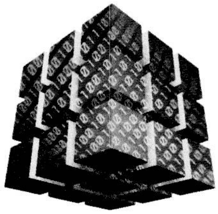
Introduction to

Probability and Statistics for Engineers and Scientists

Fourth Edition

SHELDON M. ROSS



INTRODUCTION TO PROBABILITY AND STATISTICS FOR ENGINEERS AND SCIENTISTS

▪ Fourth Edition ▪

Sheldon M. Ross
Department of Industrial Engineering and Operations Research
University of California, Berkeley

ELSEVIER

AMSTERDAM • BOSTON • HEIDELBERG • LONDON
NEW YORK • OXFORD • PARIS • SAN DIEGO
SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

AP

Academic Press is an imprint of Elsevier

Elsevier Academic Press
30 Corporate Drive, Suite 400, Burlington, MA 01803, USA
525 B Street, Suite 1900, San Diego, California 92101-4495, USA
84 Theobald's Road, London WC1X 8RR, UK

This book is printed on acid-free paper. ∞

Library of Congress Cataloging-in-Publication Data
Application submitted

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN 13: 978-0-12-370483-2

Typesetted by: diacriTech, India.

Printed in Canada
09 10 9 8 7 6 5 4 3 2 1

INTRODUCTION TO PROBABILITY AND STATISTICS FOR ENGINEERS AND SCIENTISTS

Fourth Edition

LIMITED WARRANTY AND DISCLAIMER OF LIABILITY

Academic Press, ("AP") and anyone else who has been involved in the creation or production of the accompanying code ("the product") cannot and do not warrant the performance or results that may be obtained by using the product. The product is sold "as is" without warranty of merchantability or fitness for any particular purpose. AP warrants only that the magnetic diskette(s) on which the code is recorded is free from defects in material and faulty workmanship under the normal use and service for a period of ninety (90) days from the date the product is delivered. The purchaser's sole and exclusive remedy in the event of a defect is expressly limited to either replacement of the diskette(s) or refund of the purchase price, at AP's sole discretion.

In no event, whether as a result of breach of contract, warranty, or tort (including negligence), will AP or anyone who has been involved in the creation or production of the product be liable to purchaser for any damages, including any lost profits, lost savings, or other incidental or consequential damages arising out of the use or inability to use the product or any modifications thereof, or due to the contents of the code, even if AP has been advised on the possibility of such damages, or for any claim by any other party.

Any request for replacement of a defective diskette must be postage prepaid and must be accompanied by the original defective diskette, your mailing address and telephone number, and proof of date of purchase and purchase price. Send such requests, stating the nature of the problem, to Academic Press Customer Service, 6277 Sea Harbor Drive, Orlando, FL 32887, 1-800-321-5068. AP shall have no obligation to refund the purchase price or to replace a diskette based on claims of defects in the nature or operation of the product.

Some states do not allow limitation on how long an implied warranty lasts, nor exclusions or limitations of incidental or consequential damage, so the above limitations and exclusions may not apply to you. This warranty gives you specific legal rights, and you may also have other rights, which vary from jurisdiction to jurisdiction.

The re-export of United States original software is subject to the United States laws under the Export Administration Act of 1969 as amended. Any further sale of the product shall be in compliance with the United States Department of Commerce Administration regulations. Compliance with such regulations is your responsibility and not the responsibility of AP.

For
Elise

Preface

The fourth edition of this book continues to demonstrate how to apply probability theory to gain insight into real, everyday statistical problems and situations. As in the previous editions, carefully developed coverage of probability motivates probabilistic models of real phenomena and the statistical procedures that follow. This approach ultimately results in an intuitive understanding of statistical procedures and strategies most often used by practicing engineers and scientists.

This book has been written for an introductory course in statistics or in probability and statistics for students in engineering, computer science, mathematics, statistics, and the natural sciences. As such it assumes knowledge of elementary calculus.

ORGANIZATION AND COVERAGE

Chapter 1 presents a brief introduction to statistics, presenting its two branches of descriptive and inferential statistics, and a short history of the subject and some of the people whose early work provided a foundation for work done today.

The subject matter of descriptive statistics is then considered in **Chapter 2**. Graphs and tables that describe a data set are presented in this chapter, as are quantities that are used to summarize certain of the key properties of the data set.

To be able to draw conclusions from data, it is necessary to have an understanding of the data's origination. For instance, it is often assumed that the data constitute a "random sample" from some population. To understand exactly what this means and what its consequences are for relating properties of the sample data to properties of the entire population, it is necessary to have some understanding of probability, and that is the subject of **Chapter 3**. This chapter introduces the idea of a probability experiment, explains the concept of the probability of an event, and presents the axioms of probability.

Our study of probability is continued in **Chapter 4**, which deals with the important concepts of random variables and expectation, and in **Chapter 5**, which considers some special types of random variables that often occur in applications. Such random variables as the binomial, Poisson, hypergeometric, normal, uniform, gamma, chi-square, t, and F are presented.

In **Chapter 6**, we study the probability distribution of such sampling statistics as the sample mean and the sample variance. We show how to use a remarkable theoretical result of probability, known as the central limit theorem, to approximate the probability distribution of the sample mean. In addition, we present the joint probability distribution of the sample mean and the sample variance in the important special case in which the underlying data come from a normally distributed population.

Chapter 7 shows how to use data to estimate parameters of interest. For instance, a scientist might be interested in determining the proportion of Midwestern lakes that are afflicted by acid rain. Two types of estimators are studied. The first of these estimates the quantity of interest with a single number (for instance, it might estimate that 47 percent of Midwestern lakes suffer from acid rain), whereas the second provides an estimate in the form of an interval of values (for instance, it might estimate that between 45 and 49 percent of lakes suffer from acid rain). These latter estimators also tell us the "level of confidence" we can have in their validity. Thus, for instance, whereas we can be pretty certain that the exact percentage of afflicted lakes is not 47, it might very well be that we can be, say, 95 percent confident that the actual percentage is between 45 and 49.

Chapter 8 introduces the important topic of statistical hypothesis testing, which is concerned with using data to test the plausibility of a specified hypothesis. For instance, such a test might reject the hypothesis that fewer than 44 percent of Midwestern lakes are afflicted by acid rain. The concept of the p-value, which measures the degree of plausibility of the hypothesis after the data have been observed, is introduced. A variety of hypothesis tests concerning the parameters of both one and two normal populations are considered. Hypothesis tests concerning Bernoulli and Poisson parameters are also presented.

Chapter 9 deals with the important topic of regression. Both simple linear regression — including such subtopics as regression to the mean, residual analysis, and weighted least squares — and multiple linear regression are considered.

Chapter 10 introduces the analysis of variance. Both one-way and two-way (with and without the possibility of interaction) problems are considered.

Chapter 11 is concerned with goodness of fit tests, which can be used to test whether a proposed model is consistent with data. In it we present the classical chi-square goodness of fit test and apply it to test for independence in contingency tables. The final section of this chapter introduces the Kolmogorov–Smirnov procedure for testing whether data come from a specified continuous probability distribution.

Chapter 12 deals with nonparametric hypothesis tests, which can be used when one is unable to suppose that the underlying distribution has some specified parametric form (such as normal).

Chapter 13 considers the subject matter of quality control, a key statistical technique in manufacturing and production processes. A variety of control charts, including not only the Shewhart control charts but also more sophisticated ones based on moving averages and cumulative sums, are considered.

Chapter 14 deals with problems related to life testing. In this chapter, the exponential, rather than the normal, distribution plays the key role.

In **Chapter 15** (new to the fourth edition), we consider the statistical inference techniques of bootstrap statistical methods and permutation tests. We first show how probabilities can be obtained by simulation and then how to utilize simulation in these statistical inference approaches.

About the CD

Packaged along with the text is a PC disk that can be used to solve most of the statistical problems in the text. For instance, the disk computes the p-values for most of the hypothesis tests, including those related to the analysis of variance and to regression. It can also be used to obtain probabilities for most of the common distributions. (For those students without access to a personal computer, tables that can be used to solve all of the problems in the text are provided.)

One program on the disk illustrates the central limit theorem. It considers random variables that take on one of the values $0, 1, 2, 3, 4$, and allows the user to enter the probabilities for these values along with an integer n. The program then plots the probability mass function of the sum of n independent random variables having this distribution. By increasing n, one can "see" the mass function converge to the shape of a normal density function.

ACKNOWLEDGMENTS

We thank the following people for their helpful comments on the Fourth Edition:

- Charles F. Dunkl, University of Virginia, Charlottesville
- Gabor Szekely, Bowling Green State University
- Krzysztof M. Ostaszewski, Illinois State University
- Michael Ratliff, Northern Arizona University
- Wei-Min Huang, Lehigh University
- Youngho Lee, Howard University
- Jacques Rioux, Drake University
- Lisa Gardner, Bradley University
- Murray Lieb, New Jersey Institute of Technology
- Philip Trotter, Cornell University

Contents

* Denotes optional material.

Chapter 1

INTRODUCTION TO STATISTICS

1.1 INTRODUCTION

It has become accepted in today's world that in order to learn about something, you must first collect data. *Statistics* is the art of learning from data. It is concerned with the collection of data, its subsequent description, and its analysis, which often leads to the drawing of conclusions.

1.2 DATA COLLECTION AND DESCRIPTIVE STATISTICS

Sometimes a statistical analysis begins with a given set of data: For instance, the government regularly collects and publicizes data concerning yearly precipitation totals, earthquake occurrences, the unemployment rate, the gross domestic product, and the rate of inflation. Statistics can be used to describe, summarize, and analyze these data.

In other situations, data are not yet available; in such cases statistical theory can be used to design an appropriate experiment to generate data. The experiment chosen should depend on the use that one wants to make of the data. For instance, suppose that an instructor is interested in determining which of two different methods for teaching computer programming to beginners is most effective. To study this question, the instructor might divide the students into two groups, and use a different teaching method for each group. At the end of the class the students can be tested and the scores of the members of the different groups compared. If the data, consisting of the test scores of members of each group, are significantly higher in one of the groups, then it might seem reasonable to suppose that the teaching method used for that group is superior.

It is important to note, however, that in order to be able to draw a valid conclusion from the data, it is essential that the students were divided into groups in such a manner that neither group was more likely to have the students with greater natural aptitude for programming. For instance, the instructor should not have let the male class members be one group and the females the other. For if so, then even if the women scored significantly higher than the men, it would not be clear whether this was due to the method used to teach them, or to the fact that women may be inherently better than men at learning programming skills. The accepted way of avoiding this pitfall is to divide the class members into the two groups "at random." This term means that the division is done in such a manner that all possible choices of the members of a group are equally likely.

At the end of the experiment, the data should be described. For instance, the scores of the two groups should be presented. In addition, summary measures such as the average score of members of each of the groups should be presented. This part of statistics, concerned with the description and summarization of data, is called *descriptive statistics*.

1.3 INFERENTIAL STATISTICS AND PROBABILITY MODELS

After the preceding experiment is completed and the data are described and summarized, we hope to be able to draw a conclusion about which teaching method is superior. This part of statistics, concerned with the drawing of conclusions, is called *inferential statistics*.

To be able to draw a conclusion from the data, we must take into account the possibility of chance. For instance, suppose that the average score of members of the first group is quite a bit higher than that of the second. Can we conclude that this increase is due to the teaching method used? Or is it possible that the teaching method was not responsible for the increased scores but rather that the higher scores of the first group were just a chance occurrence? For instance, the fact that a coin comes up heads 7 times in 10 flips does not necessarily mean that the coin is more likely to come up heads than tails in future flips. Indeed, it could be a perfectly ordinary coin that, by chance, just happened to land heads 7 times out of the total of 10 flips. (On the other hand, if the coin had landed heads 47 times out of 50 flips, then we would be quite certain that it was not an ordinary coin.)

To be able to draw logical conclusions from data, we usually make some assumptions about the chances (or *probabilities*) of obtaining the different data values. The totality of these assumptions is referred to as a *probability model* for the data.

Sometimes the nature of the data suggests the form of the probability model that is assumed. For instance, suppose that an engineer wants to find out what proportion of computer chips, produced by a new method, will be defective. The engineer might select a group of these chips, with the resulting data being the number of defective chips in this group. Provided that the chips selected were "randomly" chosen, it is reasonable to suppose that each one of them is defective with probability p, where p is the unknown proportion of all the chips produced by the new method that will be defective. The resulting data can then be used to make inferences about p.

In other situations, the appropriate probability model for a given data set will not be readily apparent. However, careful description and presentation of the data sometimes enable us to infer a reasonable model, which we can then try to verify with the use of additional data.

Because the basis of statistical inference is the formulation of a probability model to describe the data, an understanding of statistical inference requires some knowledge of