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RHEOLOGICAL FUNDAMENTALS OF SOIL MECHANICS

S.S. VYALOV

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PREFACE

Soil rheology is a branch of soil mechanics investigating the origin of, and the time-dependent changes in, the stressed and strained state of soil. However, the words "rheological fundamentals" in the title of the book are interpreted by the author in a wider sense. He considers rheology as the science concerned with how the state of stress and strain is formed and altered in a body on the one hand, and with the particulars of the body's behaviour failing to fit the traditional concepts of elasticity and plasticity on the other. Following Academician Sedov*[1] one may say that the objective of rheology is to produce novel models describing the behaviour of media other than idealized bodies. Soils belong to media of this kind.

As is known, soil mechanics examines and predicts the behaviour of soils in response to either external or internal forces. Engineering problems are solved by examining the physical conditions of the soil, which are determined by its geological origin, and by employing continuum mechanics mathematics. However, unlike this branch of science, proceeding from the concept of continuity of bodies, soil mechanics treats soil as a dispersed porous medium capable of irreversibly changing its volume, i.e. of compacting.

Modern soil mechanics is based on the following three principles.

(1) The pressure and volumetric changes in a soil, i.e., the changes in void ratio, are assumed to be in direct proportion as are the shear stresses and changes in shape.

(2) The compaction of soil with time (consolidation) takes place due to the migration of water through the voids according to the laws of percolation.

(3) Soil, as a dispersed medium, displays not only inter-particle cohesion but internal resistance as well; these properties determine its resistance to failure.

The role of the above principles in developing soil mechanics can hardly be overestimated. They have been instrumental in formulating the theories of linear soil deformation, percolation consolidation, and limiting equilibrium. Moreover, a wide range of engineering problems have been solved on the basis of the above principles by using mathematics of the theories concerned.

At the same time, the principles mentioned have somewhat schematized the behaviour of soil; in fact, it appears to be more complex when the soil sustains a load. For example, the deformation of clay soils is significantly influenced by time-dependent phenomena such as creep, relaxation and the deterioration of strength due to a long-term load application. In other words, it may be said that clay soils are capable of changing their state of stress and strain with time.

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*[1] For the references see Chapter 1.

Another noteworthy feature of real soils is the non-linearity of the stress–strain relation, especially if the strain is developing with time. Furthermore, one must keep in mind that internal friction – a cardinal property of soils – manifests itself in both limiting and sublimiting conditions, influencing the manner in which the deformation is developing. It will be shown that this particular aspect of soil behaviour is attributed to the difference in the resistance which the soil offers to compressional and tensional deformation. This gives rise to such anomalies as changes in volume under a shear stress (dilatancy) and changes in shape due to all-around pressure, etc.

Thus, the actual behaviour of soil differs substantially from schematized concepts. Notwithstanding the fact that schematization has frequently been helpful in arriving at results acceptable in engineering practice, many cases are recorded where ignoring all aspects of soil behaviour has led to significant departures from reality. A few examples to that end are: structural deformations induced by long-term creep; failure of slopes and retaining walls because they have been designed in terms of instantaneous strength rather than for the long term; and actual settlements being a far cry from computed values because consideration of their non-linearity was neglected.

By taking into account all peculiarities of soil deformation we can obtain precise knowledge of the soil properties. We can then improve our analytical predictions so that they approach actual soil behaviour.

From what is said here it appears appropriate that all particulars of soil behaviour be investigated and systematized. All the more so because the Building Codes and Regulations, 1972 (BC and R II-A.10.71, Structures and Foundations; Main Aspects of Design) make it a point that "the physical non-linearity, plastic and rheological properties of materials and soils be taken into account whenever necessary in analysing structures and the soil below foundations". It is exactly these problems that are tackled in this book.

To assist students in comprehending the subject, some general aspects of continuum mechanics, including well-known notions from soil mechanics, are considered in this book along with the main theme. It was the author's intention to present this material in an optimum combination with recent findings in the field of rheology and soil mechanics (including some theoretical aspects of non-linear deformation), which may also be of interest to the reader with an advanced knowledge of the subject. Accordingly, the book may be subdivided into three parts in ascending order of complication and novelty.

By referring to "rheological fundamentals" in the title of the book, the author wishes to stress the impact of rheology on soil mechanics in general.

In considering the basic concepts of classical soil mechanics, the author followed the guidelines in the work by Tsytovich (1976), a generally recognized textbook. The problems of soil physics are discussed in the light of the well-known treatise by E.M. Sergeev et al. (1961).

The three parts into which the book may be subdivided are as follows. The first part (Chapters 1–4) deals with basic rheological concepts and terms, the physics of soil, principles of stress–strain theory, elasticity, plasticity and viscosity – all cardinal rheological properties.

The second part (Chapters 5–10) explains the rheological processes taking place in soils, such as creep and long-term strength, which are examined by the author with allowance for non-linear deformation. Along with the known phenomenological theories, attention is paid to the novel kinetic (physical) theory of deformations and long-term strength.

The third part (Chapters 11 and 12) outlines the generalized theory of soil deformation. It explains why soil offers different resistances to tensional and compressional deformations and derives the generalized rheological equation of state, enabling the effect of the three stress tensor invariants on the changes in shape and volume to be taken into account.

Chapter 13 exemplifies solutions, from the standpoint of the theory discussed, of some problems facing soil mechanics.

Chapter 14, specially written for this particular edition, reviews mathematical models representing the actual behaviour of a soil under load and provides for engineering problems numerical solutions obtained with the aid of these models on a computer.

The author wishes to express his deep satisfaction with the fruitful cooperation in examining the problems outlined of Yu.K. Zaretsky, S.E. Gorodetsky, N.K. Pekarskaya, R.V. Maximyak and other colleagues from the Laboratory of Frozen Soil Mechanics, of which he is in charge, at the Foundation Research Institute. The author expresses his gratitude to Prof. G.A. Geniev, D.Sc. (Eng.), and Prof. R.S. Ziangirov, D.Sc. (Geol. Min.), for reviewing the book and also to Prof. Sergeev, Corresponding Member of the USSR Academy of Science, Head of the Department of Pedology and Engineering Geology, Moscow State University, who read the manuscript and made valuable comments. The author is also thankful to T.P. Vyalova, M.E. Slepak and V.P. Razbegin for their assistance in preparing the manuscript for printing.

S.S. VYALOV

CONTENTS

Chapter 1

RHEOLOGICAL PROPERTIES OF SOILS

1.1 PECULIARITIES OF SOIL DEFORMATION

Non-linear stress–strain relation

An all-around pressure applied to a soil irreversibly changes its volume, whereas a shear stress displaces mineral soil particles and aggregated lumps relative to each other, distorting their shape.

In the general case, a volumetric strain changes non-linearly with load, as does a shear strain (Fig. 1-1). The shear strain continuously increases with the stress, but the intensity of the volumetric strain decreases while the load increases and the strain itself approaches a certain limit at which final compaction of the soil occurs.

Real and approximate settlement curves

It is known that the real relation between a load and the settlement of a foundation is represented by a curve having two characteristic points. One indicates a critical load, p_c, under which the soil compaction phase comes to an end and the shear phase begins. The other point corresponds to an ultimate load, p_u, at which the zone of limiting state develops to its full extent and the bearing capacity of the foundation becomes completely exhausted.

The existing methods of foundation engineering are based on the assumption that a soil subjected to a load less than p_c (or, to be precise, less than R, which is the design load close to p_c) can be treated as a linearly deforming medium obeying Hooke's law. When the load reaches the value of p_u, the soil behaviour is described by the theory of limiting equilibrium and the soil can be treated in accordance with the Mohr-Coulomb law of the limiting state of stress.

Thus, the real "settlement versus load" curve illustrated in Fig. 1-2 can be approximated by dashed line *Obc*. This involves the following assumptions. In the sublimiting state, we consider segment *Oa* limited by the value p_c rather than the entire curve *Oac*. In fact, the soil is capable of effectively sustaining a load over the range $p_c \leqslant p \leqslant p_u$, commonly disregarded during analysis, providing its bearing capacity is not exhausted. Neglecting to take the above loading into consideration is neither reasonable nor logical. Although analysis in the sublimiting state of stress is aimed at limiting the settlement, S, which should not exceed a specified ultimate value, $S \leqslant S_u$, this does not imply that S_u is correlated with p_c. The ultimate settlement is dependent upon the allowable deformation of the structure, the load causing this settlement

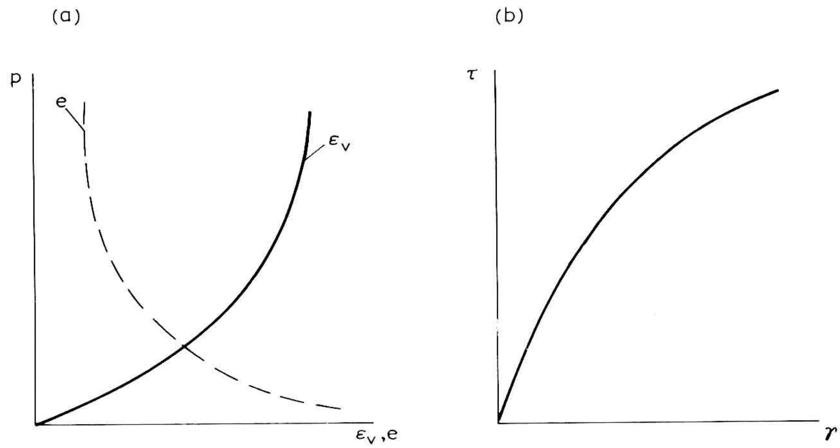

Fig. 1-1. Stress–strain relationship in soil: (a) volumetric strain; (b) shearing strain.

being either smaller or greater than p_c. Ironically, we calculate deformation operating with loads within the confines of the linear load–settlement relation. Taking the non-linear dependency of soil settlement upon load into account apparently creates the prospect of calculating settlements over the entire range of stresses.

We have approximated above the real settlement-versus-load curve by the dashed line of Fig. 1-2. This is assuming we differentiate soil behaviour. Segment *Oa* is represented by a model of a Hookean solid whose properties are expressed in terms of deformation. In this case, such a cardinal property as internal friction is ignored in spite of the unquestionable fact that frictional resistance is inherent in both limiting and sublimiting states of stress. This can be readily proved by a simple shear test in a conventional tester, using a load smaller than the ultimate one. The shear strain

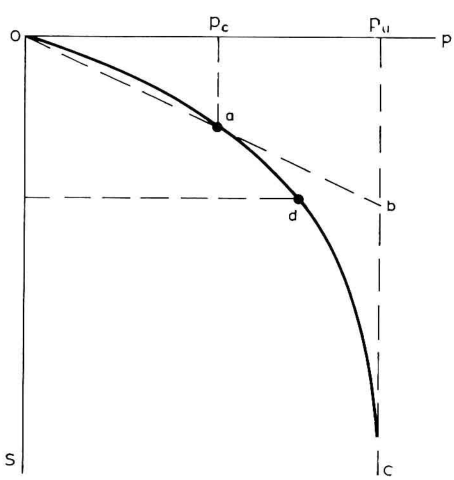

Fig. 1-2. Real and approximated settlement versus load curves.

resulting from the application of a changing normal stress will decrease with an increase in the stress. This is the manifestation of frictional resistance.

Segment *bc*, characterizing soil behaviour in the limiting state, is represented by a model of a Mohr-Coulomb body whose properties are expressed in terms of strength with due regard for the frictional resistance. The strength characteristics involved have no bearing on the deformation characteristics serving as the criterion of soil behaviour in the sublimiting state, although the strength and deformation properties of a soil are apparently correlated, being a manifestation of the same forces of soil particle interaction. Studying the sublimiting state involves ascertaining the fields of stress and strain, while investigating the limiting state requires estimating the stresses only. The problem in this latter case is thus reduced to evaluating the ultimate load, p_u, deformations being neglected. However, in many situations it is necessary to determine not only the ultimate load but also the resulting deformation, as is the case in assessing the behaviour of soils with explicit rheological properties; when exposed to an ultimate load they display a non-attenuating creep rather than the loss of stability or rapid failure.

One of the problems facing the rheology of soils is to develop a comprehensive soil model which is helpful in describing the pattern of deformation over the entire range of stresses up to the limiting value. Such a model should take account of the frictional resistance in the sublimiting state, providing thereby a correlation between the strength and deformation characteristics. All transient phenomena should also be taken into consideration while designing the model, since the process of soil deformation is accompanied by viscous drag of the particles surrounded by a film of water.

Difference between the strength of soil in tension and in compression

To take into account the frictional resistance of a soil is the equivalent of taking into consideration the difference between the strength the soil exhibits in tension and in compression.

In effect, the parameters of the equation of limiting state according to Mohr-Coulomb – cohesion and the angle of internal friction – are the parameters of a tangent to the circles representing the stresses at failure in compression and tension; the slope of tangent, which determines the angle of internal friction, indicates the difference existing between the strength of soil in compression and in tension.

It will be shown that the same holds for the sublimiting state. A soil compression under a load is smaller than the soil extension due to the same load acting in the opposite direction, and the modulus of compression will be, consequently, greater than the modulus of elongation. This point is often neglected, the modulus of compression alone being introduced into the calculations.

To give due consideration to the difference between soil compression and elongation, not only in the limiting state but in the sublimiting as well (or, which is the same, to take account of the internal frictional resistance), is equivalent to introducing into the equation of state an expression defining the shear strain as a function of all-around pressure and another expression giving the volumetric strain as a function of shear strain. In other words, the equation of state must allow for the interrelation of the invariants of stress and strain tensors. These peculiarities of soil deformation must also be represented in the generalized soil model referred to above.

1.2 BASIC CONCEPTS, RHEOLOGY DEFINED

Rheological processes

The rheological properties of a body manifest themselves in the form of creep, relaxation, and deterioration of strength due to long-term load application.

The creep of soil is understood to be a process of deformation progressing in time even under a constant load. The ability to deform with time is inherent in many substances, from colloid systems and polymers to metals and from suspensions to rocks. Although all real bodies are in principle apt to creep, this property can be observed only if, in addition to a certain load and temperature, an adequate interval of time is available. The period required to perceive the flow of a liquid is a few seconds or minutes, whereas in the case of ice it amounts to several days or months. Soils and metals can flow under a high temperature, the associated displacement becoming perceptable within hours or months. The flow of glass can be perceived not earlier than in the course of centuries, and the rocks in the earth's crust flow at a speed measurable in terms of geological time.

Thus, it seems to be quite natural that the science which studies the flow of substance is called rheology, this term stressing the truth of Heraclitus' illustrious expression "παντα ϱει" (everything flows).

A convincing proof of the flow of solids is provided by the disturbed outline of the walls of ancient Mexican temples repeating in our days the undulated contour of the terrain. Some time in the past the ground was even and the skyline of Aztecan temples straight, but the topographical changes taking place century after century have bent the walls without any failure.

Rheological phenomena in soils and rocks can be observed everywhere. Solifluction, mud flows, landslides, glacier movement, etc., are all rheological processes of a duration ranging between a few hours or days and a century or more. They cause various tectonic disturbances, such as the folding and bending of strata. Sometimes these disturbances result from a slow flow of the rock yielding to gravitational pressure in the course of an extremely long period measured by the millennium.

More frequently, however, rheological processes are triggered by the interaction of soil or rock with engineering structures and manifest themselves within periods of time shorter than the structure's life.

Many cases are known where the creep of clay soil below foundations has led to long-term and differential settlements of structures, tilting of retaining walls, instability of slopes, etc. Landslides, incurring considerable losses, are a frequent occurrence also caused by soil creep.

The phenomenon of creep and the associated stress redistribution is often observed in underground structures and mine workings drifted in rock. Rock pressure manifestation, subsidence of ground at tunnel construction sites, deterioration of roof strength and rock displacement in mines are all evidence of rock creep. Explicit forms of creep are observed in permafrost and in ice-soil retaining structures built by the artificial freezing technique in order to sink or drift workings in loose soils.

The above enumeration, although far from being a comprehensive one, provides ample evidence that none of the basic problems facing soil and rock mechanics can be tackled unless the rheological properties of soils are given due consideration. The requirement to be fulfilled in analysing the limiting state with allowance for these properties is as follows: the analysis in terms of deformation should be aimed at estimating the load which induces a deformation not exceeding an allowable limit during a given interval of time (say, the useful life of the structure).

Whenever a non-attenuating creep is present, impairing the resistance of soil, the analysis of the limiting state in terms of the bearing capacity can be reduced to ascertaining the load causing stresses in the soil which reach the long-term ultimate strength at a given moment.

Objective of rheology

In the narrow sense of the word, rheology is the science studying the flow of various substances. However, in recent years this term has acquired a broader meaning.

According to the classical theories of elasticity and plasticity, the state of stress and strain of a body is explicitly defined by the magnitude of the applied load and the manner of its application. If the load remains unchanged, the resulting stresses and strains remain also unchanged. In real bodies, the stress–strain behaviour changes with time and is dependent upon the history of the preceding load. As a result, the stress–strain relation is many-valued rather than single-valued, for even if one of the values – be it stress or strain – is constant the other one is changing with time. Investigations of the mechanism of the state of stress and strain and of the ways this state is changing with time is the objective of rheology.

In examining simple idealized bodies, the classical theory of elasticity and plasticity postulates that the patterns of deformation due to combined and simple stress are identical. However, as far as real bodies are concerned, the relation between stress, strain and rate of strain is non-linear and, moreover, dependent upon the arrangement of stresses and loading conditions. The study of these problems, which are outside the scope of elasticity and plasticity theory, is also rheology's concern. In other words, rheology must give the answer to the question what stresses and strains will come about at a given point depending on any kind of relation between their components and time.

Macro- and microrheology

The study of those processes in real bodies which are externally perceivable, i.e. can be observed with the aid of conventional measuring means (e.g. the increase in