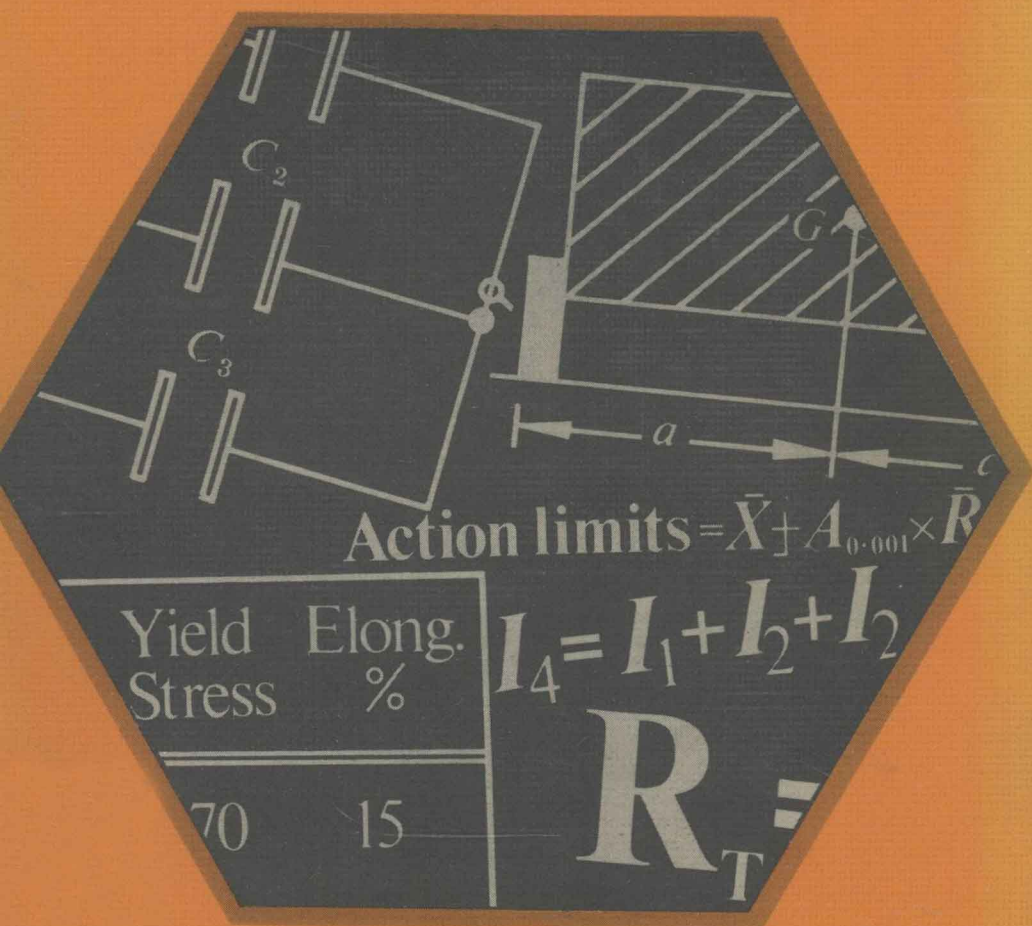


Tables, Data and Formulae for ENGINEERS

A.GREER and D.J.HANCOX



Tables, Data and Formulae for **ENGINEERS**

compiled by

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Mathematical signs and abbreviations

Symbol	Term	Symbol	Term
$\{ () \}$	brackets	$\lim y$	limit of y
$+$	plus	$\rightarrow a$	approaches a
$-$	minus	∞	infinity
$\pm$	plus or minus	Σ	sum of
$ a-b $	modulus of difference between a and b	Π	product of
$\times$ or $\cdot$	multiplied by	$\sqrt{x}, x^{\frac{1}{2}}$	square root of x
$\div$ or $/$	divided by	$x^{\frac{1}{3}}$	cube root of x
$=$	is equal to	e	base of natural logarithms
$\neq$	is not equal to	$\log_a x$	logarithm to the base a
$\equiv$	is identical with	$\ln x, \log_e x$	natural logarithm of x
$\cong$	corresponds to	$\lg x, \log_{10} x$	common logarithm of x
$\approx$	is approximately equal to	antilog	antilogarithm
$\sim$	is asymptotically equal to	$\exp x, e^x$	exponential function of x
$\propto$	varies directly as	$n!$	factorial n
$>$	is greater than	$\binom{n}{p}, {}^nC_p$	binomial coefficient
$<$	is less than	Δ, δ	increment or finite difference operator
$>$	is equal to or greater than	D	operator $\frac{d}{dx}$
$\leq$	is equal to or less than	$\int y dx$	indefinite integral
$\gg$	is much greater than	$\int_a^b y dx$	integral between the limits of a and b
$\ll$	is much less than	$\oint y dx$	around a closed contour
i, j	complex number $i = j = \sqrt{-1}$	σ	standard deviation of a distributed variate
$ z $	modulus of z	s	standard deviation for a sample
$\arg z$	argument of z	n	number in a sample
x_i	i th value of the variate x	w	range
$\bar{x}$	average of several values of x	$\therefore$	therefore
ρ	correlation coefficient	$\triangle$	triangle
r	correlation coefficient for a sample	$\cup$	union
p	probability	$\cap$	intersection
$\angle$	angle	A^T	transpose of the matrix A
$\subset$	one subset		
$\mathcal{U}$	universal set		
A^{-1}	inverse of the matrix A		
$\parallel$	parallel to		
$\perp$	perpendicular to		

Multiples and Submultiples

Multiplying factor	Prefix	Symbol	Multiplying factor	Prefix	Symbol
10^{12}	tera	T	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^6	mega	M	10^{-12}	pico	p
10^3	kilo	k	10^{-15}	femto	f
10^{-3}	milli	m	10^{-18}	atto	a

Greek letters

$A \alpha$ alpha	$E \epsilon$ epsilon	$I \iota$ iota	$N \nu$ nu	$P \rho$ rho	$\Phi \phi$ phi
$B \beta$ beta	$Z \zeta$ zeta	$K \kappa$ kappa	$\Xi \xi$ xi	$\Sigma \sigma$ sigma	χ chi
$\Gamma \gamma$ gamma	$H \eta$ eta	$\Lambda \lambda$ lambda	$O \omicron$ omicron	$T \tau$ tau	$\Psi \psi$ psi
$\Delta \delta$ delta	$\Theta \theta$ theta	$M \mu$ mu	$\Pi \pi$ pi	$Y \upsilon$ upsilon	$\Omega \omega$ omega

Standard symbols and units for physical quantities

Quantity	Symbol	Unit	Quantity	Symbol	Unit
Acceleration—gravitational	g	m/s ²	Frequency	f	Hz
Acceleration—linear	a	m/s ²	Frequency, resonant	f_r	Hz
Admittance	Y	S	Gravitational acceleration	g	m/s ²
Altitude above sea level	z	m	Gibbs function	G	J
Amount of substance	n	mol	Gibbs function, specific	g	kJ/kg
Angle—plane	$\alpha, \beta, \theta, \phi$	rad	Heat capacity, specific	c	kJ/kg K
Angle—solid	Ω, ω	steradian	Heat flow rate	ϕ	W
Angular acceleration	α	rad/s ²	Heat flux intensity	ϕ	kW/m ²
Angular velocity	ω	rad/s	Illumination	E	lx
Area	A	m ²	Impedance	Z	Ω
Area—second moment of	I	m ⁴	Inductance, self	L	H
Bulk modulus	K	N/m ² , Pa	Inductance, mutual	M	H
Capacitance	C	μ F	Internal energy	U, E	J
Capacity	V	ℓ	Internal energy, specific	u, e	kJ/kg
Coefficient of friction	μ	no unit	Inertia, moment of	I, J	kg m ²
Coefficient of linear expansion	α	/°C	Kinematic viscosity	ν	m ² /s, St
Conductance, electrical	G	S	Length	l	m
Conductance, thermal	h	kW/m ² K	Light—velocity of	c	m/s
Conductivity, electrical	σ	kS/mm	Light—wavelength of	λ	m
Conductivity, thermal	λ	W/m K	Linear expansion—coefficient of	α	/°C
Cubical expansion—coefficient of	β	/°C	Luminance	L	cd/m ²
Current, electrical	I	A	Luminous flux	ϕ	lm
Current density	J	A/mm ²	Luminous intensity	I	cd
Density	ρ	kg/m ³	Magnetic field strength	H	A/m
Density, relative	d	no unit	Magnetic flux	Φ	Wb
Dryness fraction	x	no unit	Magnetic flux density	B	T
Dynamic viscosity	η	Ns/m ² , cP	Magnetomotive force	F	A
Efficiency	η	no unit	Mass, macroscopic	m	kg
Elasticity, modulus of	E	N/m ² , Pa	Mass, microscopic	M	u
Electric field strength	E	V/m	Mass, rate of flow	V	m ³ /s
Electric flux	ϕ	C	Mass, velocity	G	kg/m ² s
Electric flux density	D	C/m ²	Modulus, bulk	K	N/m ²
Energy	W	J	Modulus of elasticity	E	N/m ²
Energy, internal	U, E	J	Modulus of rigidity	G	N/m ²
Energy, specific internal	u, e	kJ/kg	Modulus of section	Z	m ³
Enthalpy	H	J	Molar mass of gas	M	kg/k mol
Enthalpy, specific	h	kJ/kg	Molar volume	V_m	m ³ /k mol
Entropy	S	kJ/K	Moment of force	M	Nm
Expansion—coefficient of cubical	β	/°C	Moment of inertia	I, J	kg m ²
Expansion—coefficient of linear	α	/°C	Mutual inductance	M	H
Field strength, electric	E	V/m	Number of turns in a winding	N	no unit
Field strength, magnetic	H	A/m	Periodic time	T	s
Flux density, electric	D	C/m ²	Permeability, absolute	μ	μ H/m
Flux density, magnetic	B	T	Permeability, absolute of free space	μ_0	μ H/m
Flux, electric	ψ	C	Permeability, relative	μ_r	
Flux, magnetic	Φ	Wb	Permeance	Λ	H
Force	F	N	Permittivity, absolute	ϵ	pF/m
Force, resisting	R	N			

Quantity	Symbol	Unit	Quantity	Symbol	Unit
Permittivity of free space	ϵ_0	pF/m	Stress, direct	σ	N/m ² , Pa
Permittivity, relative	ϵ_r	no unit	Shear modulus of rigidity	G	N/m ² , Pa
Poisson's ratio	ν	no unit	Surface tension	γ	N/m
Polar moment of area	J	m ⁴	Susceptance	B	S
Power, apparent	S	VA	Temperature value	θ	°C
Power, active	P	W	Temperature coefficients of resistance	α, β, γ	/°C
Power, reactive	Q	VA _r	Thermodynamic temperature value	T	K
Pressure	p	N/m ² , Pa	Time	t	s
Quantity of heat	Q	J	Torque	T	Nm
Quantity of electricity	Q	Ah, C	Vapour velocity	C	m/s
Reactance	X	Ω	Velocity	v	m/s
Reluctance	S	/H, A/Wb	Velocity, angular	ω	rad/s
Relative density	d	no unit	Velocity of light	c	Mm/s
Resistance, electrical	R	Ω	Velocity of sound	a	m/s
Resisting force	R	N	Voltage	V	V
Resistance, temperature coefficients of	α, β, γ	/°C	Volume	V	m ³
Resistivity, conductors	ρ	M Ω mm	Volume, rate of flow	V	m ³ /s
Resistivity, insulators	ρ	M Ω mm	Viscosity, dynamic	η	Ns/m ² , cP
Resonant frequency	f_r	Hz	Viscosity, kinematic	ν	m ² /s, cSt
Second moment of area	I	m ⁴	Wavelength	λ	m
Self inductance	L	H	Work	W	J
Shear strain	γ	no unit	Young's modulus of elasticity	E	N/m ² , Pa
Shear stress	τ	N/m ² , Pa			
Specific gas constant	R	kJ/kg K			
Specific heat capacity	c	kJ/kg K			
Specific volume	v	m ³ /kg			
Strain, direct	ϵ	no unit			

Abbreviations for units

Unit	abb.	Unit	abb.	Unit	abb.	Unit	abb.
metre	m	steradian	sr	newton	N	mole	mol
ångström	Å	radian per second	rad/s	bar	bar	watt	W
square metre	m ²	hertz	Hz	millibar	mb	decibel	dB
cubic metre	m ³	revolution per minute	rev/min	standard atmosphere	atm	kelvin	K
litre	ℓ	kilogramme	kg	millimetre of mercury	mm Hg	centigrade	°C
second	s	gramme	g	poise	P	coulomb	C
minute	min.	tonne (= 1 Mg)	t	joule	J	ampere	A
hour	h	seimen	S	kilowatt hour	kW h	volt	V
lumen	lm	atomic mass unit	u	electron volt	eV	ohm	Ω
candela	cd	pascal	Pa	calorie	cal	farad	F
lux	lx					henry	H
day	d					weber	Wb
year	a					tesla	T
radian	rad						

Chemical symbols and atomic weights

Element	Symbol	Atomic number	Atomic weight	Element	Symbol	Atomic number	Atomic weight
Actinium	Ac	89	(227)	Molybdenum	Mo	42	95.9
Aluminium	Al	13	26.9815	Neodymium	Nd	60	144.2
Americium	Am	95	(243)	Neon	Ne	10	20.179
Antimony	Sb	51	124.7	Neptunium	Np	93	237.0482
Argon	A	18	39.948	Nickel	Ni	28	58.7
Arsenic	As	33	74.9216	Niobium	Nb	41	92.9064
Astatine	At	85	~ 210	Nitrogen	N	7	14.0067
Barium	Ba	56	137.3	Nobelium	No	102	(254)
Berkelium	Bk	97	(247)	Osmium	Os	76	190.2
Beryllium	Be	4	9.01218	Oxygen	O	8	15.999
Bismuth	Bi	83	208.9806	Palladium	Pd	46	106.4
Boron	B	5	10.81	Phosphorus	P	15	30.9738
Bromine	Br	35	79.904	Platinum	Pt	78	195.0
Cadmium	Cd	48	112.40	Plutonium	Pu	94	(244)
Californium	Cf	98	(251)	Potassium	K	19	29.102
Calcium	Ca	20	40.08	Praseodymium	Pr	59	140.907
Carbon	C	6	12.011	Protoactinium	Pa	91	231.0359
Cerium	Ce	58	140.12	Polonium	Po	84	(210)
Cesium	Cs	55	132.9055	Promethium	Pm	61	(145)
Chlorine	Cl	17	35.453	Radium	Ra	88	226.0254
Chromium	Cr	24	51.996	Radon	Rn	86	(~ 222)
Cobalt	Co	27	58.9332	Rhenium	Re	75	186.2
Copper	Cu	29	63.546	Rhodium	Rh	45	102.9055
Curium	Cm	96	(247)	Rubidium	Rb	37	85.467
Dysprobium	Dy	66	162.50	Ruthenium	Ru	44	101.0
Erbium	Er	68	167.26	Samarium	Sm	62	150.4
Europium	Eu	63	151.96	Scandium	Sc	21	44.9559
Fermium	Fm	100	(257)	Selenium	Se	34	78.96
Fluorine	F	9	18.9984	Silicon	Si	14	28.086
Francium	Fr	87	223.0	Silver	Ag	47	107.868
Gadolinium	Gd	64	157.2	Sodium	Na	11	22.9898
Gallium	Ga	31	69.72	Strontium	Sr	38	87.62
Germanium	Ge	32	72.59	Sulphur	S	16	32.06
Gold	Au	79	196.9665	Tantalum	Ta	73	180.947
Hafnium	Hf	72	178.49	Technetium	Tc	43	98.9062
Helium	He	2	4.00260	Tellurium	Te	52	127.60
Holmium	Ho	67	164.9303	Terbium	Tb	65	158.9254
Hydrogen	H	1	1.0080	Thallium	Tl	81	204.37
Indium	In	49	114.82	Thorium	Th	90	232.0381
Iodine	I	53	126.9045	Thulium	Tm	69	168.9342
Iridium	Ir	77	193.2	Tin	Sn	50	118.6
Iron	Fe	26	55.84	Titanium	Ti	22	49.9
Krypton	Kr	36	83.86	Tungsten	W	74	183.8
Lanthanum	La	57	138.905	Uranium	U	92	238.029
Lawrencium	Lr	103	(257)	Vanadium	V	23	50.941
Lead	Pb	82	207.2	Xenon	Xe	54	131.30
Lithium	Li	3	6.941	Ytterbium	Yb	70	173.0
Lutetium	Lu	71	174.97	Yttrium	Y	39	88.9059
Magnesium	Mg	12	24.305	Zinc	Zn	30	65.3
Manganese	Mn	25	54.9380	Zirconium	Zr	40	91.22
Mendelevium	Md	101	(256)				
Mercury	Hg	80	200.5				

Specific heat of various substances

Substance	S.H. capacity kJ/kgK	Substance	S.H. capacity kJ/kgK	Substance	S.H. capacity kJ/kgK
Alcohol	2.604	Graphite	0.842	Quartz	0.787
Aluminium	0.896	Ice	2.110	Sand	0.816
Antimony	0.214	Iron, cast	0.544	Silica	0.800
Benzine	1.884	Iron, wrought	0.461	Silver	0.234
Brass	0.394	Kerosene	2.093	Soda	0.967
Brickwork	0.837	Lead	0.130	Steel, mild	0.486
Cadmium	0.239	Limestone	0.909	Steel, high carbon	0.490
Charcoal	0.837	Magnesia	0.930	Stone	0.837
Chalk	0.900	Marble	0.879	Sulphur	0.745
Coal	1.005	Masonry, brick	0.837	Sulphuric acid	1.382
Coke	0.850	Mercury	0.138	Tin	0.234
Copper	0.394	Naptha	1.298	Turpentine	1.976
Corundum	0.829	Nickel	0.456	Water	4.187
Ether	2.106	Oil, machine	1.675	Wood, fir	2.721
Fusel oil	2.361	Oil, olive	1.465	Wood, oak	2.387
Glass	0.812	Phosphorus	0.791	Wood, pine	1.955
Gold	0.130	Platinum	0.134	Zinc	0.398

Boiling points at atmospheric pressure

Substance	B.P. °C.	Substance	B.P. °C.	Substance	B.P. °C.
Alcohol	78	Ether	38	Brine	108
Alcohol wood	66	Linseed oil	264	Sulphuric acid	310
Ammonia	— 33	Mercury	358	Water, pure	100
Benzine	80	Napthalene	220	Water, sea	100.7
Bromine	63	Nitric acid	120		
Chloroform	60	Turpentine	157		

Loudness of sounds

Source	Intensity in decibels	Source	Intensity in decibels
Threshold of hearing	0	Loud conversation	70
Virtual silence	10	Door slamming	80
Quiet room	20	Riveting gun	90
Average home	30	Loud motor horn	100
Motor car	40	Thunder	110
Ordinary conversation	50	Aero-engine	120
Street traffic	60	Threshold of pain	130

Densities of various substances

Substance	Density kg/m ³	Substance	Density kg/m ³	Substance	Density kg/m ³
Alcohol	790	Emery	4000	Palm oil	970
Ammonia	890	Ether, sulphuric	720	Phosphorus	1800
Asbestos	2800	Fluoric acid	1500	Petroleum oil	820
Benzine	690	Gasoline	700	Phosphoric acid	1780
Borax	1750	Glass	2600	Quartz	2600
Brick, common	1800	Granite	2650	Rape oil	920
Brick, fire	2300	Gravel	1750	Salt, common	2100
Brick, hard	2000	Gypsum	2200	Sand, dry	1600
Brick, pressed	2150	Ice	900	Sand, wet	2000
Brickwork in mortar	1600	Ivory	1850	Sandstone	2300
Brickwork in cement	1800	Kerosene	800	Slate	2800
Cement	3100	Limestone	2600	Soapstone	2700
Chalk	2600	Linseed oil	920	Soil, black	2000
Charcoal	400	Marble	2700	Sulphur	2000
Coal, anthracite	1500	Masonry	2400	Sulphuric acid	1840
Coal, bituminous	1270	Mica	2800	Tar	1000
Concrete	2200	Mineral oil	920	Tile	1800
Carbolic acid	960	Mortar	1500	Turpentine	870
Carbon disulphide	1260	Muriatic acid	1200	Vinegar	1080
Cotton seed oil	930	Naptha	760	Water	1000
Earth, loose	1200	Nitric acid	1220	Water, sea	1030
Earth, rammed	1600	Olive oil	920	Whale oil	930

Densities of Metals

Metal	Density kg/m ³	Metal	Density kg/m ³	Metal	Density kg/m ³
Aluminium	2700	Lead	11370	Silver	10530
Copper	8900	Mercury	13580	Tungsten	19300
Gold	19300	Nickel	8800	Zinc	6860
Iron	7850	Platinum	21040		

Latent heat of evaporation

Liquid	kJ/kg	Liquid	kJ/kg	Liquid	kJ/kg
Alcohol, ethyl	863	Bisulphide of		Sulphur dioxide	381
Alcohol, methyl	1119	carbon	372	Turpentine	309
Ammonia	1230	Ether	379	Water	2248

Latent heat of fusion

Substance	kJ/kg	Substance	kJ/kg	Substance	kJ/kg
Aluminium	387	Paraffin	147.2	Sulphur	39.2
Bismuth	52.9	Phosphorus	21.1	Tin	59.7
Cast iron, grey	96.3	Lead	23.3	Zinc	117.8
Cast iron, white	138.2	Silver	88.2	Ice	334.9
Copper	180	Nickel	309	Magnesium	372

Abbreviations for words

Word	abb.	Word	abb.	Word	abb.	Word	abb.
absolute	abs.	crystalline	cryst.	infra-red	i.r.	relative	
alternating		decompo-	decomp.	magneto-		humidity	r.h.
current	a.c.	sition		motive		root mean	
anhydrous	anhyd.	dilute	dil.	force	m.m.f.	square	r.m.s.
aqueous	aq.	direct current	d.c.	maximum	max.	temperature	temp.
boiling point	b.p.	electromotive		melting point	m.p.	standard	
calculated	calc.	force	e.m.f.	minimum	min.	temp. and	
concentrated	conc.	equation	eqn.	potential		pressure	s.t.p.
constant	const.	equivalent	equiv.	difference	p.d.	ultra violet	u.v.
corrected	corr.	experiment(al)	expt.	recrystallised	recryst.		
critical	crit.	freezing point	f.p.				

Commonly used constants

Constant	Numerical value	Logarithm	Constant	Numerical value	Logarithm
π	3.141593	0.4972	$1/\pi$	0.318310	$\bar{1}.5029$
2π	6.283185	0.7982	$\sqrt{\pi}$	1.772454	0.2486
$\pi/4$	0.785398	$\bar{1}.8951$	e	2.71828	0.4343
π^2	9.869604	0.9943	g	9.81	0.9917

Fixed points

Boiling point of liquid oxygen	-182.97°C
Melting point of ice (secondary point)	0.00°C
Triple point of water	0.01°C
Boiling point of water	100°C
Freezing point of zinc (secondary point)	419.505°C
Boiling point of liquid sulphur	444.60°C
Freezing point of liquid antimony	630.50°C
Melting point of silver	960.80°C
Melting point of gold	1063.00°C

Atmospheric pressure = 760 mm Hg = 1013 mb 1 bar = 10^5 kPa

Diameter of the earth = 12 750 km at the equator and 12 710 km at the poles.

Average radius of the earth = 6371 km

Speed of rotation of the earth = 1670 km/h

Calculator check

Not all calculators have the same logic and the keys and stores do not always work in the same way. Therefore before starting to perform strings of calculations it pays to check that the calculator is working correctly. The following can be used to check the logic of the calculator.

$$67.84 + 91.92 + 71.85 = 231.61$$

$$66.32 - 19.85 = 46.47$$

$$88.56 - 13.84 + 24.31 = 99.03$$

$$77.3 \times 64.8 = 5009.04$$

$$91.76 \times 3.84 + 817.52 = 1169.8784$$

$$(7.85 + 3.91) \times 83.64 = 983.6064$$

$$91.3 \times 43.2 \times 68.0 = 268\,202.88$$

$$\frac{91.76}{1.85} = 49.6$$

$$\frac{81.32 \times 14.63}{76.51} = 15.549\,752$$

$$\frac{84.3}{91.2} + \frac{76.51}{3.84} = 20.848\,821$$

for calculators without a memory this may be calculated thus

$$\left(\frac{84.3 \times 3.84}{91.2} + 76.51 \right) \div 3.84 = 20.848\,821$$

$$\frac{816.1}{94.3} - \frac{36.2}{14.7} = 6.191\,7098$$

for calculators without a memory this may be calculated thus

$$\left(\frac{-36.2 \times 94.3}{14.7} + 816.1 \right) \div 94.3 = 6.191\,7098$$

$$17.62 - \frac{8.54}{3.61} = 15.254\,35$$

Mathematics

Factors

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$${}^m\sqrt{a^n} = a^{m/n}$$

$$\frac{1}{a^n} = a^{-n}$$

$$a^0 = 1$$

Logarithms

$$\text{If } N = a^x \text{ then } \log_a N = x \text{ and } N = a^{\log_a N}$$

$$\log_a N = \frac{\log_b N}{\log_b a}$$

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^n = n \log a$$

$$\log {}^n\sqrt{a} = \frac{1}{n} \log a$$

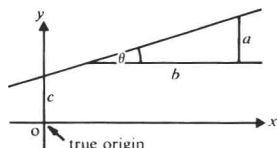
$$\log_a 1 = 0$$

$$\log_e N = 2.3026 \log_{10} N$$

Quadratic equation

$$\text{If } ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graphs



$$m = \tan \theta = \frac{a}{b}$$

The equation of a straight line can be written in the form $y = mx + c$ where m is the gradient of the line and c is the intercept on the y -axis.

Non-linear relationships can sometimes be converted into linear relationships.

The most common of these are given in the table below:

Equation	Plot	Gradient	Intercept
$y = ax^n + b$	$y \vee x^n$	a	b
$y = \frac{a}{x^n} + b$	$y \vee \frac{1}{x^n}$	a	b
$y = a\sqrt[n]{x} + b$	$y \vee \sqrt[n]{x}$	a	b
$y = ax^n + bx^{n-1}$	$\frac{y}{x^{n-1}} \vee x$	a	b
$y = ax^n$	$\log y \vee \log x$	n	$\log a$
$y = ab^x$	$\log y \vee x$	$\log b$	$\log a$
$y = ae^{bx}$	$\log y \vee x$	$b \log e$	$\log a$

Variation

If $y \propto x$ then $y = kx$. This is direct variation.

If $y \propto \frac{1}{x}$ then $y = \frac{k}{x}$. This is inverse variation.

If p varies directly as t and inversely as v then $p = \frac{kt}{v}$.

This is joint variation.

Arithmetic Progression

$$a, a+d, a+2d, a+3d, \dots, a+(n-1)d$$

$$\text{Sum of } n \text{ terms} = \frac{n}{2} [2a + (n-1)d]$$

Geometric Progression

$$a, ar, ar^2, ar^3, \dots, ar^{n-1}$$

$$\text{Sum of } n \text{ terms} = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad \frac{a(1 - r^n)}{1 - r}$$

$r > 1$ $r < 1$

$$\text{Sum of infinity when } r < 1 = \frac{a}{1 - r}$$

Binomial theorem

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!} a^{n-2}b^2 + \dots + b^n$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \dots + x^n$$

If n is a positive integer the series is finite and is true for all values of x .

If n is negative or fractional the series is infinite and is valid only if x lies between -1 and $+1$.

Series

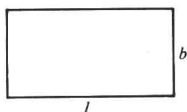
$$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

Areas of plane figures

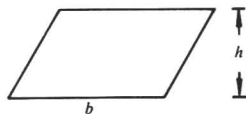
Rectangle



$$\text{Area} = lb$$

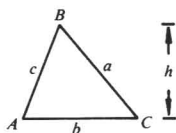
$$\text{Perimeter} = 2l + 2b$$

Parallelogram



$$\text{Area} = bh$$

Triangle

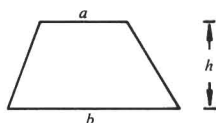


$$\text{Area} = \frac{1}{2}bh = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \frac{1}{2}ab \sin C$$

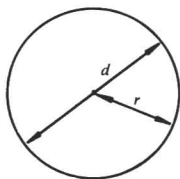
$$\text{Where } s = \frac{a+b+c}{2}$$

Trapezium



$$\text{Area} = \frac{1}{2}h(a+b)$$

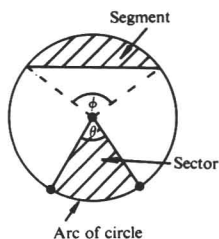
Circle



$$\text{Area} = \pi r^2 = \frac{\pi d^2}{4}$$

$$\text{Circumference} = \pi d = 2\pi r$$

Segment of a circle



$$\text{Area} = \frac{1}{2}r^2(\phi - \sin \phi)$$

(ϕ in radians)

Sector of a circle

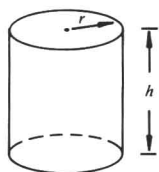
$$\text{Area} = \pi r^2 \times \frac{\theta}{360}$$

$$\text{Length of arc} = 2\pi r \times \frac{\theta}{360}$$

(θ in degrees)

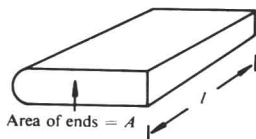
Volumes and surface areas

Cylinder



$$\begin{aligned}\text{Volume} &= \pi r^2 h \\ \text{Curved surface area} &= 2\pi r h \\ \text{Total surface area} &= 2\pi r h + 2\pi r^2 \\ &= 2\pi r(r + h)\end{aligned}$$

Any solid having
a uniform
cross-section



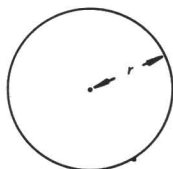
$$\begin{aligned}\text{Volume} &= Al \\ \text{Curved surface area} &= \text{perimeter of cross-section} \times \text{length} \\ \text{Total surface area} &= \text{curved surface area} + \text{area of ends}\end{aligned}$$

Cone



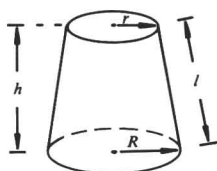
$$\begin{aligned}\text{Volume} &= \frac{1}{3}\pi r^2 h \\ \text{Curved surface area} &= \pi r l \\ \text{Total surface area} &= \pi r l + \pi r^2\end{aligned}\quad \begin{array}{l}(h = \text{vertical height}) \\ (l = \text{slant height})\end{array}$$

Sphere



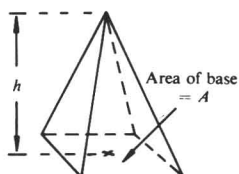
$$\begin{aligned}\text{Volume} &= \frac{4}{3}\pi r^3 \\ \text{Surface area} &= 4\pi r^2\end{aligned}$$

Frustum of a cone



$$\begin{aligned}\text{Volume} &= \frac{1}{3}\pi h(R^2 + Rr + r^2) \\ \text{Curved surface area} &= \pi(R + r)l \\ \text{Total surface area} &= \pi(R + r)l + \pi R^2 + \pi r^2\end{aligned}$$

Pyramid



$$\text{Volume} = \frac{1}{3}Ah$$

Prism

Any solid with two
faces parallel and hav-
ing a constant cross-
section. The end faces
must be triangles,
quadrilaterals or poly-
gons.

$$\text{Volume} = \text{area of cross-section} \times \text{length of prism}$$

Angles

$$1 \text{ revolution} = 360^\circ = 2\pi \text{ radians}$$

$$60' = 1^\circ$$

$$60'' = 1'$$

$$1^\circ = \frac{2\pi}{360} \text{ radians}$$

$$1 \text{ radian} = \frac{360}{2\pi} = 57.3^\circ$$

$$45^\circ = \frac{\pi}{4} \text{ radians}$$

$$90^\circ = \frac{\pi}{2} \text{ radians}$$

$$60^\circ = \frac{\pi}{3} \text{ radians}$$

$$180^\circ = \pi \text{ radians}$$

$$120^\circ = \frac{2\pi}{3} \text{ radians}$$

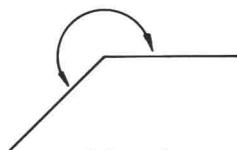
$$270^\circ = \frac{3\pi}{2} \text{ radians}$$



Acute angle
(less than 90°)



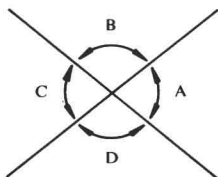
Obtuse angle
(between 90° and 180°)



Reflex angle
(greater than 180°)

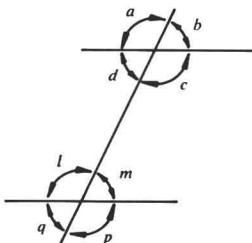
Complementary angles are angles whose sum is 90°

Supplementary angles are angles whose sum is 180°



The vertically opposite angles are equal

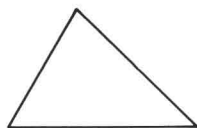
$$\angle A = \angle C \text{ and } \angle B = \angle D$$



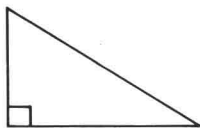
When two parallel lines are cut by a transversal

- (i) The corresponding angles are equal:
 $a = l; b = m; c = p; d = q$
- (ii) The alternate angles are equal: $d = m; c = l$
- (iii) The interior angles are supplementary:
 $d + l = 180^\circ; c + m = 180^\circ$

Triangles



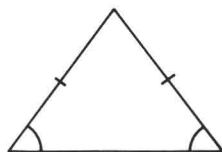
Acute-angled (all angles less than 90°)



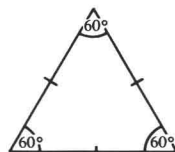
Right-angled (one angle = 90°)



Obtuse angled (one angle greater than 90°)



Isosceles triangle (two sides and two angles equal)

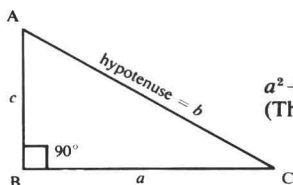


Equilateral triangle (all sides and all angles equal)

The sum of the angles of a triangle equals 180°

Pythagoras' theorem

In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.



$$a^2 + c^2 = b^2$$

(The hypotenuse lies opposite to the right-angle)

Congruency

Two triangles are congruent if they are equal in every respect. Any of the following are sufficient to prove that two triangles are congruent:

- One side and two angles in one triangle equal to one side and two similarly located angles in the second triangle.
- Two sides and the angle between them in one triangle equal to two sides and the angle between them in the second triangle.
- Three sides of one triangle equal to three sides in the second triangle.
- In right-angled triangles the hypotenuses are equal and one other side in each triangle also equal.

Similar triangles

Two triangles are similar if they are equi-angular. If in $\triangle$ s ABC and XYZ, $\angle A = \angle X$, $\angle B = \angle Y$ and $\angle C = \angle Z$ then

$$\frac{AB}{XY} = \frac{AC}{XZ} = \frac{BC}{YZ}$$

Any of the following is sufficient to prove that two triangles are similar:

- Two angles in one triangle equal to two angles in the second triangle.
- Two sides in one triangle are proportional to two sides in the second triangle and the angle between these sides in each triangle is equal.
- Three sides in one triangle are proportional to the three sides in the second triangle.