scale invariant, similar as fully developed turbulent states in hydrodynamics exhibit a selfsimilar dynamics on a large range of scales [Bohr et al. (1998); Frisch (1995); Arad et al. (2001); Pope (2000); Ruelle (1982)]. In fact, chaotic strings behave very much like a turbulent quantum state. The probabilistic aspects of our model can be related to a generalized version of statistical mechanics, the formalism of nonextensive statistical mechanics [Tsallis (1988); Tsallis et al. (1998); Abe (2000); Abe et al. (2001); Beck (2001b); Beck et al. (2001); Plastino et al. (1995); Wilk et al. (2000); Pennini et al. (1995); Johal (1999); Cohen (2002)].

What can we learn from these types of statistical models? We will show that the assumption of a dynamical origin of vacuum fluctuations, due to chaotic strings on a small scale, can help to explain and reduce the large number of free parameters of the standard model. The guiding principle for this is the minimization of vacuum energy of the chaotic string. We will provide numerical evidence that the vacuum energy is minimized for certain distinguished string coupling constants. These couplings are numerically observed to coincide with running standard model couplings as well as with gravitational couplings, taking for the energy scales the masses of the known quarks, leptons, and gauge bosons. In this way our approach can help to understand many of the free parameters of the standard model, using concepts from generalized statistical mechanics.

The approach described in this book is new and different from previous attempts to calculate, e.g., the fine structure constant [Eddington (1948); Gilson (1996)]. It is much more in line with a suggestion made by R.S. MacKay in his book [MacKay (1993)] (p. 291), namely that the fine structure constant might be derived as a property of a fixed point of an appropriate renormalization operator. As we shall see in chapter 7, the relevant dynamical systems are indeed the chaotic strings, the renormalization operator is a scale transformation, and the renormalization flow corresponds to an evolution equation for possible standard model couplings in the fictitious time of the Parisi-Wu approach. This renormalization flow is not only relevant for the fine structure constant but provides information on all the other standard model parameters as well.

The minima of the vacuum energy of chaotic strings can be determined quite precisely and allow for high-precision predictions of various running electroweak, strong, Yukawa and gravitational coupling constants. These can then be translated into high-precision estimates of the masses of the particles involved. Moreover, evolving the couplings to higher energies grand unification scenarios can be constructed. In this sense the approach described in this book yields an interesting amendment of the usual formulation of the standard model. Based on the assumption that chaotic noise strings exist in addition to the continuous standard model fields, we obtain high-precision predictions of the free parameters of the standard model (see Tab. 12.4 in chapter 12), which can be checked by experiments. Our chaotic models yield rapidly evolving dynamical models of vacuum fluctuations which, as we will show in detail in the following chapters, have minimum vacuum energy for the observed standard model parameters.

Can we further embed the chaotic strings into other theories, for example superstring and M-theory [Green et al. (1987); Kaku (1988); Polchinski (1998); Polchinski (1999); Witten (1997); Banks et al. (1997); Gauntlett (1998); Susskind (1995); Antoniadis et al. (1999a); Gubser et al. (2001)], or relate them to models of 2-dimensional quantum gravity [Gross et al. (1990)] or string cosmology [Ghosh et al. (2000); Melchiorri et al. (1999); Veneziano (1997); Lidsey (1998)]? Could the very recently established contact between string field theory and stochastic quantization yield a suitable embedding [Polyakov (2001); Baulieu et al. (2001); Periwal (2000); Ennyu et al. (1999)]? All this is possible but open at the moment. Generally it should be clear that chaotic strings are very different from superstrings. The latter ones evolve in a regular way, the former ones in a chaotic way. Still it is reasonable to look for possible connections with candidate theories of quantum gravity, such as superstring theory or M-theory. These theories require an extension of ordinary 4-dimensional space-time to 10 (or 11) space-time dimensions. The 6 extra dimensions are thought to be 'compactified', i.e. they are curled up on small circles with periodic boundary conditions. One possible way to embed chaotic strings is to assume that they live in the compactified space of superstring theory. The couplings of the chaotic strings can then be regarded as a kind of inverse metric in the compactified space, determining the strength of the Laplacian coupling. The analogue of the Einstein equations as well as suitable scalar field equations then lead to the observed standard model coupling constants, fixed and stabilized as equilibrium metrics in the compactified space.

Let us give an overview over the following chapters. In chapter 1 we will generalize the stochastic quantization method to a chaotic quantization method, where the noise is generated by a discrete chaotic dynamics on a small time scale. In chapter 2 we will introduce chaotic strings and discuss some of their symmetry properties. Two types of vacuum energies associated with chaotic strings are discussed in chapter 3, namely the self energy and the interaction energy of chaotic strings. Spontaneous symmetry breaking phenomena for chaotic strings and their higher-dimensional extensions will be investigated in chapter 4. In chapter 5 we will show why chaotic strings can be regarded as simple selfsimilar dynamical models of vacuum fluctuations, and introduce webs of Feynman graphs that describe this physical interpretation. In chapter 6 we will relate the chaotic string dynamics to a thermodynamic description of the vacuum, using concepts from generalized statistical mechanics and information theory. In chapter 7 we will consider analogues of Einstein field equations that make *a priori* arbitrary standard model couplings evolve to the stable zeros of the interaction energy of chaotic strings. We will provide extensive numerical evidence that the smallest stable zeros of the interaction energy numerically coincide with running electroweak and strong coupling strengths, evaluated at the smallest fermionic and bosonic mass scales. In chapter 8 we will consider suitable self-interacting scalar field equations for possible standard model couplings, which make *a priori* arbitrary couplings evolve to the local minima of the self energy of the chaotic strings. We will present extensive numerical evidence that the self energy has local minima that numerically coincide with various Yukawa, gravitational, electroweak and strong couplings at energy scales given by masses of the three families of quarks and leptons. In chapter 9 we extend the analysis to bounded quark states, and provide numerical evidence that the total vacuum energy has minima for running strong coupling constants that correspond to the mass spectrum of light mesons and baryons. The precision results of chapter 7 and 8 will be used in chapter 10 to evolve the standard model couplings to much higher energies and to construct grand unification scenarios. In chapter 11 we will discuss the connection with extra dimensions and describe possible scenarios at the Planck scale and beyond. Finally, chapter 12 is a detailed, self-contained summary of the most important concepts and results described in chapter 1-11.

Contents