Pre-Calculus

DUMIES

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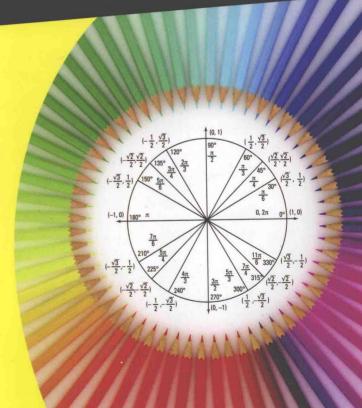
- Grasp essential pre-calculus topics
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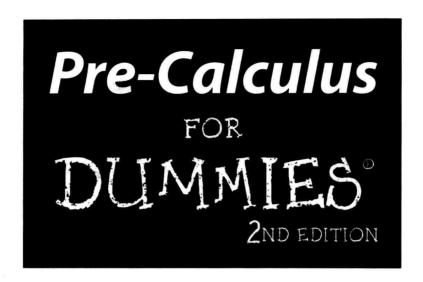
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by Yang Kuang and Elleyne Kase 计算

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About the Authors

Yang Kuang is a proud father of three, Youny, Foris, and Belany. He has been a professor of mathematics at Arizona State University since 1988. He entered the University of Science and Technology of China in 1980, at age 14, went to the University of Oxford in 1984 for M.Phil study, completed a PhD in mathematics in 1988, and became a mathematics professor at ASU at age 22.

Yang Kuang has published over 130 refereed publications and authored and edited many books. He has directed a dozen PhD dissertations and many funded research projects in mathematical biology and medicine. He has founded and continues to edit the *Journal of Mathematical Biosciences and Engineering*. He has given mathematical research talks all over the world and organized many international mathematical biology and medicine meetings. He is well known for his pioneering work in the application of delay differential equation in models of biology and medicine as well as his ongoing work in establishing a solid foundation and framework to build accurate and dynamic rich population models that explicitly include resource quality dynamics.

His more recent interest is to formulate scientifically well-grounded and computationally tractable mathematical models to describe the rich and intriguing dynamics of various within-host diseases and their treatments, including various types of cancer, diabetic diseases, and virus-induced diseases such influenza and HBV. He hopes that these models will help speed up the much-needed personalized medicine development.

Yang Kuang teaches and uses calculus all the time. He is keenly aware of the many challenges facing students and the instructors. He hopes this book will be helpful to many of those who want to confront those challenges.

Elleyne Kase is a multi-disciplined artist and graphics professional whose creative expertise has been applied commercially in many different avenues, such as fine art designs for the commercial interior design industry, Wayfinding programs for large hospital systems, 3D displays, website design, and national brand identity. A bachelor's degree in graphic design from San Jose State University began her professional career in the San Francisco Bay Area, which continued in San Diego design agencies as her family grew.

Book design, a favorite of hers, began early on in her career and has now been applied to two of her own exam prep publications, *Visual Quick Notes for Life Insurance* and *Visual Quick Notes for Life and Health*. Both use her mind mapping system, an evolution of her study on how the mind absorbs data, to explain simple terms and concepts in a visual matrix of graphic elements that improve memory retention.

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I would like to thank Elleyne Kase again for inviting me to take part in this interesting and timely book project and all the people involved in the process. It was a joy and rewarding experience working with all of you. Of course, I am forever grateful to my wife, Aijun, for her beautiful creation of and the great attention devoted to our family so that I can have time to do my work, including carrying out this time-consuming project. —Yang Kuang

Throughout my varied career I have had the great pleasure of working with professionals of many disciplines. The *For Dummies* series combines the skills of many very talented people. I want to express my gratitude to Bill Gladstone, who gave me this rare opportunity to participate; to the editors at Wiley, Tracy Boggier, Tim Gallan, and Caitie Copple, for their professionalism, understanding, and skill at bringing such a complex product to market; and to Yang Kuang, who juggled his demanding teaching schedule to accommodate this project. Above all, I want to thank my dear mother who has always supported me in all my varied interests, and to my three children, Seth, Shane, and Mimi, who have grown to become wonderful, nurturing, caring adults and my best friends. —Elleyne Kase

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Introduction

elcome to *Pre-Calculus For Dummies*, a nondiscriminatory, equalopportunity book. You're welcome to participate whether you are a genius or (like us) you need a recipe to make ice. Don't let the title throw you. If you've gotten this far in math, in no way are you a dummy! You may be reading this book for a few perfectly great reasons. Maybe you need a reference book that you can actually *understand* (we've never met a pre-calc text that we liked). Perhaps your guidance counselor told you that taking pre-calc would look good on your college application, but you couldn't care less about the subject and just want to get a good grade. Or, maybe you're contemplating buying this book and you want to check us out to see if we're a good match (not unlike looking at your blind date through the window before you walk into the restaurant). Regardless of why you opened up this book, it will help you navigate the tricky path that is pre-calc.

You may also be wondering, "When will I ever really use pre-calculus?" You're not alone. Some of our students have referred to it as pre-calc-uselessness. Well, they quickly found out how wrong they were. The concepts throughout this book are used in many real-world applications.

This book has one goal and one goal only — to teach you pre-calculus in as painless a way as possible. If you thought that you could never tackle this subject and you end up with a decent grade in this class, would you mind sending us a letter? E-mail's good too. We love to hear our students' success stories!

About This Book

This book is not necessarily meant to be read from beginning to end. It's structured in a way that you can flip to a particular chapter and get your needs met (those pesky needs we all have). Sometimes we may tell you to look in another chapter to get a more in-depth explanation, but we have tried to allow each chapter to stand on its own.

All vocabulary is mathematically correct and clear. We have taken liberties at some points throughout this book to make the language more approachable and likable. It's just more fun that way.

Pre-calc is its own special math topic. You see, some states, like California, don't have any set standards that students need to learn to officially master pre-calculus. As a result, the subject of pre-calc varies between districts, schools, and individual teachers. Because we don't know what your teacher is going to

want you to take away from this course, we've covered pretty much every concept in pre-calc. We may have covered areas that you'll never be required to tackle. That's okay. Just use this book according to your individual needs.

If you use this book only to prop open a door or as a bug smoosher, you won't get what you need from it. We suggest two alternatives:

- Look up only what you need to know when you need to know it. This book is handy for this. Use the index, the table of contents, or better yet, the quickie contents at a glance found in the very front of this book to find what you need.
- ✓ Start at the beginning and read through the book, chapter by chapter. This approach is a good way to tackle this subject because the topics sometimes build on previous ones. Even if you're a math god and you want to skim through a section that you feel you know, you may be reminded of something that you forgot. We recommend starting at the beginning and slowly working your way through the material. The more practice you have, the better.

Conventions Used in This Book

For consistency and ability to navigate easily, this book uses the following conventions:

- Math terms are italicized to indicate their introduction and to help you find their definition.
- ✓ Variables are also italicized to distinguish them from common letters.
- The step-by-step problems are always **bold** to help you identify them more easily.
- ▶ The symbol for imaginary numbers is a lowercase i.

Foolish Assumptions

We can't assume that just because we absolutely love math that you share the same enthusiasm for the subject. We can assume, however, that you opened this book for a reason: You need a refresher on the subject, need to learn it for the first time, are trying to relearn it for college, or have to help your kid understand it at home. We can also assume that you have been exposed, at least in part, to many of the concepts found in this subject because pre-calc really takes geometry and Algebra II concepts to the next level.

We also assume that you're willing to do some work. Although pre-calculus isn't the end-all to math courses out there, it's still a higher level math course. You're going to have to work a bit, but you knew that, didn't you?

We also are pretty sure that you're an adventurous soul and have chosen to take this class, because pre-calculus is not necessarily a required high school course (in most U.S. high schools, anyway). Maybe it's because you love math like we do, or you have nothing better to do with your life, again like us, or because the course will enhance your college application. Obviously, you managed to get through some pretty complex concepts in geometry and Algebra II. We can assume that if you made it this far, you'll make it even farther. We're going to help!

How This Book Is Organized

This book is broken down into four sections dealing with the most frequently taught and studied concepts in pre-calc.

Part 1: Set It Up, Solve It, Graph It

The chapters in Part I begin with a review of material you should already know from Algebra II. Then we review real numbers and how to operate with them. From there we cover functions, including polynomial, rational, exponential, and logarithmic functions and graphing them, solving them, and performing operations on them.

Part 11: The Essentials of Trigonometry

The chapters in Part II begin with a review of angles, right triangles, and trig ratios. Then we build the glorious unit circle. Graphing trig functions may or may not be a review, depending on the Algebra II course you've taken, so we show you how to graph the parent graph of the six basic trig functions and then explain how to transform those graphs to get to the more complicated ones.

This part also covers the harder formulas and identities for trig functions, breaking them down methodically so you can internalize each identity and truly understand them. We then move right along into simplifying trig expressions and solving for an unknown variable using those formulas and identities. Finally, this part covers how to solve triangles that are not right triangles using the Law of Sines and Law of Cosines.